CS311 Data Structures
Lecture 11 — Red-Black Trees

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In a 1978 paper "A Dichromatic Framework for Balanced Trees", Leonidas J. Guibas and Robert Sedgewick derived red-black tree from symmetric binary B-tree. The color "red" was chosen because it was the best-looking color produced by the color laser printer...

- Wikip: Red-black tree
- TreeSet and TreeMap in the Java Collections are implemented using red-black trees.
A Binary Search Tree with 4 additional properties

1. Every node is red or black
2. The root is black
3. If a node is red, its children are black
4. Every path from root to null has the same number of black nodes

Frequently drawn/reasoned about with null colored black
A Sample RB Tree (?)

- Is this a red-black tree?
- Discounting color, is it an AVL tree?
Immediate Implications for Height Difference

Red-black properties

1. Every node is red or black
2. The root is black
3. If a node is red, its children are black
4. Every path from root to null has the same number of black nodes

Question
From root to a null in the left subtree of a red-black tree, 8 black nodes are crossed (don’t count the null at bottom)

- What is the max/min height of the left subtree?
- What is the max/min height of the right subtree?
- What is the max/min height of the whole tree?
- What is the maximum difference between left/right subtrees?
**Lemma:** The height $h$ of a red-black tree with $n$ internal nodes is no greater than $2 \log(n + 1)$.

**Proof:**

- Every root-to-leaf path in the tree has the same number of black nodes; let this number be $B$.
- So there are no leaves in this tree at depth less than $B$, which means the tree has at least as many internal nodes as a complete binary tree of height $B$.
- Therefore, $n \geq 2^{B-1}$. This implies $B \leq \log(n + 1)$.
- Because, two red nodes must not adjacent to each other, at most every other node on a root-to-leaf path is red. Therefore, $h \leq 2B$.
- Putting these together, we have $h \leq 2 \log(n + 1)$. 
Preserving Red Black Properties

Basics
- Insert data as in standard binary trees as a node initially
- If two consecutive reds result, fix it
- Gets complicated fast

Insertion Strategy: Bottom-up
- Insert the node like a regular BST node
- What should be the color of this node?
- Change the color of nodes or rotate the nodes to maintain the red-black properties
- Implement recursively
General ideas

Basics

- Insert **red** at a leaf
- If **black** parent, then done
- If **red** parent, we will have to check if uncle is red or black
- If uncle and parent are both **red**, change colors.
- If uncle is **black** and parent is **red**, single/double rotation.
- Unwind back up fixing any **red-red** occurrences
Examples

- Insert 25: node 25 is a red right-child of 20; 20 is black; done.
- Insert 3: node 3 is a red left-child of 5; 5 is red, so rotate?
Rotations

Another way of looking at this
Examples: Leaves Easy

- Insert 25 and 68: **black** parent, easy
Examples: Rotate and Recolor

- Insert 3 red
Examples: Rotate and Recolor

- Insert 3 red

- right rotation at 10, recolor 5 black 10 red

  Why not skip rotation, recolor 3 red 5 black 10 red?

  - INCORRECT: Problem with black null child of 10
Examples: Uncles Matter

Insert 82 red
  ▶ Recolor parent 80 black
  ▶ Recolor grandparent 85 red
  ▶ Recolor uncle 90 black
Problems with Red Subtree Roots

If a fix (recolor+rotation) makes a subtree root red, then we may have created two consecutive red nodes

- Insertion parent was red
- Insertion grandparent must be black
- New root is at grandparent position
- Insertion great-grandparent may be red

If this happens

- Must detect and percolate up performing additional fixes
- Can always change the root to black for a final fix
- Strategy 1 requires down to insert, up to fix via rotation/recoloring
Examples: Must Percolate Fixes Up

Insert 45 red
- Recoloring alone won’t work
- Must also rotate right 70
- Lots of recoloring also but involves trip back up the tree
Try this out

More Examples

Red-Black Tree Insertion Example
Insert the letters ALGORITHM in order into a red-black tree.
AVL Tree v Red Black Tree

**AVL**
- (+) Conceptually simpler
- (+) Stricter height bound: fast lookup
- (-) Stricter height bound: more rotations on insert/delete
- (-) Simplest implementation is recursive: down/up

**Red Black**
- (-) More details/cases
- (-) Implementation is nontrivial
- (-) Looser height bound: slower lookup
- (+) Looser height bound: faster insert/delete
- (+) Tricks can yield iterative down-only implementation
Balanced BSTs keep contents in order and provided guarantee $O(\log N)$ find/add/remove

Reproduce them in sorted order via an in-order traversal

In Java, get a `tree.iterator()` and walk it through data

Can also visit sorted subsets of data by locating a record in $O(\log N)$ time then proceeding with an in-order traversal from there.

In Java, `TreeSet<T>` provides `tailSet(T start)` to get a subset ”view” of the set