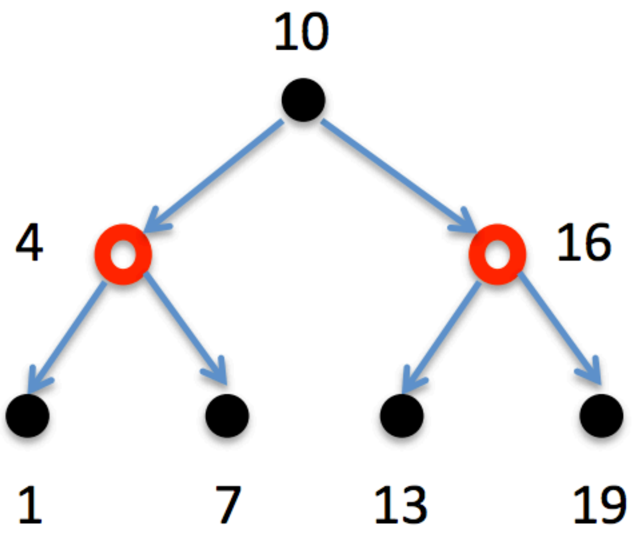
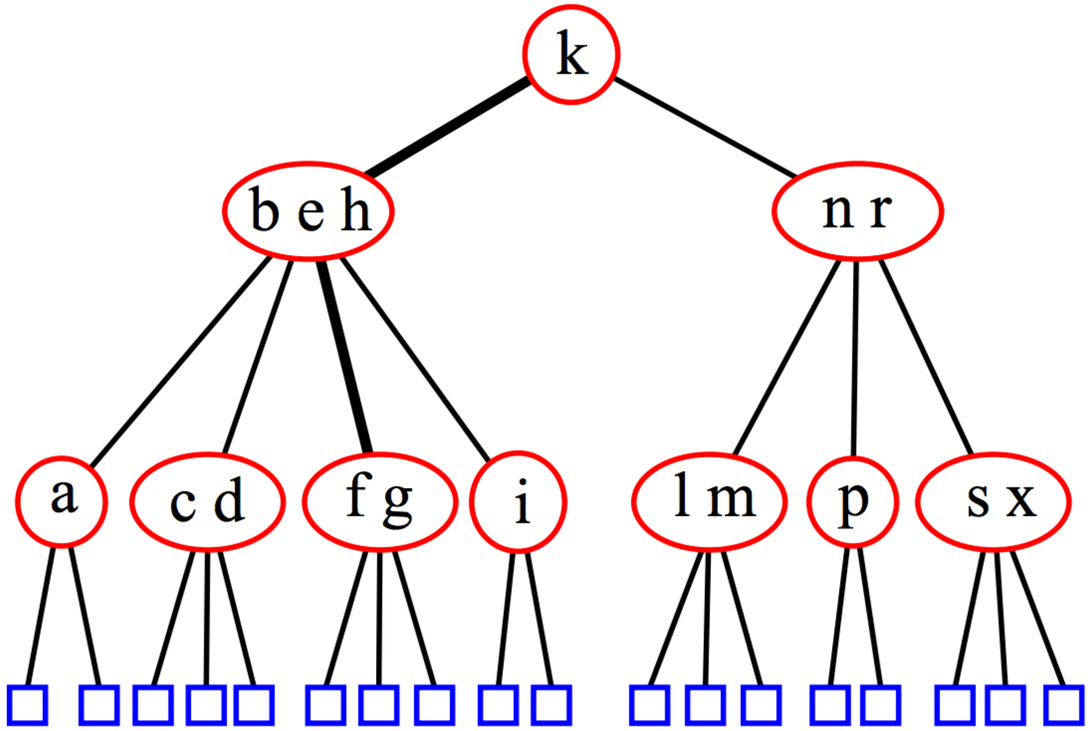
Insert 14, 15, delete root



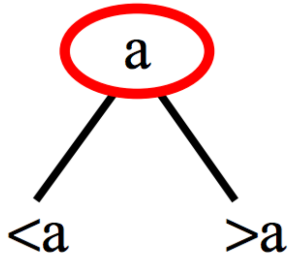
2-3-4 tree

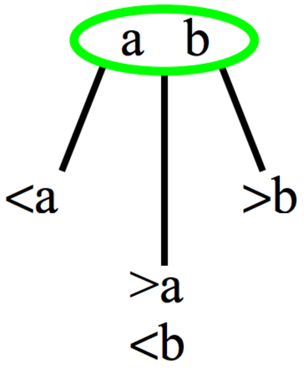
- Nodes store 1, 2, or 3 keys and have 2, 3, or 4 children, respectively
- All leaves have the same depth

