CS451 Deformation

free-form deformation

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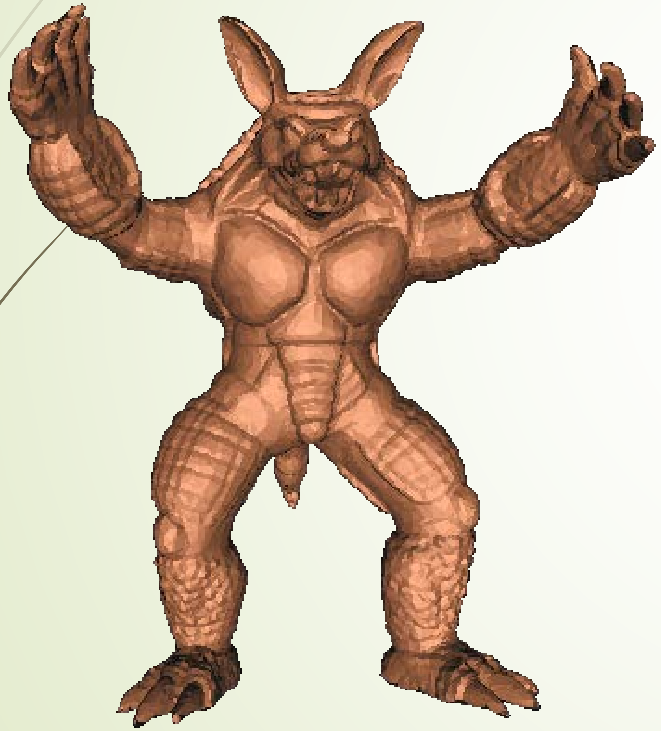
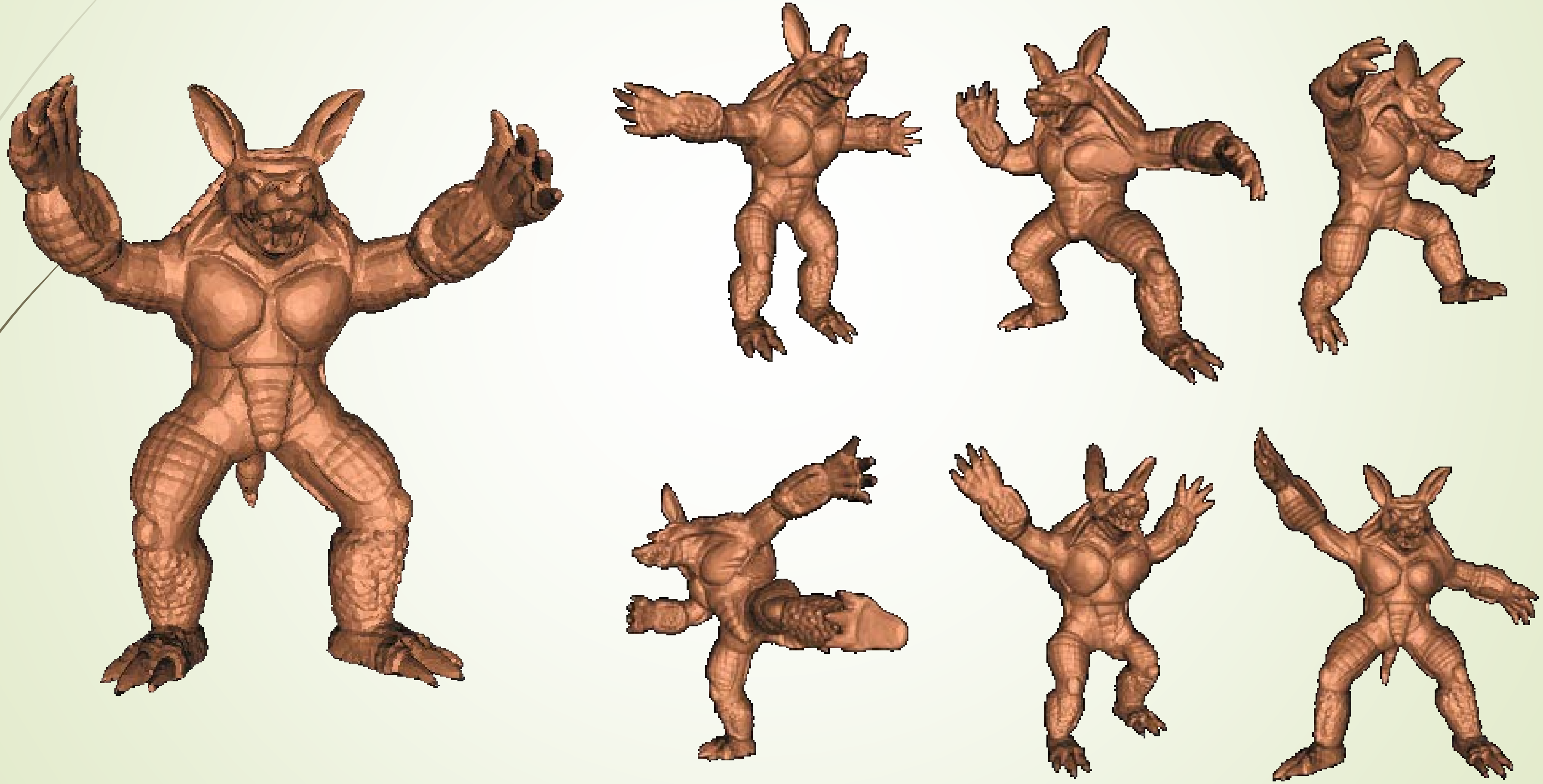
Based on Scott Schaefer's lecture note

