Graph Coverage Criteria (Ch. 7.2)
Testing & Covering Graphs (7.2)

We use graphs in testing as follows:
- Develop a model of the software as a graph
- Require tests to visit or tour specific sets of nodes, edges, or subpaths

**Test requirements (TR):** Describe properties of test paths

**Test Criterion:** Rules that define test requirements

**Satisfaction:** Given a set $TR$ of test requirements for a criterion $C$, a set of tests $T$ satisfies $C$ on a graph if and only if for every test requirement in $TR$, there is a test path in $\text{path}(T)$ that meets the test requirement $tr$.

**Structural Coverage Criteria:** Defined on a graph just in terms of nodes and edges
The first (and simplest) two criteria require that each node and edge in a graph be executed.

**Node Coverage (NC)**: Test set $T$ satisfies node coverage on graph $G$ iff for every syntactically reachable node $n$ in $N$, there is some path $p$ in $\text{path}(T)$ such that $p$ visits $n$.

This statement is a bit cumbersome, so we abbreviate it in terms of the set of test requirements.

**Node Coverage (NC)**: TR contains each reachable node in $G$. 
Node and Edge Coverage

Edge coverage is slightly stronger than node coverage.

**Edge Coverage (EC):** TR contains each reachable path of length up to 1, inclusive, in G

The phrase “length up to 1” allows for graphs with one node and no edges. NC and EC are only different when there is an edge and another subpath between a pair of nodes (as in an “if-else” statement).

Node Coverage
- \( TR = \{ 1, 2, 3 \} \)
- Test Path = [ 1, 2, 3 ]

Edge Coverage
- \( TR = \{ (1, 2), (1, 3), (2, 3) \} \)
- Test Paths = [ 1, 2, 3 ]
  [ 1, 3 ]
In-class Exercise

Graph Criteria EC

Answer the following questions for the graph on the left:

1. List test paths that satisfy edge coverage.
2. Write the set of test requirements for edge-pair coverage.
3. List test paths that satisfy edge-pair coverage.
4. Write the set of test requirements for prime path coverage.
5. List test paths that satisfy prime path coverage.
In-class Exercise

Graph Criteria EC

Answer the following questions for the graph on the left.

1. List test paths that satisfy edge coverage.

   - [1, 2, 4]
   - [1, 3, 5, 6, 3, 4]
   - [1, 3, 5, 7]
Node and Edge Coverage

A graph with only one node will not have any edges

It may seem trivial, but formally, Edge Coverage needs to require Node Coverage on this graph

Otherwise, Edge Coverage will not subsume Node Coverage

- So we define “length up to 1” instead of simply “length 1”

We have the same issue with graphs that have only one edge - for Edge-Pair Coverage…
Covering Multiple Edges

Edge-pair coverage requires **pairs of edges**, or subpaths of length 2

**Edge-Pair Coverage (EPC)**: TR contains each reachable path of length up to 2, inclusive, in G

The phrase “**length up to 2**” is used to include graphs that have less than 2 edges

```
1 -- 2 -- 3
     \   |
      4   5
     /   /|
    6   4
```

**Edge-Pair Coverage**: 

TR = \{ [1,4,5], [1,4,6], [2,4,5], \\
      [2,4,6], [3,4,5], [3,4,6] \}

The logical extension is to require **all paths**...
Answer the following questions for the graph on the left:

1. List test paths that satisfy edge coverage.
2. Write the set of test requirements for edge-pair coverage.
3. List test paths that satisfy edge-pair coverage.
4. Write the set of test requirements for prime path coverage.
5. List test paths that satisfy prime path coverage.
In-class Exercise

Graph Criteria EPC

Answer the following questions for the graph on the left:

1. List test paths that satisfy edge coverage.
2. Write the set of test requirements for edge-pair coverage.
3. List test paths that satisfy edge-pair coverage.

\[ TR = \{[1, 2, 4], [1, 3, 4], [1, 3, 5], [3, 5, 6], [5, 6, 3], [6, 3, 4], [6, 3, 5], [3, 5, 7]\} \]
Covering Multiple Edges

**Complete Path Coverage (CPC):** TR contains all paths in $G$.

Unfortunately, this is **impossible** if the graph has a loop, so a weak compromise make the tester decide which paths:

**Specified Path Coverage (SPC):** TR contains a set $S$ of test paths, where $S$ is supplied as a parameter.
## Covering Multiple Edges

**Node Coverage**

TR = \{ 1, 2, 3, 4, 5, 6, 7 \}

Test Paths: [ 1, 2, 3, 4, 7 ] [ 1, 2, 3, 5, 6, 5, 7 ]

**Edge Coverage**

TR = \{ (1,2), (1, 3), (2, 3), (3, 4), (3, 5), (4, 7), (5, 6), (5, 7), (6, 5) \}

Test Paths: [ 1, 2, 3, 4, 7 ] [ 1, 3, 5, 6, 5, 7 ]

