Parallel Sorting

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CS 499: Spring 2016 GMU

Logistics

Today

- Trailing questions on HW2?
- Review Parallel Performance Theory
- Parallel Sorting

Normal Office Hours

- Tue 3/1 3:30-5:30

Reading: Grama Ch 9

- Sorting
- Focus on 9.4: Quicksort

Schedule

Tue 2/23	PageRank & MPI
Thu 2/25	Performance Analysis
Mon 2/29	HW 2 Due 11:59pm
Tue 3/1	Performance, Parallel Sorting
Thu 3/3	Guest Lecture, Mini-Exam 2
3/2-3/4	HW 2 Interviews

Quick Review

- What is Amdahl's law? What does it say about the speedup achievable by parallel programs?
- How does one calculate the following for a parallel algorithm
  - S: Speedup
  - E: Efficiency
  - C: Cost
- How does the Efficiency of a parallel usually change if the problem size increases but the number of processors P stays the same?
- How does the Efficiency of a parallel usually change if the number of processors P increases but the problem size stays the same?
- What is Parallel Overhead?
- What is Isoefficiency?

Sorting

- Much loved computation problem
- What is the best complexity of general purpose (comparison-based) sorting algorithms?
- What are some algorithms which have this complexity?
- What are some other sorting algorithms which aren't so hot?
- What issues need to be addressed to parallelize any sorting algorithm?

Partition and Quicksort

- Quicksort has $O(N \log N)$ average complexity
- In-place, low overhead sorting, recursive

Partition

- Partition: select `pivot` value
- On completion
  - Left array is $\leq$ `pivot`
  - Right array is $>$ `pivot`
  - `pivot` is in "middle"

```
algorithm partition(A, lo, hi) is
  pivot := A[hi]
  boundary := lo
  for j := lo to hi - 1 do
    if A[j] <= pivot then
      swap A[boundary] with A[j]
      boundary++
  swap A[i] with A[hi]
  return boundary
```

Quicksort

- Partition into two parts
- Recurse on both halves
- Bail out when boundaries `lo`/`hi` cross

```
algorithm quicksort(A, lo, hi) is
 if lo < hi then
   p := partition(A, lo, hi)
   quicksort(A, lo, p - 1)
   quicksort(A, p + 1, hi)
```

Practical Parallel Sorting Setup

- Input array A of size N is already spread across P processors (no need to scatter)

```
P0: A[] = { 84 31 21 28 }
P1: A[] = { 17 20 24 84 }
P2: A[] = { 24 11 31 99 }
P3: A[] = { 13 32 26 75 }
```

- Goal: Numbers sorted across processors. Smallest on P0, next smallest on P1, etc.

```
P0: A[] = { 11 13 17 20 }
P1: A[] = { 21 24 24 26 }
P2: A[] = { 28 31 32 33 }
P3: A[] = { 75 84 84 99 }
```

- Want to use P processors as effectively as possible
- Bulk communication preferred over many small messages

Exercise: Parallel Quicksort

- Find a way to parallelize quicksort
- Hint: The last step is each processor sorting its own data using a serial algorithm. Try to arrange data so this is possible.

```
START:
P0: A[] = { 84 32 21 28 }
P1: A[] = { 17 20 25 85 }
P2: A[] = { 24 11 31 99 }
P3: A[] = { 13 32 26 75 }

GOAL
P0: A[] = { 11 13 17 20 }
P1: A[] = { 21 24 25 26 }
P2: A[] = { 28 31 32 33 }
P3: A[] = { 75 84 85 99 }
```

```
SERIAL ALGORITHM
algorithm quicksort(A, lo, hi) is
 if lo < hi then
   p := partition(A, lo, hi)
   quicksort(A, lo, p - 1)
   quicksort(A, p + 1, hi)

algorithm partition(A, lo, hi) is
  pivot := A[hi]
  boundary := lo
  for j := lo to hi - 1 do
    if A[j] <= pivot then
      swap A[boundary] with A[j]
      boundary++
  swap A[i] with A[hi]
  return boundary
```

Parallel Quicksort Ideas 1

```
A[] = { 84 32 21 11 | 17 20 25 85 | 24 28 31 99 | 13 32 26 75 }
        P0            P1            P2            P3
Partition(pivot=26) on each processor
A[] = { 21 11 84 32 | 17 20 25 85 | 24 28 31 99 | 13 26 32 75 }
Boundary:   ^                ^       ^                ^
Counts: P0: 2         P1: 3         P2: 1         P3: 2
Calculate which data goes where
A[] = { 21 11 84 32 | 17 20 25 85 | 24 28 31 99 | 13 26 32 75 }
        P0 P0 P2 P2   P0 P0 P1 P2   P1 P2 P3 P3   P1 P1 P3 P3

Re-arrange so values <= 26 on P0 and P1, > 26 on P2 and P3
A[] = { 21 11 17 20 | 25 24 13 25 | 84 32 85 28 | 31 99 23 75 }
        P0            P1            P2            P3

Split the world: 2 groups
A[] = { 21 11 17 20 | 25 24 13 25}|{84 32 85 28 | 31 99 23 75 }
        P0            P1            P2            P3
```

