CS311 Data Structures
Lecture 11 — Red-Black Trees (Deletion)

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Red-Black Tree Review

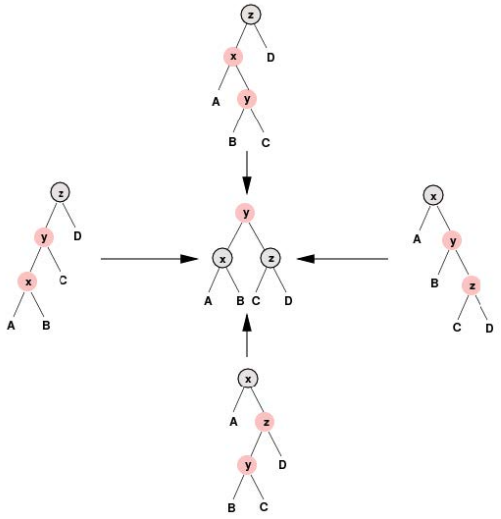
Properties

A Binary Search Tree with 4 additional properties

1. Every node is **red** or **black**
2. The root is **black**
3. If a node is **red**, its children are **black**
4. Every path from root to `null` has the same number of **black** nodes

Insertion and Invariants

1. Color swap between grandparent and (parent and uncle) does not change **black height**
2. Rotation at grandparent followed by recoloring does not change **black height**



Red-Black Tree Deletion

number of kids

Let x be the node in RB tree to be deleted

1. x has two children nodes
   - swap value with the node y with the largest value in the left subtree
   - then delete node y
   - node y must have one or zero child node
2. x has one child
3. x has zero child

color of node

Let x be the node in RB tree to be deleted

1. x is red, delete as usual
   - black depth remains the same
   - no two adjacent nodes are red
2. x is black, many hairy cases
   - black depth along the path containing x is one less
   - two adjacent nodes may be red

Case Analysis

Four possible cases

1. x is red
   - 1.1 x has one child
   - 1.2 x has no child
2. x is black
   - 2.1 x has one child
   - 2.2 x has no child

x is black

1. deleting x is going to reducing the black height of the path containing x by one
2. key idea: recoloring x's sibling s to red, and reducing the black height of the path containing s by one

- Question: is it possible that s is null?
- Question: what to do if s is already red?

Some simple practice

Questions

1. How to get your sibling?
2. How to get your uncle?

More Case Analysis

Now we reduced our analysis to: x is black and x's kid k is also black (or null). We further let s be the sibling, L and R are the left and right children of s, respectively, and let P be the parent.

Five possible cases

1. s is red
   - 1.1 What are the colors of P, L, R?
2. s is black
   - 2.1 Both kids L, and R are all black
   - 2.2 Both R and L are red
   - 2.3 R is red, L is black
   - 2.4 L is red, R is black