**Edge-Pair Coverage**

TR = \{ [1,2,3], [1,3,4], [1,3,5], [2,3,4], [2,3,5], [3,4,7], [3,5,6], [3,5,7], [5,6,5], [5,6], [5,7] \}

Test Paths: [ 1, 2, 3, 4, 7 ] [ 1, 2, 3, 5, 7 ] [ 1, 2, 3, 5, 6, 5, 7 ]

[ 1, 2, 3, 5, 6, 5, 5, 7 ]

**Complete Path Coverage**

Test Paths: [ 1, 2, 3, 4, 7 ] [ 1, 2, 3, 5, 7 ] [ 1, 2, 3, 5, 6, 5, 7 ] [ 1, 2, 3, 5, 6, 5, 6, 5, 7 ] [ 1, 2, 3, 5, 6, 5, 6, 5, 6, 5, 7 ] ...

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*Write down the TRs and Test Paths for these criteria*
Handling Loops in Graphs

If a graph contains a loop, it has an infinite number of paths.

Thus CPC is not feasible.

SPC is not satisfactory because the results are subjective and vary with the tester.

Attempts to “deal with” loops:
- **1970s**: Execute cycles once
- **1980s**: Execute each loop, exactly once
- **1990s**: Execute loops 0 times, once, more than once
- **2000s**: Prime paths (touring, sidetrips, detours)
Simple Paths & Prime Paths

**Simple path:** A path from node $n_i$ to $n_j$ is simple if no node appears more than once, except possible the first and last nodes are the same

- No internal loops
- A loop is a simple path

**Prime path:** A simple path that does not appear as a proper subpath of any other simple path.

Write down the simple and prime paths for this graph

Simple Paths: $[1,2,4,5], [1,3,4,2], [1,3,4,5], [1,2,4], [1,3,4], [2,4,2], [2,4,5], [3,4,2], [3,4,5], [4,2,4], [1,2], [1,3], [2,4], [3,4], [4,2], [4,5], [1], [2], [3], [4], [5]$

Prime Paths: $[1,2,4,5], [1,3,4,2], [1,3,4,5], [2,4,2], [4,2,4]$
Simple Paths & Prime Paths

What if we change the graph?

Write down the simple and prime paths for this graph

**Simple Paths:** [1,2,4,1], [1,3,4,1], [2,4,1,2], [2,4,1,3], [3,4,1,2], [3,4,1,3], [4,1,2,4], [4,1,3,4], [1,2,4], [1,3,4], [2,4,1], [3,4,1], [4,1,2], [4,1,3], [1,2], [1,3], [2,4], [3,4], [4,1], [1], [2], [3], [4]

**Prime Paths:** [1,2,4,1], [1,3,4,1], [2,4,1,2], [2,4,1,3], [3,4,1,2], [3,4,1,3], [4,1,2,4], [4,1,3,4]
Prime Path Coverage

A simple, elegant and finite criterion that requires loops to be executed as well as skipped

Prime Path Coverage (PPC): TR contains each prime path in G.

Will tour all paths of length 0, 1,…

That is, it subsumes node and edge coverage

PPC almost, but not quite, subsumes EPC…
In-class Exercise

Graph Criteria PPC

Answer the following questions for the graph on the left

1. List test paths that satisfy edge coverage.
2. Write the set of test requirements for edge-pair coverage.
3. List test paths that satisfy edge-pair coverage.
4. Write the set of test requirements for prime path coverage.
5. List test paths that satisfy prime path coverage.
In-class Exercise

Graph Criteria PPC

Answer the following questions for the graph on the left

1. List test paths that satisfy edge coverage.
2. Write the set of test requirements for edge-pair coverage.
3. List test paths that satisfy edge-pair coverage.
4. Write the set of test requirements for prime path coverage.
5. List test paths that satisfy prime path coverage.
PPC does not subsume EPC

If a node has an edge to itself (self edge), EPC requires \([n, n, m]\) and \([m, n, n]\)

\([n, n, m]\) is not prime

Neither \([n, n, m]\) nor \([m, n, n]\) are simple paths (not prime)

EPC Requirements: ?
TR = \{ [1,2,3], [1,2,2], [2,2,3], [2,2,2] \}

PPC Requirements: ?
TR = \{ [1,2,3], [2,2] \}
Prime Path Example

The previous example has 38 simple paths

Only nine prime paths

Write down all 9 prime paths

Prime Paths
- [1, 2, 3, 4, 7]
- [1, 2, 3, 5, 7]
- [1, 2, 3, 5, 6]
- [1, 3, 4, 7]
- [1, 3, 5, 7]
- [1, 3, 5, 6]
- [6, 5, 7]
- [6, 5, 6]
- [5, 6, 5]

Execute loop 0 times
Execute loop once
Execute loop more than once
Touring, Sidetrips, and Detours