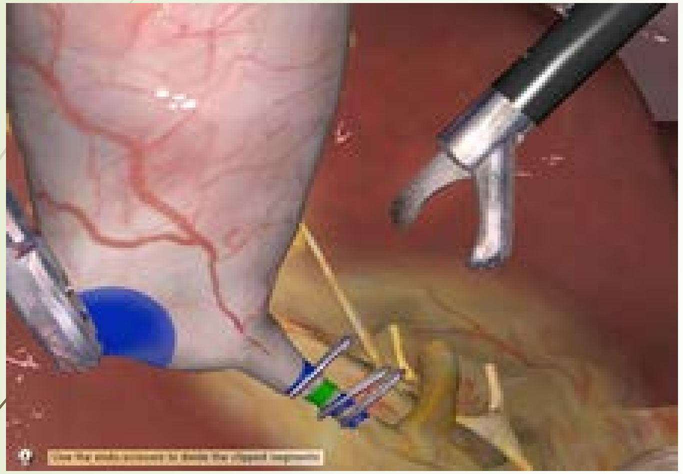
Deformation

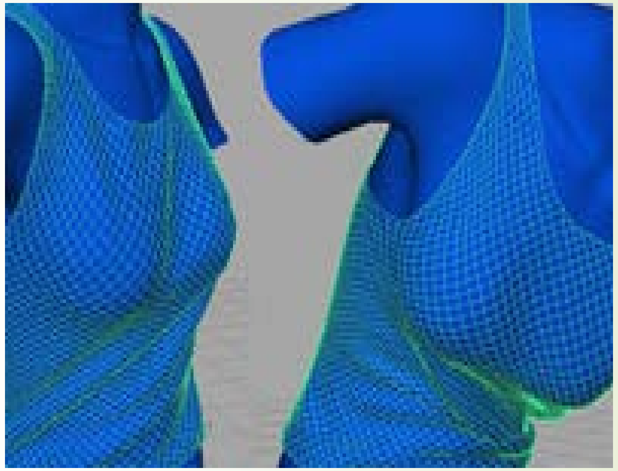


Deformation



Deformation Applications

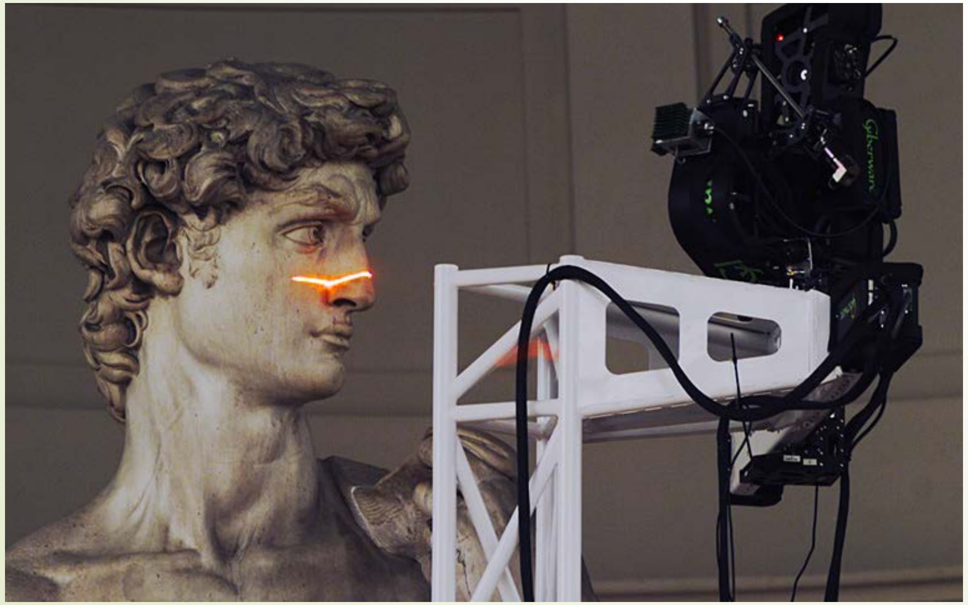




Toy Story © Disney / Pixar

Challenges in Deformation

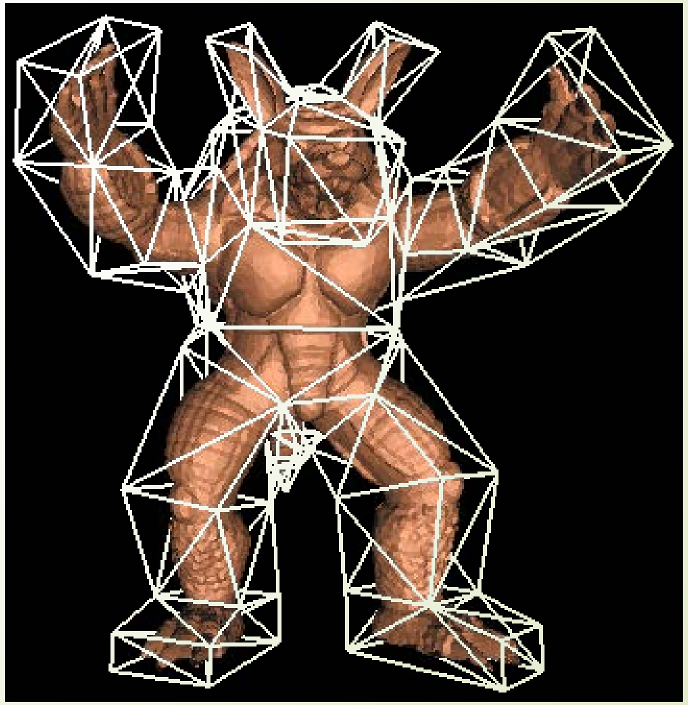
- Large meshes – millions of polygons
- Need efficient techniques for computing and specifying the deformation



Digital Michelangelo Project

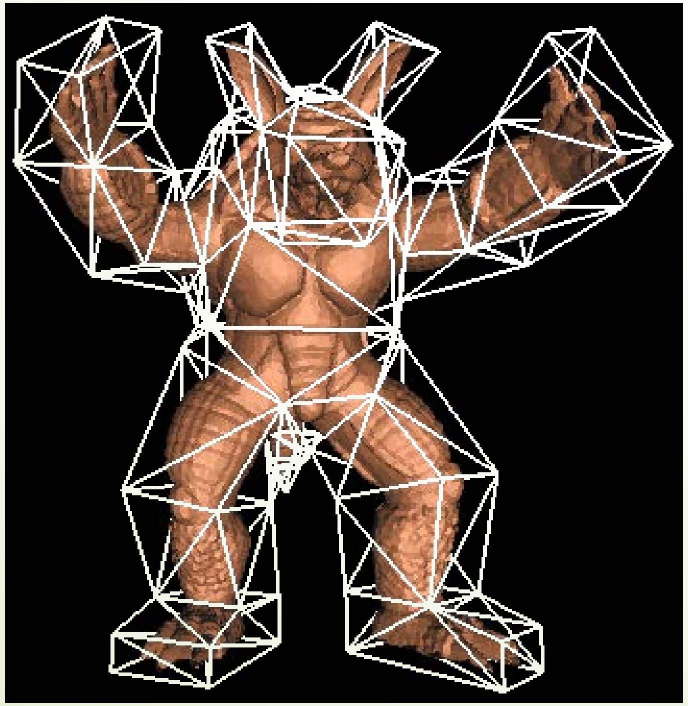
Deformation Handles

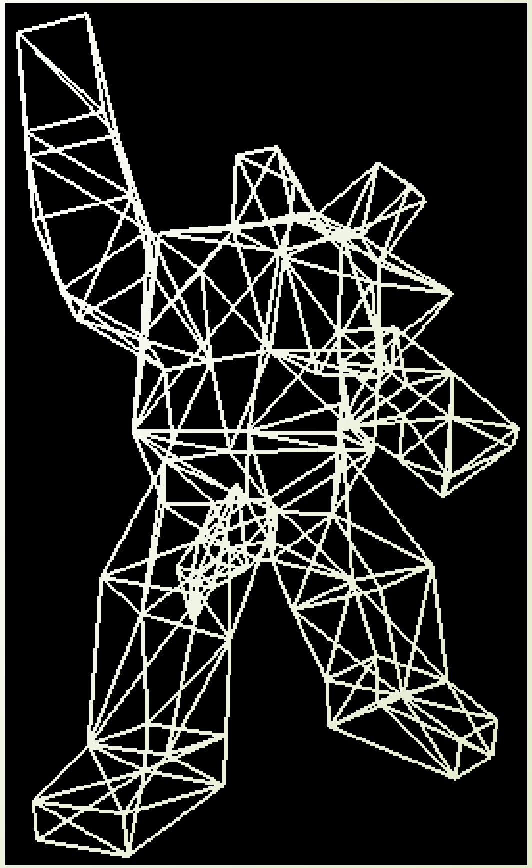
- Low-resolution auxiliary shape controls deformation of high-resolution model