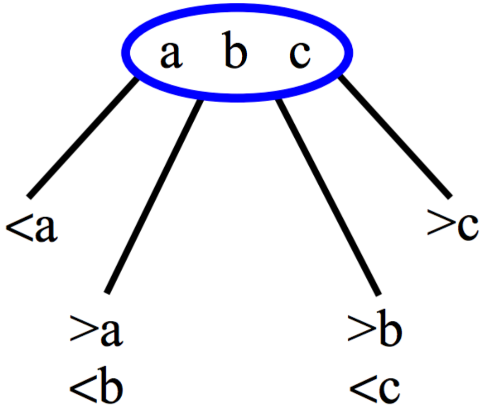


2-3-4 tree

- 2-node
  - 1 key, 2 kids
  - Same a binary tree
- 3-node
  - 2 keys and 3 kids
- 4-node
  - 3 keys and 4 kids

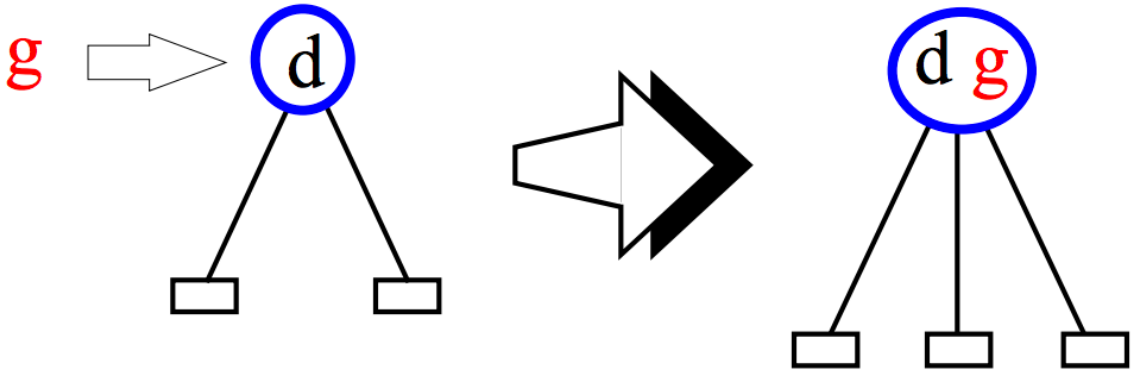




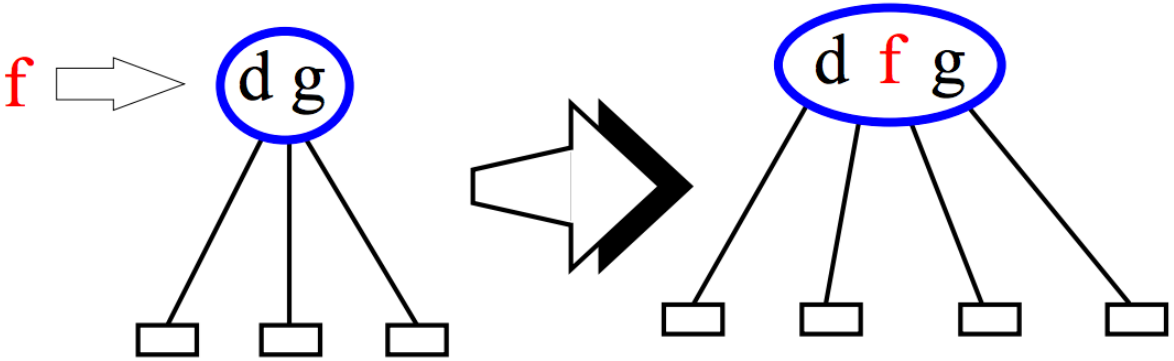


Insert to 2-3-4 tree

- Insert the **new key** at the **lowest internal node** reached in the search
  - 2-node becomes 3-node