Parallel Quicksort Ideas 2

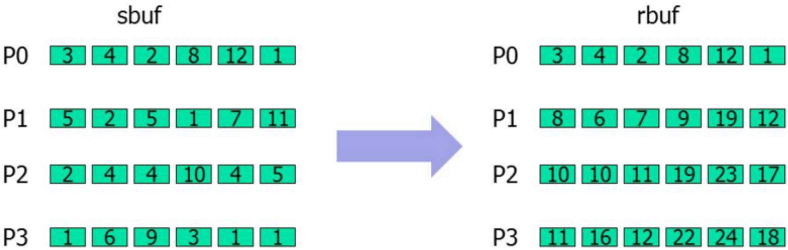
```
Each half partitions on different value
P0-P1: Partition(pivot=20)
P2-P3: Partition(pivot=32)
A[] = { 11 17 20 21 | 13 25 24 25}|{28 32 84 85 | 31 23 99 75 }
Boundary:      ^       ^                ^              ^
Counts: P0: 3         P1: 1          P2: 2          P3: 2
Calculate which data goes where
A[] = { 11 17 20 21 | 13 25 24 25}|{28 32 84 85 | 31 23 99 75 }
        P0 P0 P0 P1   P0 P1 P1 P1   P2 P2 P3 P3   P2 P2 P3 P3
Re-arrange values to proper processors
A[] = { 11 17 20 13 | 21 25 24 25}|{28 32 31 23 | 84 85 99 75 }
        P0            P1             P2             P3
Split the world: 4 groups
A[] = { 11 17 20 13}|{21 25 24 25}|{28 32 31 23}|{84 85 99 75 }
        P0            P1             P2             P3

4 groups == 4 processors, all processors sort locally
A[] = { 11 13 17 20}|{21 24 25 25}|{23 28 31 32}|{75 84 85 99 }
        P0            P1             P2             P3
Done
```

Issues

- Pivots were cherry-picked to get even distribution
- Generally not possible to do: processors might have uneven portions of the array after partitioning
- Will require
- Must figure out how to communicate which elements to each processor
- Must split the world into smaller groups

Prefix Sums / Scan

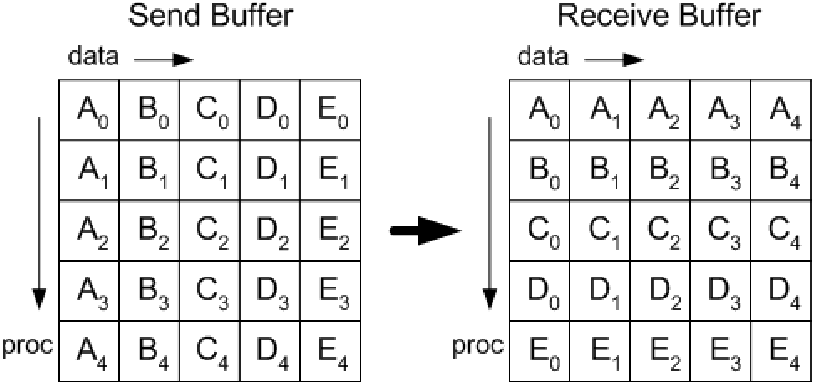



```
int MPI_Scan(const void *sendbuf, void *recvbuf, int count,
             MPI_Datatype datatype, MPI_Op op, MPI_Comm comm)
```

- Similar to reduction
- Change: only add on values from procs <= `proc_id`
- op can be sum/max/min/etc.
- In Quicksort, use All-gather to get an array of counts of small values on each proc, follow with Prefix Sum to calculate how much to send to each processor

All-to-All Personalized Communication

All-to-all personalized communication: like every processor scattering to every other processor.



Source: Cornell University Center for Advanced Computing

MPI_Alltoall

- Standard version: every processor gets a slice of `sendbuf`, same sized data
- Vector version allows different sized slices (appropriate for quicksort)

```
int MPI_Alltoall(
 void *sendbuf, int sendcount, MPI_Datatype sendtype,
 void *recvbuf, int recvcount, MPI_Datatype recvtype,
 MPI_Comm comm);

int MPI_Alltoallv(
 void *sendbuf, int sendcounts[], int sdispls[], MPI_Datatype sendtype,
 void *recvbuf, int recvcounts[], int rdispls[], MPI_Datatype recvtype,
 MPI_Comm comm);
```

Splitting the World

```
int MPI_Comm_split(MPI_Comm comm, int color, int key,
                   MPI_Comm *newcomm);
```

- `comm` is the old communicator (start with `MPI_COMM_WORLD`
- `color` is which sub-comm to go into
- `key` establishes rank in new sub-comm, usually `proc_id`
- `newcomm` is filled in with a new communicator
- Examine `mpi-code/comm-split.c`

Ultimate Complexity of Parallel Quicksort

Take a moment to calculate O complexity based on

- N elements
- P processors