Deformation Handles

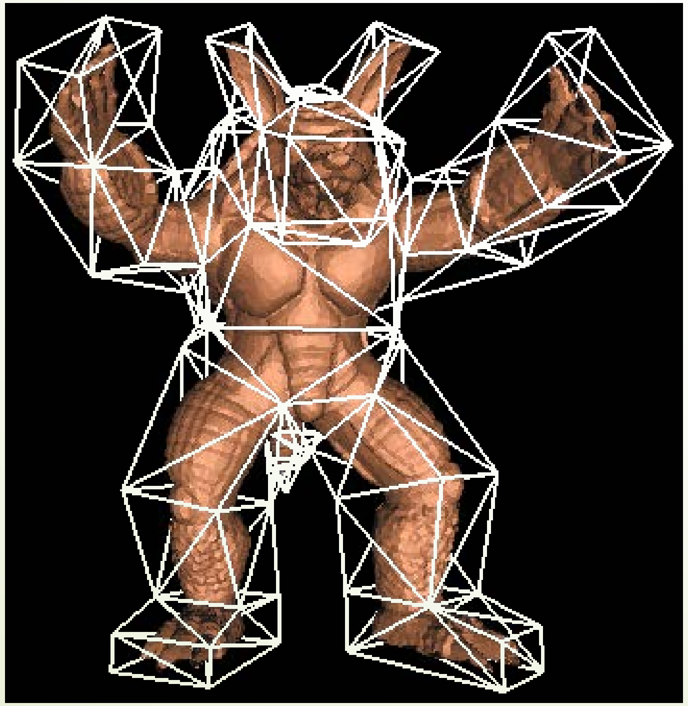
- Low-resolution auxiliary shape controls deformation of high-resolution model

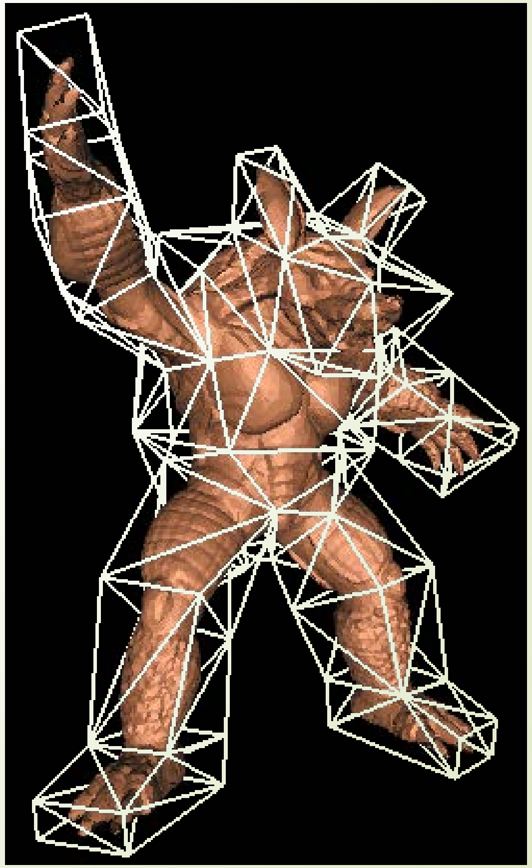




Deformation Handles

- Low-resolution auxiliary shape controls deformation of high-resolution model





Smoothness of Deformation

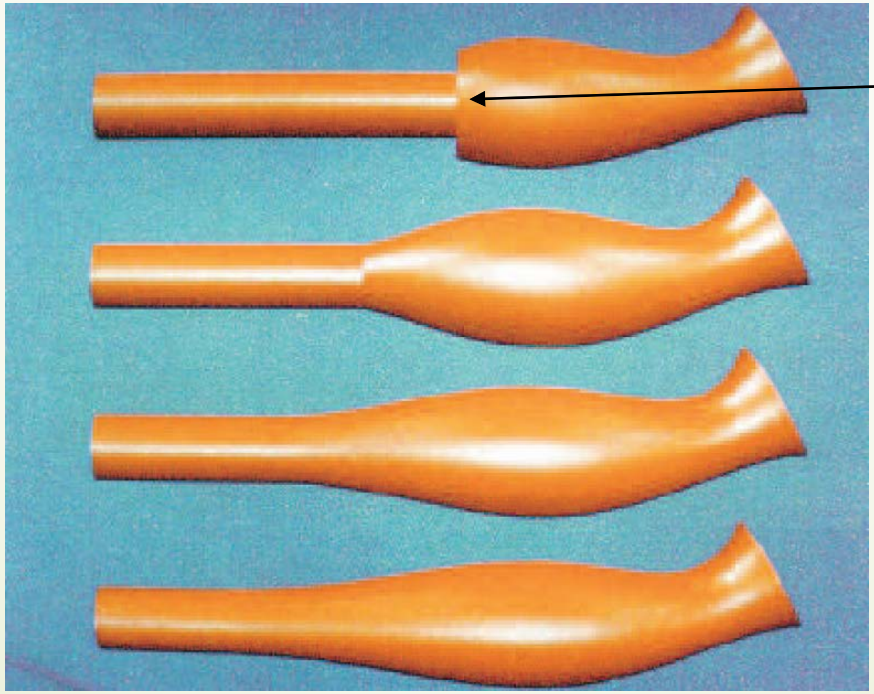
- Constraining Bezier control points controls smoothness

C^{-1}

C^{0}

C^{1}

C^{2}



disconnection

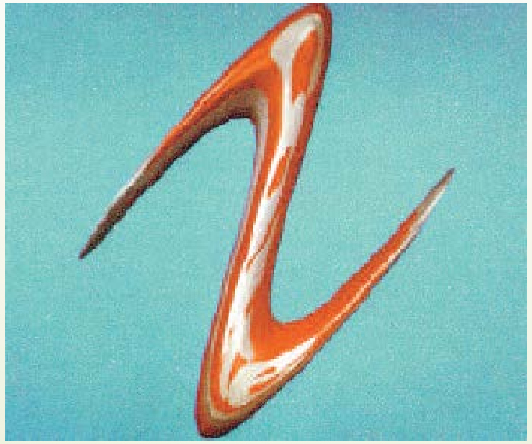
Image taken from “Free-form Deformations of Solid Geometric Models”

Volume Preservation

- Must ensure that the jacobian of the deformation is 1 everywhere

$$(\hat{x}, \hat{y}, \hat{z}) = \big(F(x,y,z), G(x,y,z), H(x,y,z)\big)$$

$$\begin{vmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} & \frac{\partial F}{\partial z} \\ \frac{\partial G}{\partial x} & \frac{\partial G}{\partial y} & \frac{\partial G}{\partial z} \\ \frac{\partial H}{\partial x} & \frac{\partial H}{\partial y} & \frac{\partial H}{\partial z} \end{vmatrix} = 1$$