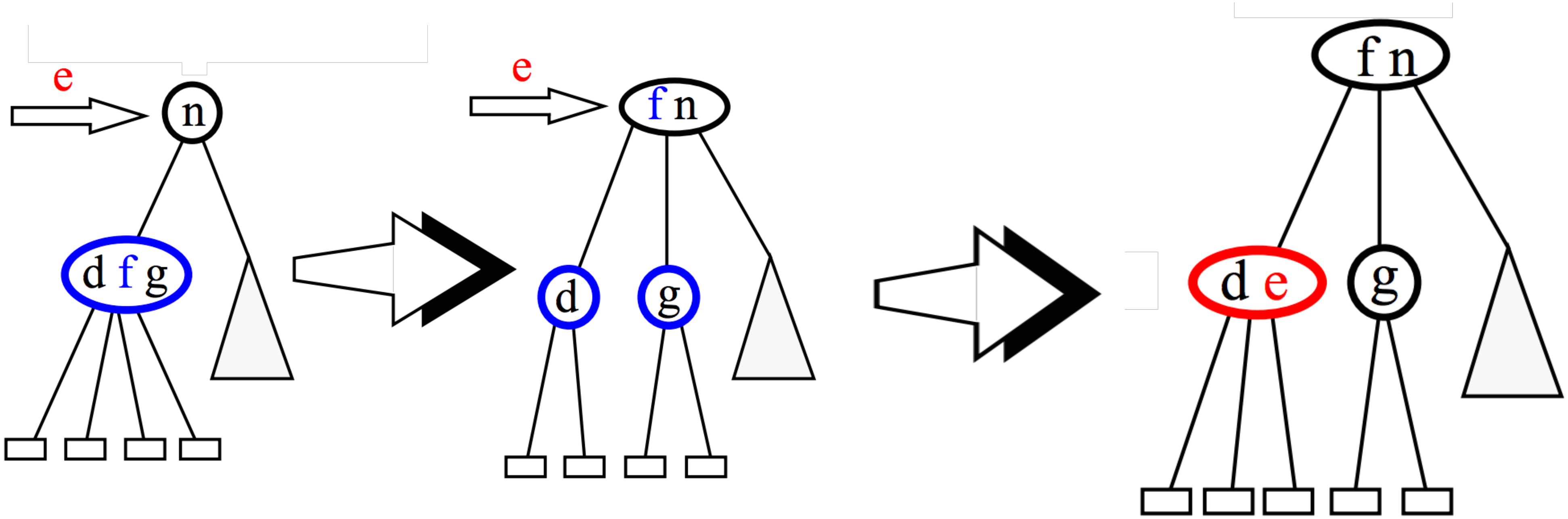
  - 3-node becomes 4-node



  - What about a 4-node?

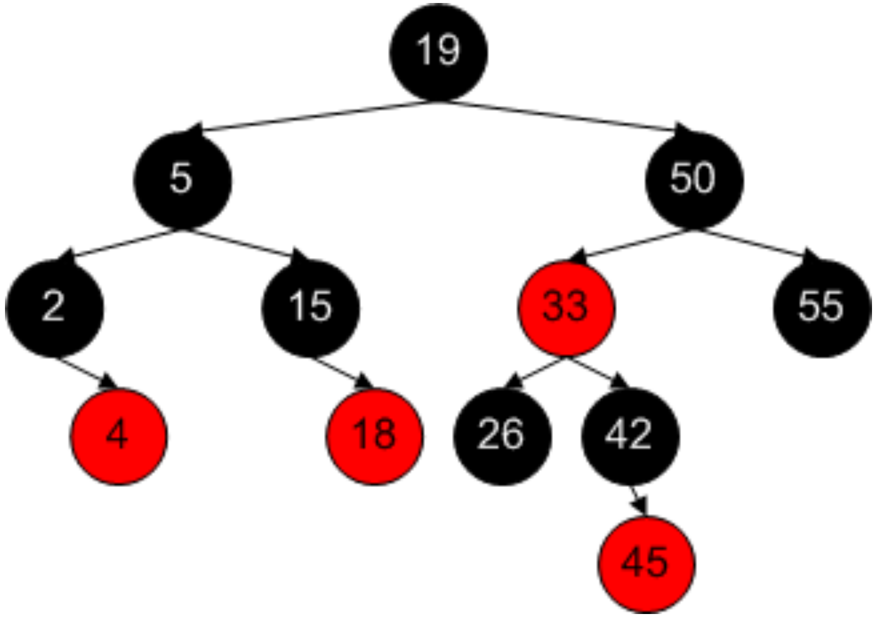
Insert to 2-3-4 tree

- In our way down the tree, whenever we reach a **4-node**, we break it up into **two 2- nodes**, and move the middle element up into the parent node



2-3-4 tree and Red-black tree

- 2-3-4 tree and red-black tree are closely related



Matrix Sum

Given an M by N matrix X, sum its elements

- M rows, N columns

- What is the difference?
- What is the complexity of each?
- Should the execution speed be different?