Prime paths do not have **internal loops** … test paths might

**Tour**: A test path $p$ tours subpath $q$ if $q$ is a subpath of $p$

**Tour with sidetrips**: A test path $p$ tours subpath $q$ with sidetrips iff every edge in $q$ is also in $p$ in the same order
- The tour can include a sidetrip, as long as it comes back to the same node

**Tour with detours**: A test path $p$ tours subpath $q$ with detours iff every node in $q$ is also in $p$ in the same order
- The tour can include a detour from node $n_i$, as long as it comes back to the prime path at a successor of $n_i$
Sidetrips and Detours Example

Touring with a sidetrip

Touring with a detour

Touring the prime path [1, 2, 3, 5, 6] without sidetrips or detours
Infeasible Test Requirements

An infeasible test requirement cannot be satisfied.
- Unreachable statement (dead code)
- Subpath that can only be executed with a contradiction ($x > 0$ and $x < 0$)

Most test criteria have some infeasible test requirements

It is usually undecidable whether all test requirements are feasible

When sidetrips are not allowed, many structural criteria have more infeasible test requirements

However, always allowing sidetrips weakens the test criteria

Practical recommendation—Best Effort Touring
- Satisfy as many test requirements as possible without sidetrips
- Allow sidetrips to try to satisfy remaining test requirements
Simple & Prime Path Example

- **Simple paths**
- **Write paths of length 0**

- '!' means path terminates

```
Len 0 [1] [2] [3] [4] [5] [6] [7]!
```
Simple & Prime Path Example

Simple paths

Write paths of length 1

Len 1
[1, 2]
[1, 3]
[2, 3]
[3, 4]
[3, 5]
[4, 7]!
[5, 7]!
[5, 6]
[6, 5]
Simple & Prime Path Example

Write paths of length 2

Len 2
[1, 2, 3]
[1, 3, 4]
[1, 3, 5]
[2, 3, 4]
[2, 3, 5]
[3, 4, 7]!
[3, 5, 7]!
[3, 5, 6]!
[5, 6, 5]*
[6, 5, 7]!
[6, 5, 6]*

'*' means path cycles

Simple paths

Graph

1
2
3
4
5
6
7
Simple & Prime Path Example

Write paths of length 3

Len 3

[1, 2, 3, 4]
[1, 2, 3, 5]
[1, 3, 4, 7]!
[1, 3, 5, 7]!
[1, 3, 5, 6]!
[2, 3, 4, 7]!
[2, 3, 5, 6]!
[2, 3, 5, 7]!
Simple & Prime Path Example

Write paths of length 4

[1, 2, 3, 4, 7]!
[1, 2, 3, 5, 7]!
[1, 2, 3, 5, 6]!
Simple & Prime Path Example

Simple paths

Simple paths

Len 1
[1, 2]
[1, 3]
[2, 3]
[3, 4]
[3, 5]
[4, 7]!
[5, 7]!
[5, 6]
[6, 5]

Len 2
[1, 2, 3]
[1, 3, 4]
[1, 3, 5]
[2, 3, 4]
[2, 3, 5]
[3, 4, 7]!
[3, 5, 7]!
[3, 5, 6]!
[5, 6, 5]*
[6, 5, 7]!
[6, 5, 6]*

Len 3
[1, 2, 3, 4]
[1, 2, 3, 5]
[1, 3, 4, 7]!
[1, 3, 5, 7]!
[1, 3, 5, 6]!
[2, 3, 4, 7]!
[2, 3, 5, 6]!
[2, 3, 5, 7]!

Len 4
[1, 2, 3, 4, 7]!
[1, 2, 3, 5, 7]!
[1, 2, 3, 5, 6]!
Round Trips

Round-Trip Path: A prime path that starts and ends at the same node

Simple Round Trip Coverage (SRTC): TR contains at least one round-trip path for each reachable node in G that begins and ends a round-trip path.

Complete Round Trip Coverage (SRTC): TR contains all round-trip paths for each reachable node in G.

These criteria omit nodes and edges that are not in round trips

Thus they do not subsume edge-pair, edge, or node coverage
Graph Coverage Criteria
Subsumption

- Complete Path Coverage (CPC)
- Prime Path Coverage (PPC)
- Complete Round Trip Coverage (CRTC)
- Simple Round Trip Coverage (SRTC)
- Edge-Pair Coverage (EPC)
- Edge Coverage (EC)
- Node Coverage (NC)
- All-defs Coverage (ADC)
- All-uses Coverage (AUC)
- All-DU-Paths Coverage (ADUP)
Graph Coverage Summary (7.1-7.2)

Graphs are a very powerful abstraction for designing tests.

The various criteria allow lots of cost/benefit tradeoffs.

These two sections are entirely at the “design abstraction level” from chapter 2.

Graphs appear in many situations in software.

-As discussed in the rest of chapter 7.