Images taken from “Free-form Deformations of Solid Geometric Models”

Free-Form Deformation Contributions

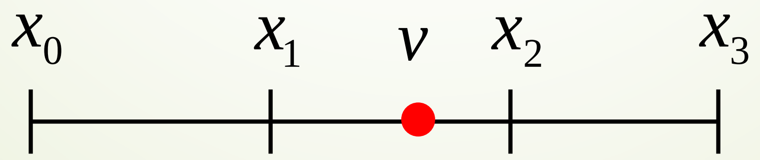
- Smooth deformations of arbitrary shapes
- Local control of deformation
- Performing deformation is fast

- Widely used
  - Game/Movie industry
  - Part of nearly every 3D modeler

Free-Form Deformations

- Embed object in uniform lattice
- Represent each point in space as a weighted combination of grid vertices

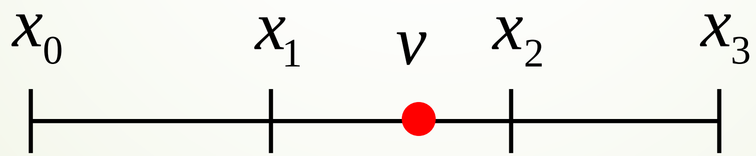
$$v = \sum_i w_i x_i$$



Free-Form Deformations

- Assume x_i are equally spaced
- Use Bernstein basis functions

$$v = \sum_i w_i x_i = \sum_i \binom{d}{i} t^i (1-t)^{d-i} x_i$$

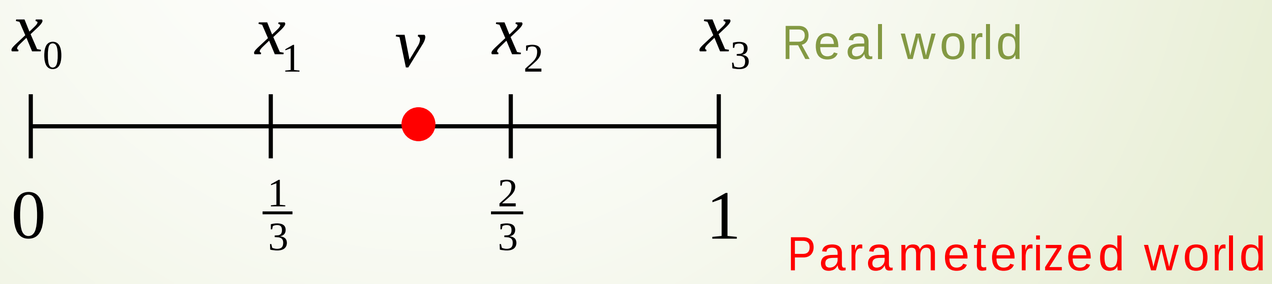


$\binom{d}{i}$ Is the binomial coefficient

Free-Form Deformations

- Assume x_i are equally spaced
- If we normalize the coordinate between 0 and 1

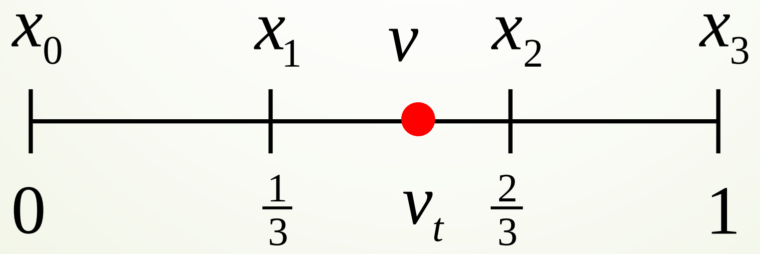
$$v = \sum_i \binom{d}{i} t^i (1-t)^{d-i} x_i = \sum_i \binom{d}{i} t^i (1-t)^{d-i} \frac{i}{d} = t$$



Free-Form Deformations

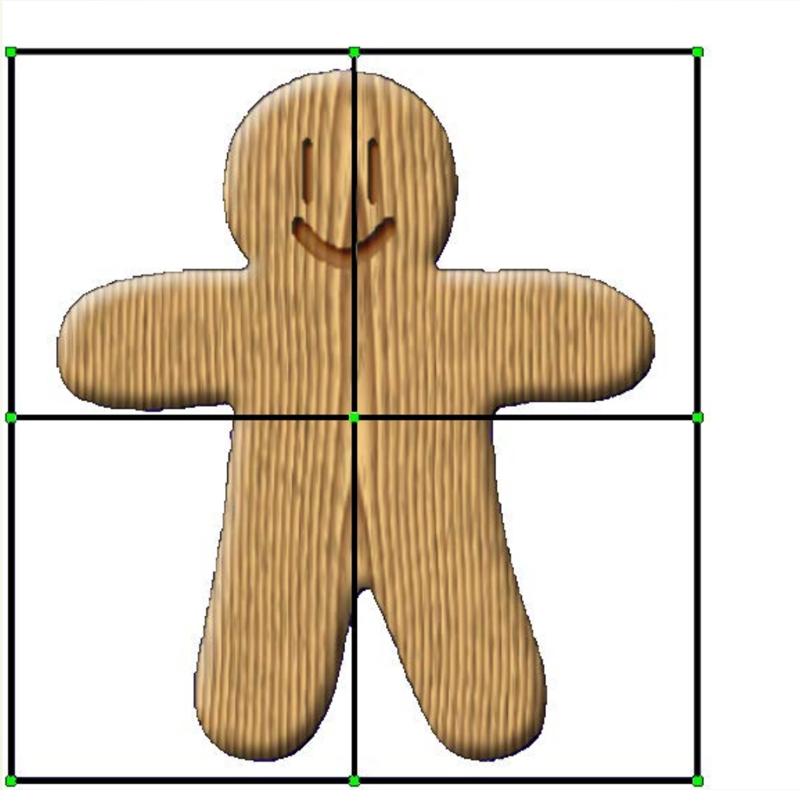
- Assume x_i are equally spaced
- If we normalize the coordinate between 0 and 1