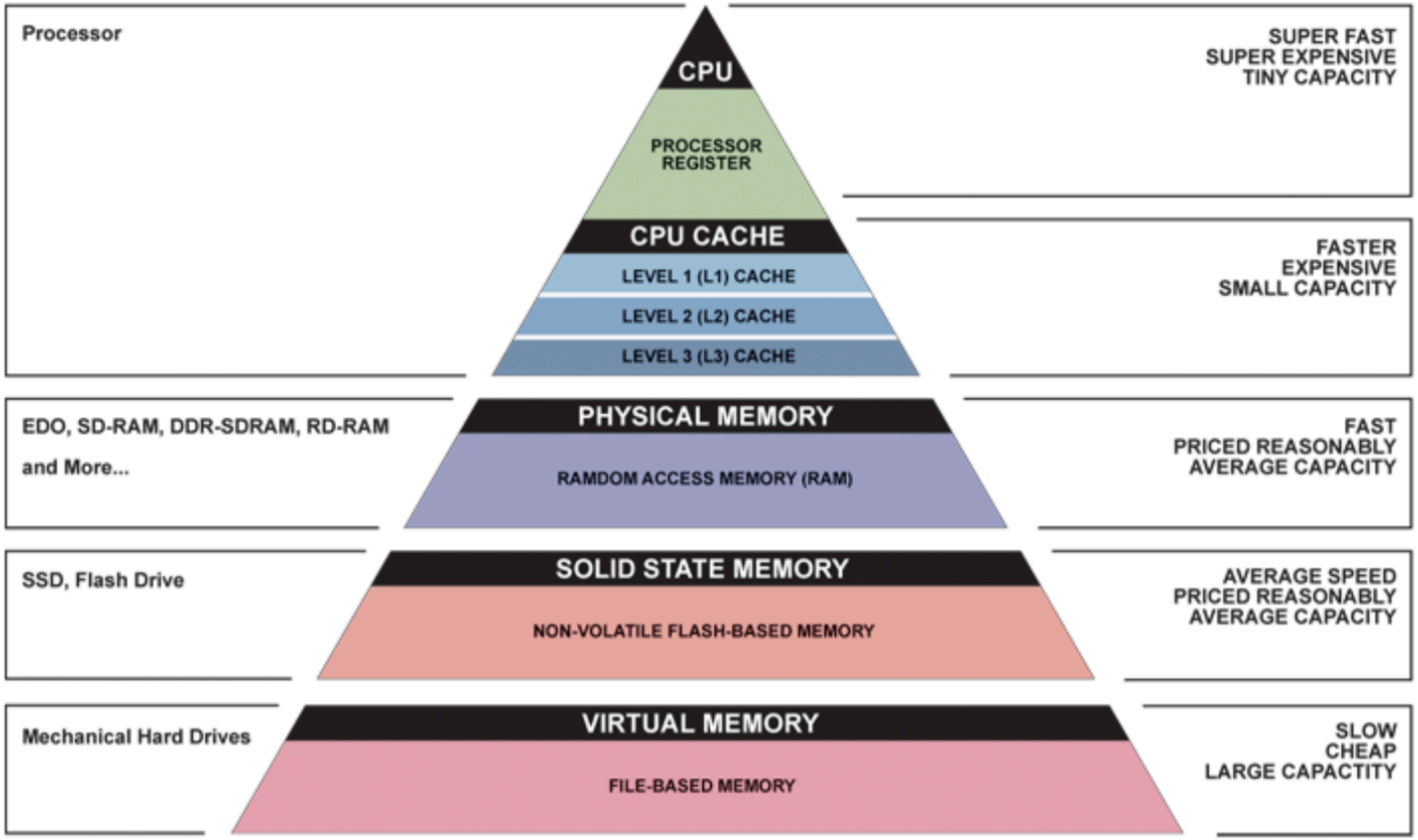
Sum R

```
given X, M, N
sum = 0
for i=0 to M-1{
  for j=0 to N-1 {
    sum += X[i][j]
  }
}
```

Sum C

```
given X, M, N
sum = 0
for j=0 to N-1{
  for i=0 to M-1 {
    sum += X[i][j]
  }
}
```

The memory pyramid



▲ Simplified Computer Memory Hierarchy
Illustration: Ryan J. Leng

Edited Excerpt of Jeff Dean's talk on data centers.

Reference	Time	Analogy
Register	-	Your brain
L1 cache reference	0.5 ns	Your desk
L2 cache reference	7 ns	Neighbor's Desk
Main memory reference	100 ns	This Room
Disk seek	10,000,000 ns	Salt Lake City

Does Big-O analysis capture these effects?