$$w_i = \binom{d}{i}(1-v_t)^{d-i}v_t^{\,i}$$

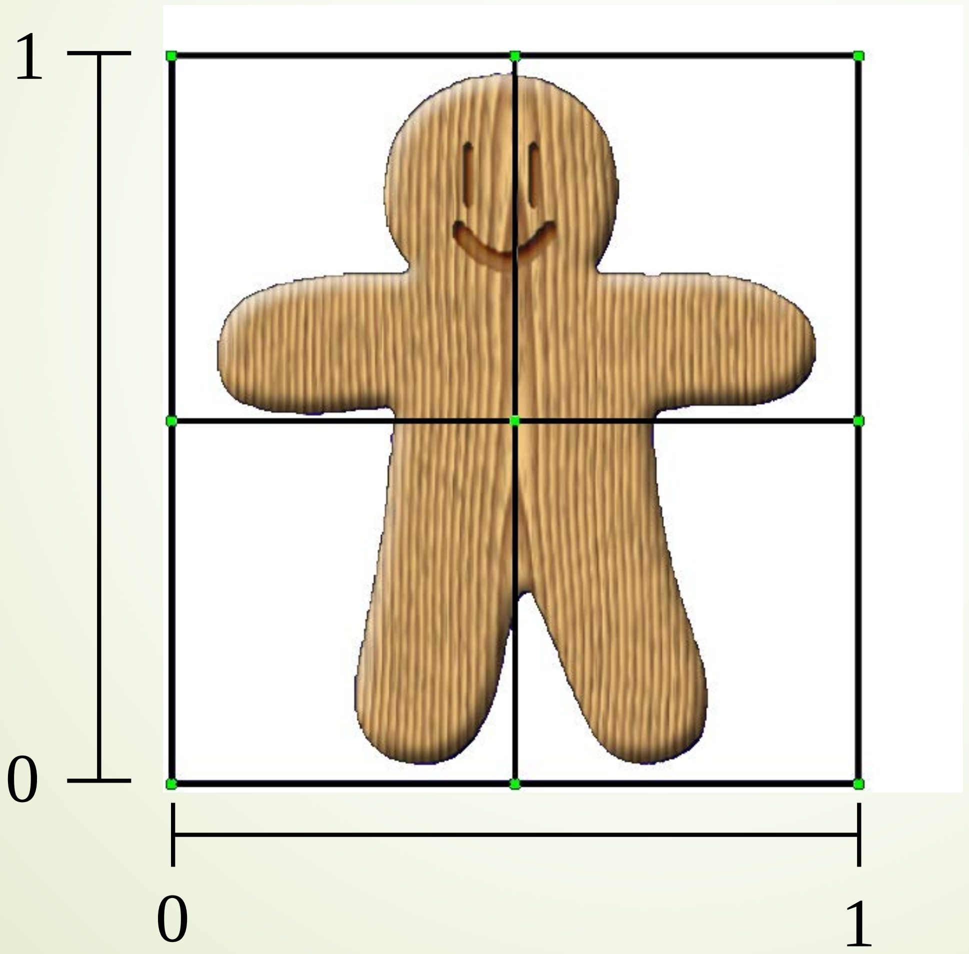


Parameterized world

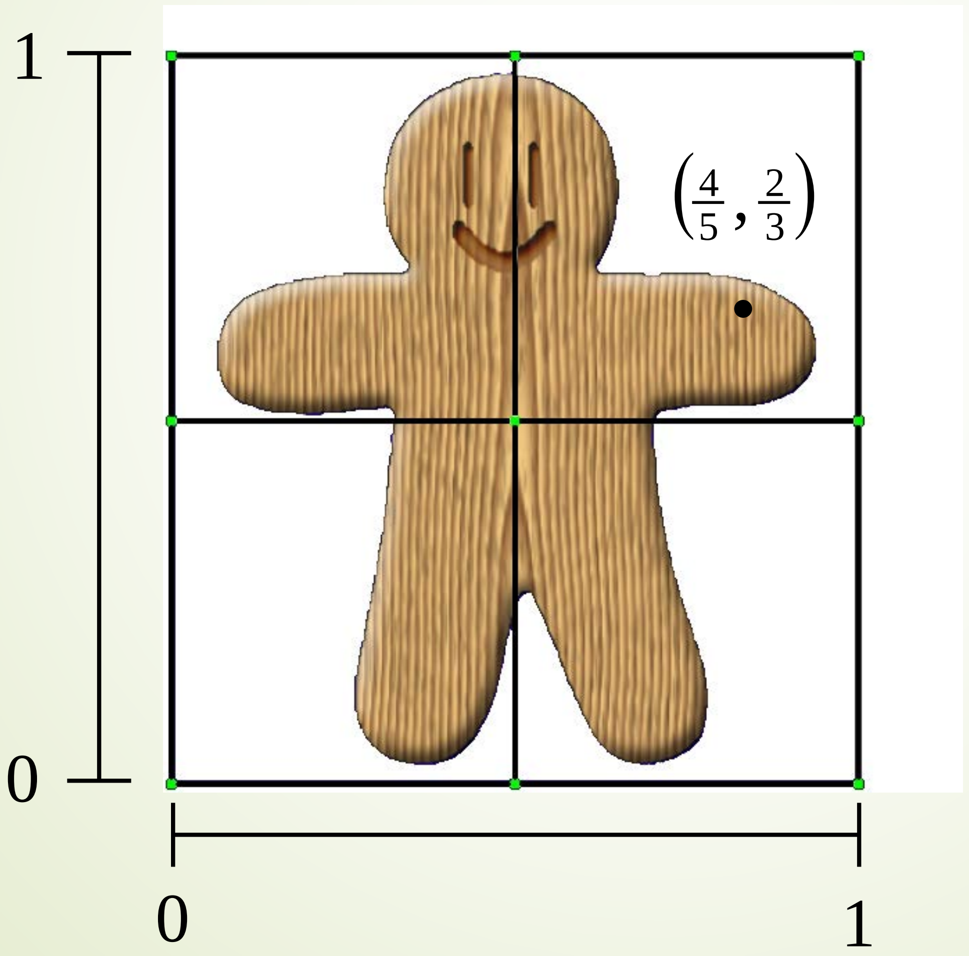
2D Example



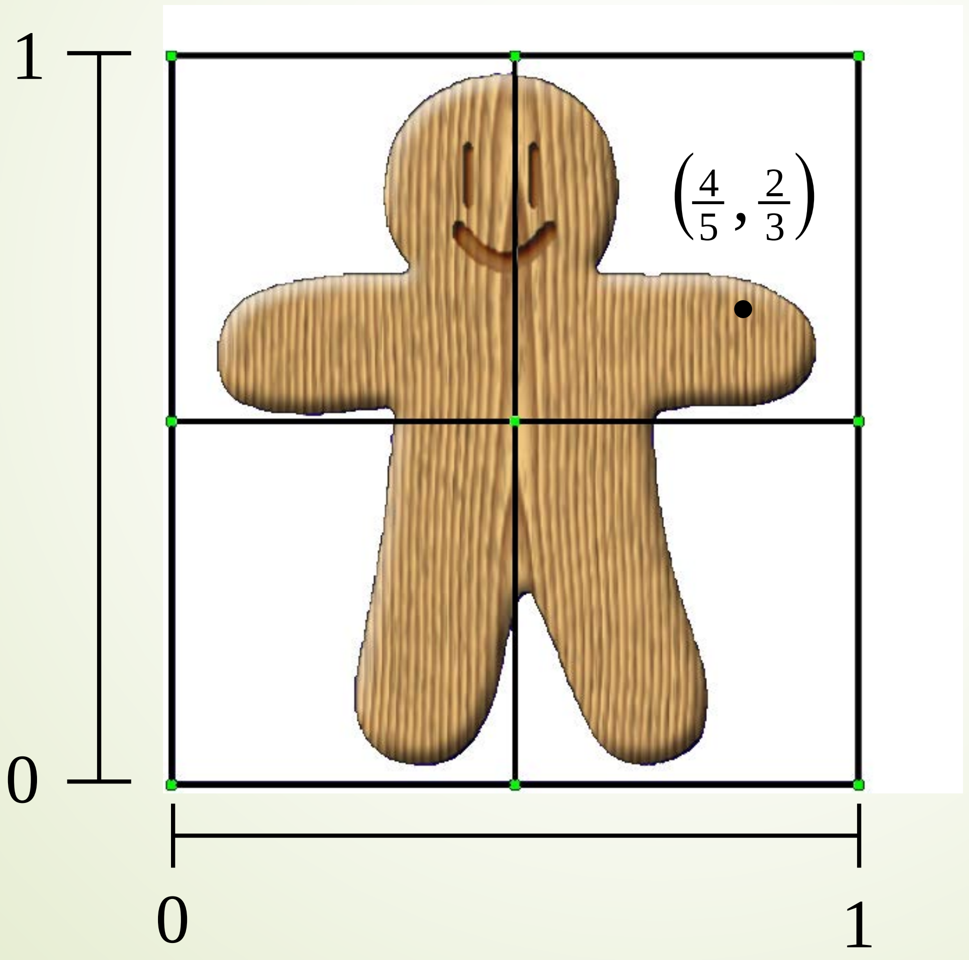
2D Example



2D Example



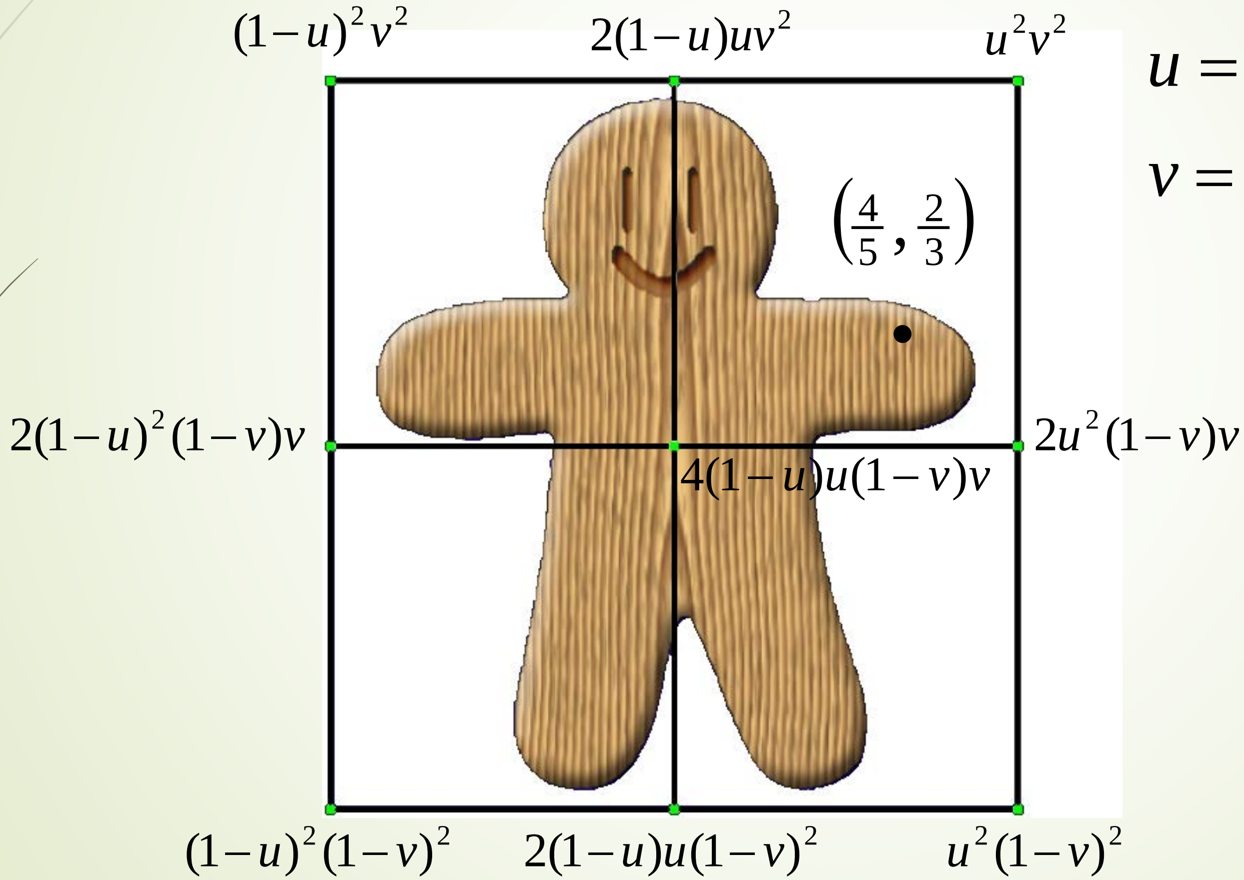
2D Example



$u = \frac{4}{5}$

$v = \frac{2}{3}$

2D Example



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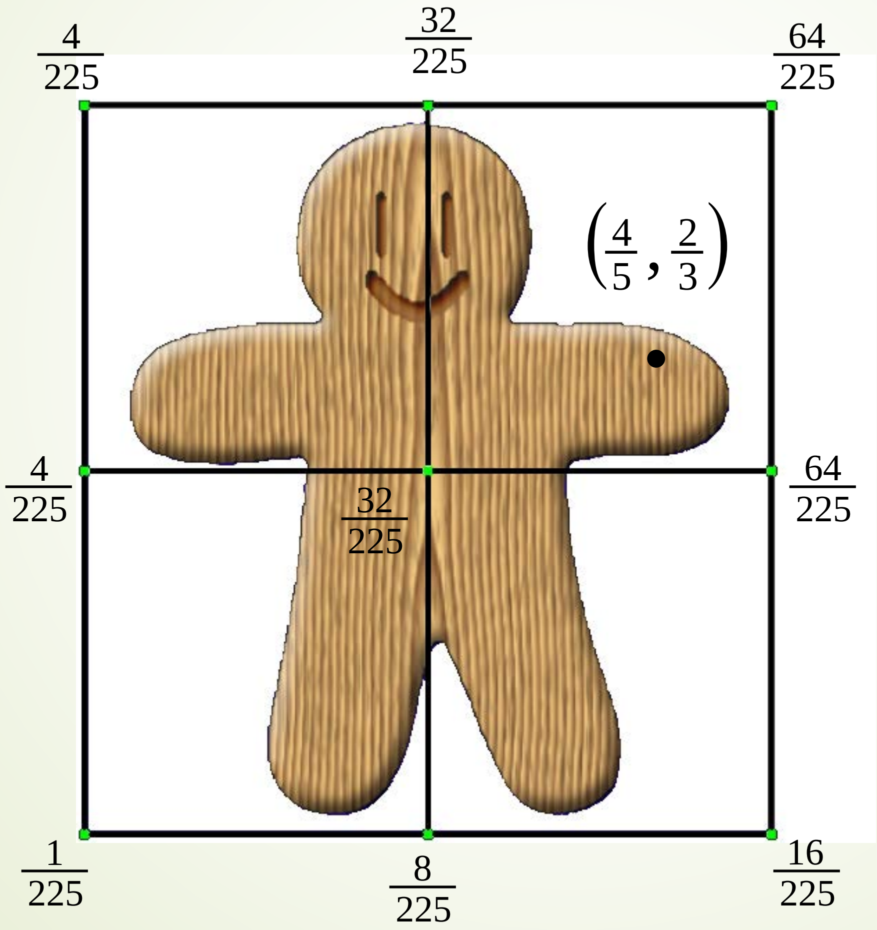