Problem

- Secondary storage devices is 1 million times (at least) slower than the primary storage and even more so than L1 and L2 catch!
    - Reading each node that stores on the disk might take 1/10 second
    - Even for a red-black tree it will take $\mathrm{O}(\log_2 n)$ reads from the disk
    - For 10 million record, the height of a red-black tree is about 25
    - That is 25 disk accesses
- Solution?

Problem

- Data (record) size might not be uniform (some can be much bigger)
  - Store index (key or address)
- Properties
  - Disk seek time is very slow, but reading data (once it is found on disk) is much faster
  - So, we should read a much as possible once a record is found
  - Usually a block of data will be read from the disk anyway, even if you just ask for a byte
  - This implies that, each node should store entire disk block (page size)
  - Each node should also store index of the record instead of the data itself
  - So, we are talking about something like 512-ary tree
- Finally, how do we balance, insert, delete data from such a tree?

B-tree

- “B” means several things: balanced, broad, Boeing, Bayer, etc.
- Motivated by
  - Limited resources in early years
  - Data that cannot fit into the main memory at once
  - Even today, we have gigabytes of memories, we still face similar problem
    - Better data acquisition techniques (many more senosrs)
    - More data capturing devices (your cellphone)
    - More people are connected
    - Mobile devices also have limited resources
    - Resources can be shared by many people (cloud computing/storage)

B-tree

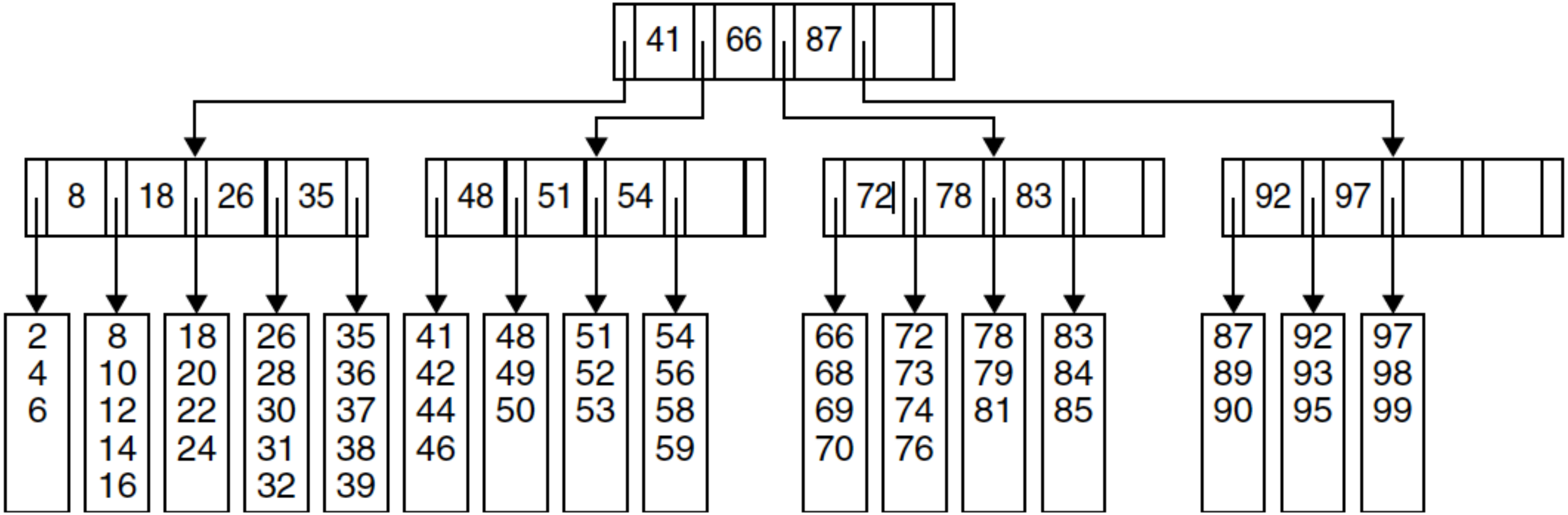
- Data items are all in the leaves
- Root is a leaf or has 2 to M children
- Non-leaf node has $M-1$ indices (keys)
  - Key i is the smallest key in $(i+1)$ −th subtree
  - Must have $\left\lceil \frac{M}{2} \right\rceil$ to M children
- All leaves are at the same level and have $\left\lceil \frac{L}{2} \right\rceil$ to L data
- Note M and L are user inputs