$u = \frac{4}{5}$

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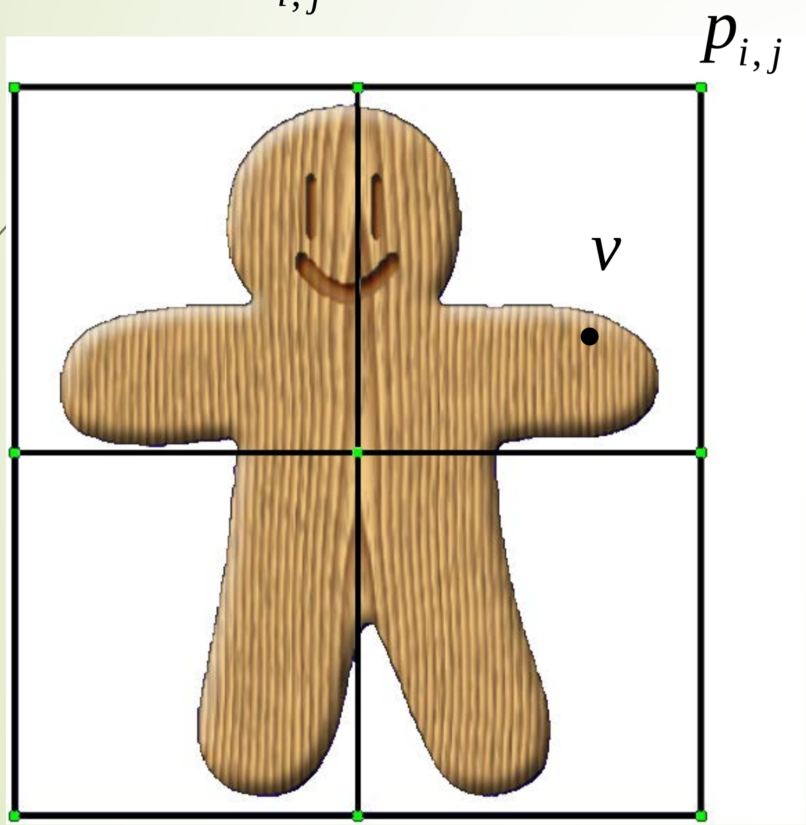
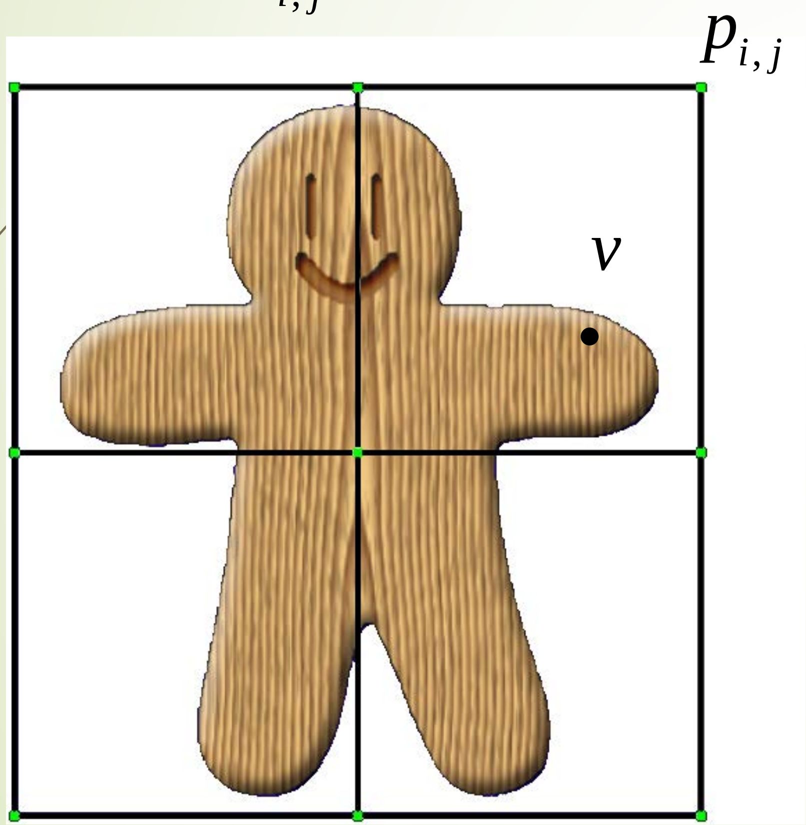
Applying the Deformation