B-tree

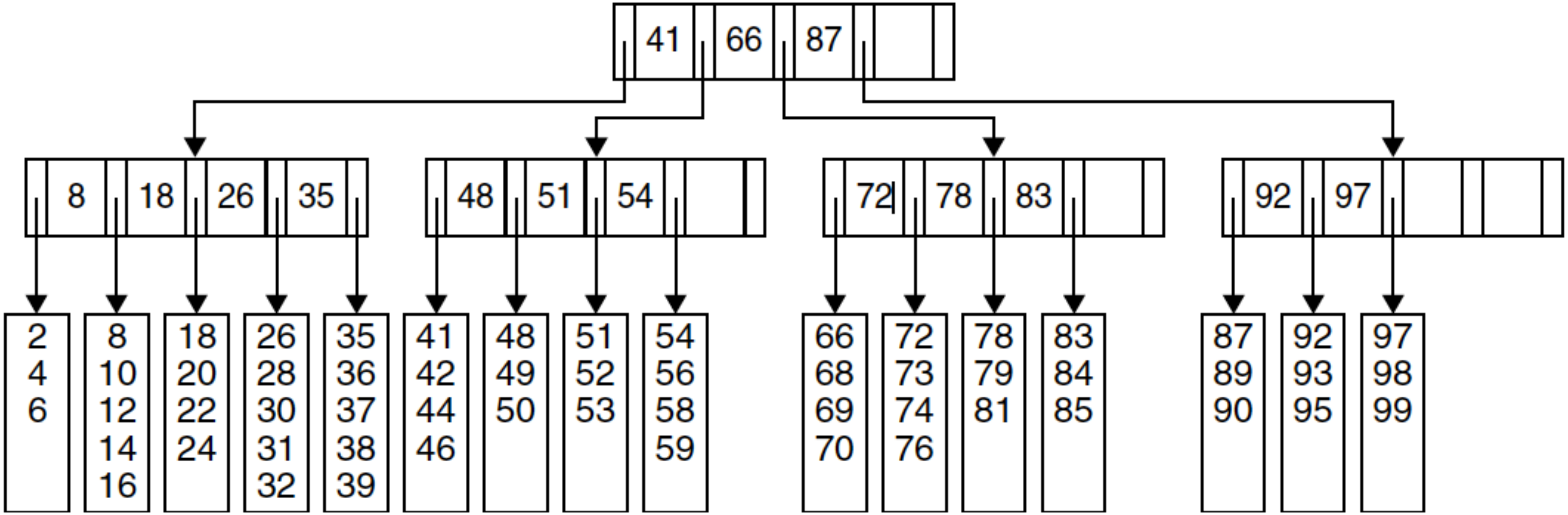
- Insert(x)
  - Find the **leaf** that x correspond to
  - Adjust the tree so it does not violate the B-tree properties
    - Split a node into two nodes
    - Promote the index (key) to a parent node

- Delete(x)
  - Find the **leaf** that x correspond to
  - Adjust the tree so it does not violate the B-tree properties