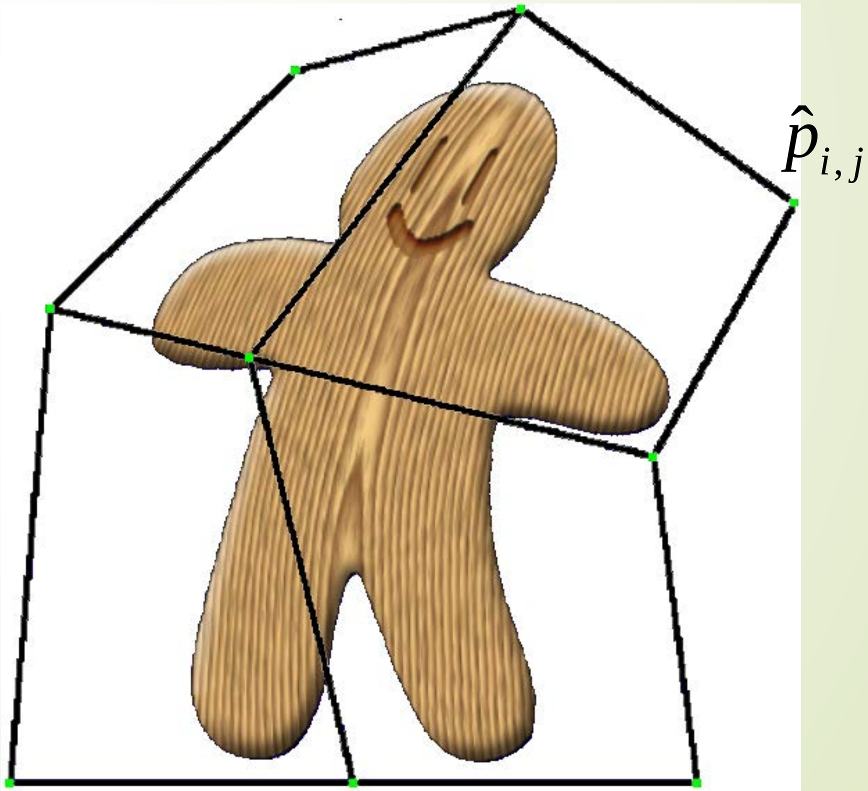
$$v = \sum_{i,j} w_{i,j} p_{i,j}$$



Applying the Deformation

$$v = \sum_{i,j} w_{i,j} p_{i,j}$$

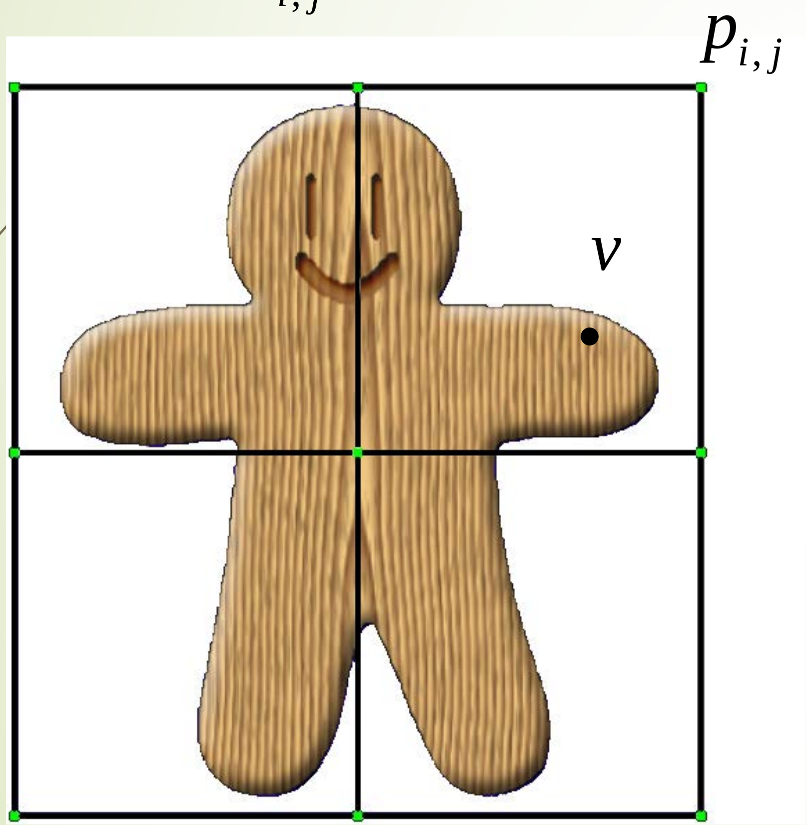


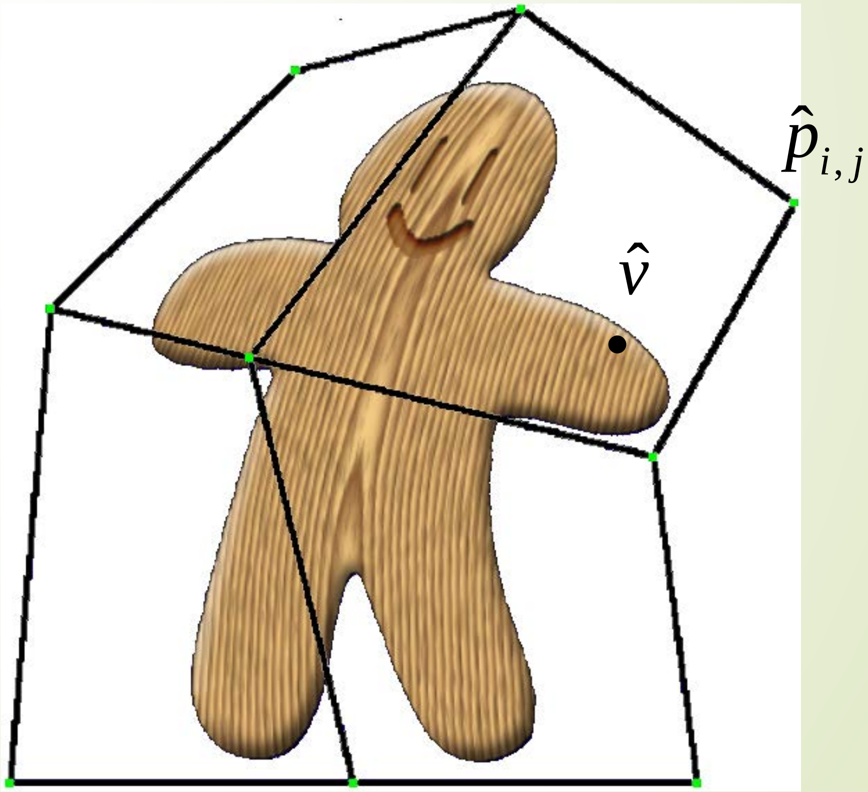


Applying the Deformation

$$v = \sum_{i,j} w_{i,j} p_{i,j}$$

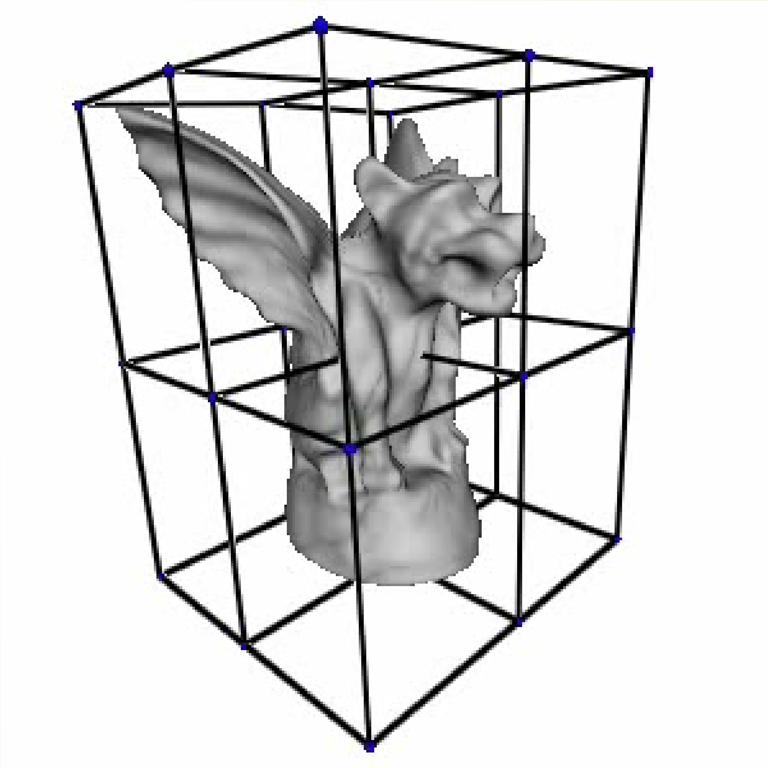
$$\hat{v} = \sum_{i,j} w_{i,j} \hat{p}_{i,j}$$





Advantages

- Smooth Deformation of arbitrary shapes
- Local control of deformations
- Computing the deformation is easy
- Deformations are very fast



Disadvantages

- Must use boxes for deformation
- Restricted to uniform grid
- Deformation warps space… not surface
  - Does not take into account geometry/topology of surface
- May need many FFD’s to achieve a simple deformation

Summary

- Widely used deformation technique
- Fast, easy to compute
- Some control over volume preservation/smoothness

- Uniform grids are restrictive