A B-tree of order 5

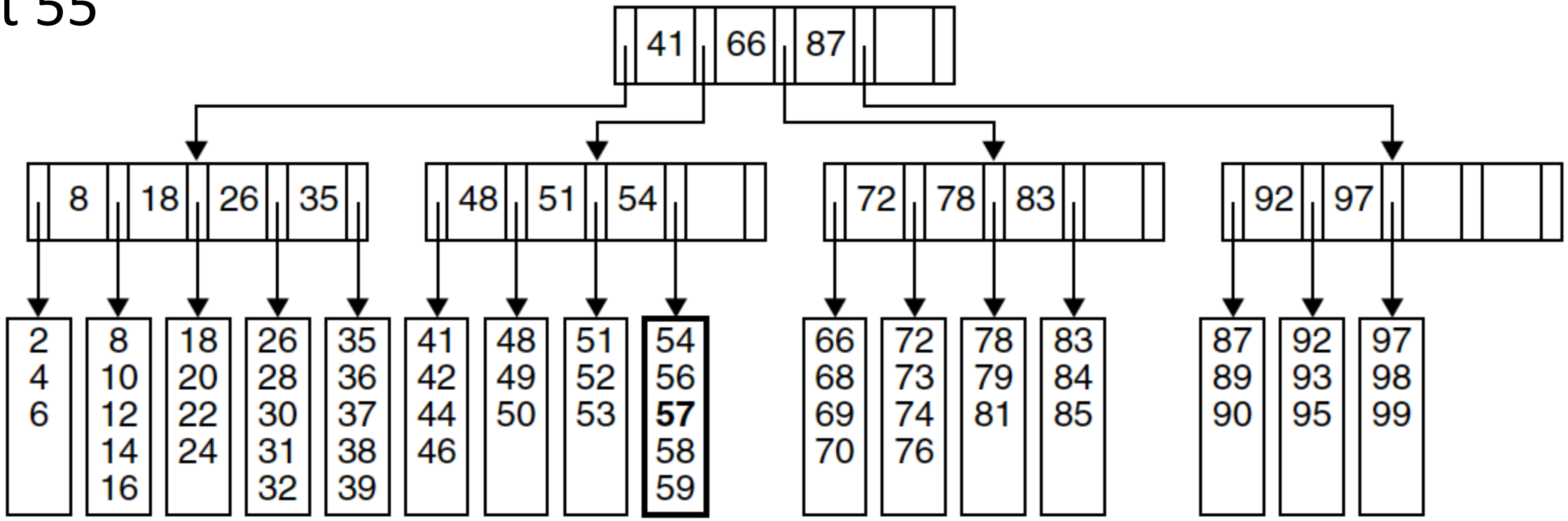


Insert 57

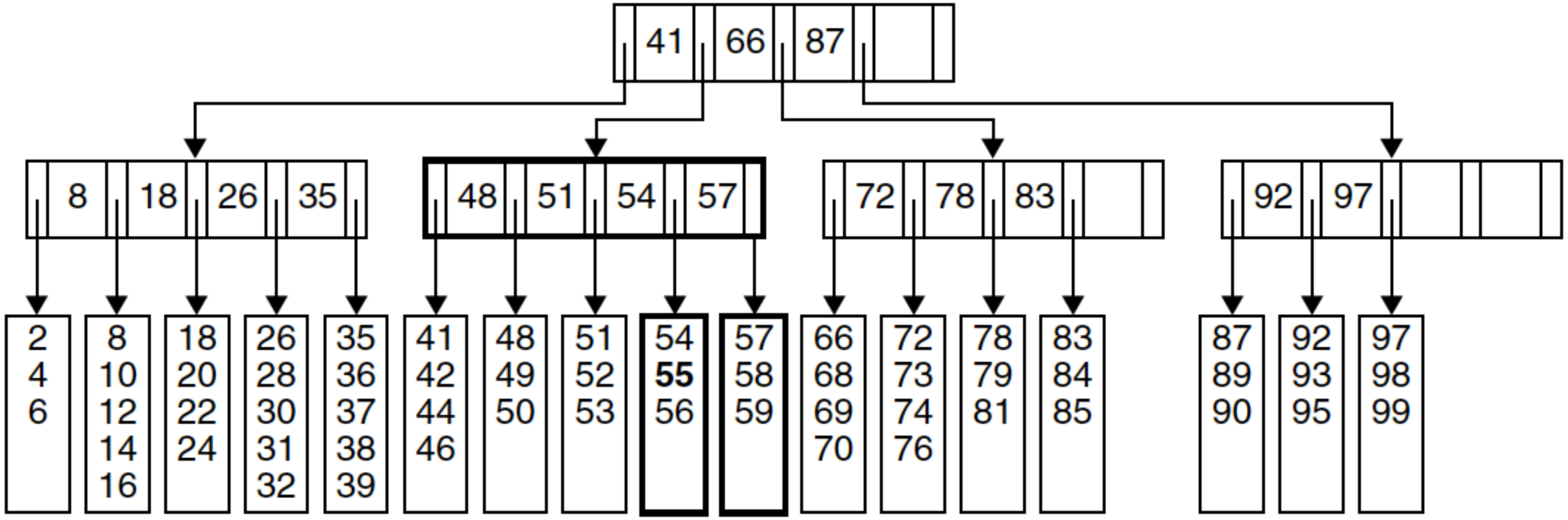


41
66
87
8
18
26
35
48
51
54
72
78
83
92
97
2 4 6
8 10 12 14 16
18 20 22 24
26 28 30 31 32
35 36 37 38 39
41 42 44 46
48 49 50
51 52 53
54 56 58 59
66 68 69 70
72 73 74 76
78 79 81
83 84 85
87 89 90
92 93 95
97 98 99

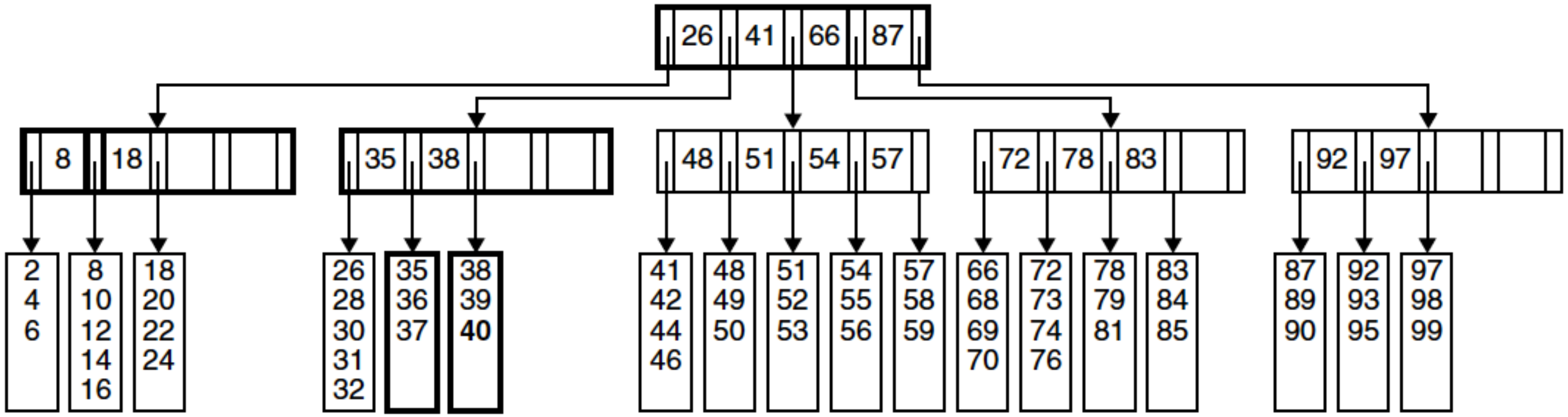
- After inserting 57
- Insert 55



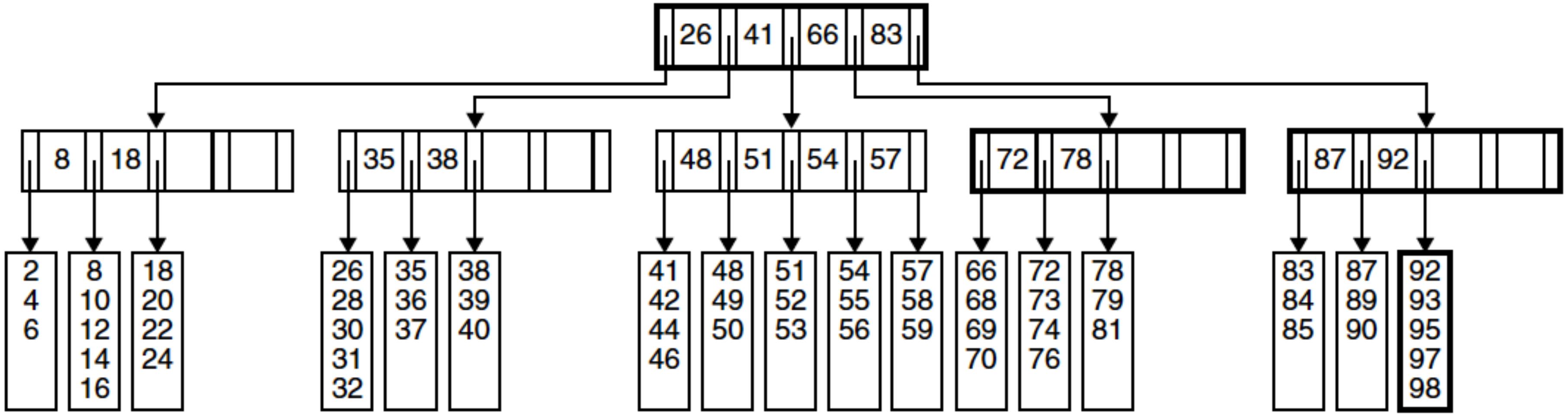
- After inserting 55
- Insert 40



- After inserting 40
- Delete 99



- After deleting 99



- ADD(x,bt)
  - find the right leaf in bt
  - if space in leaf
    - add x to leaf else
  - if parent has room
    - new leaf
    - split data
    - add x to leaf
  - else
    - recurse up
    - split internal
    - new leaves
    - split data
    - back down to add x

- REMOVE(x,bt)
  - find leaf with x
  - remove x
  - if leaf < 1/2 full
    - merge with neighbor leaf
    - steal leaves if needed
    - recurse up to adjust

B-tree Take-home

- Multi-way tree
- If order-M nodes are all ½ full $O(\log_M N)$ height
- Hybrid of array/tree
- Good for data that doesn't fit in memory
  - Large database
  - Filesystems (e.g., BFS, NTFS uses B+ trees for directory)
  - Sensitive to memory
- Simple idea, complex implementation (many cases)
- Many variants on the idea (B-tree, B+ tree, B* tree, etc.)