More on template matching

Wide baseline matching

Region based Similarity Metric

- Sum of squared differences
  \[ SSD(h) = \sum_{\tilde{x} \in W(x)} \| I_1(\tilde{x}) - I_2(h(\tilde{x})) \|^2 \]

- Normalize cross-correlation
  \[ NCC(h) = \frac{\sum_{\tilde{x} \in W(x)} (I_1(\tilde{x}) - I_1)(I_2(h(\tilde{x})) - I_2))}{\sqrt{\sum_{\tilde{x} \in W(x)} (I_1(\tilde{x}) - I_1)^2 \sum_{\tilde{x} \in W(x)} (I_2(h(\tilde{x})) - I_2)^2}} \]

- Sum of absolute differences
  \[ SAD(h) = \sum_{\tilde{x} \in W(x)} | I_1(\tilde{x}) - I_2(h(\tilde{x})) | \]

NCC score for two widely separated views
Reducing the comp. cost of correlation matching

- A number of factors lead to large costs in correlation matching:
  - the image $N$ is much larger than the template $M$, so we have to perform correlation matching of $M$ against every $n \times n$ window of $N$
  - we might have many templates, $M_i$, that we have to compare against a given image $N$
  - face recognition - have a face template for every known face; this might easily be tens of thousands
  - character recognition - template for each character
  - we might not know the orientation of the template in the image
  - template might be rotated in the image $N$ - example: someone tilts their head for a photograph
  - would then have to perform correlation of rotated versions of $M$ against $N$

Template matching

<table>
<thead>
<tr>
<th>Database</th>
<th>Position</th>
<th>Orientation</th>
<th>Scale</th>
</tr>
</thead>
</table>

New image

Recognition by finding patterns

- We have seen very simple template matching (under filters)
- Some objects behave like quite simple templates
- Frontal faces

- Strategy:
  - Find image windows
  - Correct lighting
  - Pass them to a statistical test (a classifier) that accepts faces and rejects non-faces
Bayesian Decision Making

- Use a histogram to represent the class-conditional densities
  - (i.e. $p(x|1)$, $p(x|2)$, etc)
- Advantage: estimates become quite good with enough data!
- Disadvantage: Histogram becomes big with high dimension
  - but maybe we can assume feature independence?

Histogram based classifiers

Finding skin

- Skin has a very small range of (intensity independent) colours, and little texture
- Compute an intensity-independent colour measure, check if colour is in this range, check if there is little texture (median filter)
- See this as a classifier - we can set up the tests by hand, or learn them.
- Get class conditional densities (histograms), priors from data (counting)

Classifier is
  - if $p(\text{skin} | x) > \theta$, classify as skin
  - if $p(\text{skin} | x) < \theta$, classify as not skin
  - if $p(\text{skin} | x) = \theta$, choose classes uniformly and at random
Reducing the cost of template matching

- Reducing the number of image windows that need to be compared against the database
  - find "objects" in the image
- Reducing the number of database objects that need to be matched against any window
  - index templates by features such as moments that are not changed by rotations and scale changes
  - measure moments of candidate windows
  - only match "similar" templates
- Reducing the number of operations needed to match a given template to the image

Reducing the comp. cost of correlation matching

- Two basic techniques for reducing the number of operations associated with correlation
  - reduce the number of pixels in M and N
    - multi-resolution image representations
    - principal component or "feature selection" reductions
  - match a subset of M against a subset of N
    - random subsets
    - boundary subsets - edge correlation

Multi-resolution correlation

- Multi-resolution template matching
  - reduce resolution of both template and image by creating an image pyramid
  - match small template against small image
  - identify locations of strong matches
  - expand the image and template, and match higher resolution template selectively to higher resolution image
  - iterate on higher and higher resolution images
- Issue:
  - how to choose detection thresholds at each level
    - too low will lead to too much cost
    - too high will miss match
Image pyramids

- Base of the pyramid, level 0, is the full resolution image - say $2^n \times 2^n$
- Level $i$ of the pyramid is obtained from level $i-1$ as follows
  - partition level $i-1$ into non-overlapping $2^k \times 2^k$ blocks
    - typically, $k = 1$ or $2$
  - compute an average grey level in each of these blocks
    - unweighted average
    - Gaussian weighted average more typical
    - assign that average grey level to the corresponding level $i$ pixel
- For you to think about: How many pixels are there in an image pyramid having an $n \times n$ base and a reduction by neighborhoods of size $2^k \times 2^k$?

Example

- Representation of images at multiple levels of resolution
- Importance – at different resolutions different features look differently
- Used for localization properties, motion computation and matching, biological motivation

Pyramid construction

- Take an original image – convolve with a blurring
- Filter and subsample to get an image at lower resolution

Reduction – how is the signal at level $l+1$ related to level $l$

$$f^{l+1}[x, y] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} g[k, l] f^l[2x - k, 2y - l]$$
**Pyramid construction/reconstruction**

Expansion – how to reconstruct the signal at level \( l \) given related to level \( l-1 \) – notation

\[
f^{l,k+1}[x, y] = \sum_{k=-\frac{1}{2}}^{\frac{1}{2}} \sum_{l=-\frac{1}{2}}^{\frac{1}{2}} \phi[k, l] f^l[(x - k)/2, (y - l)/2]
\]

Signal at level \( l \), expanded \( k \) times

Idea - take the smaller signal fill every second entry

With zero and convolve with the blurring filter

---

**“Drop” vs “Smooth and Drop”**

Drop every second pixel

Smooth and Drop every second pixel

Aliasing problems

---

**Gaussian Pyramid**

Consecutive smoothing and sub-sampling

---

**Laplacian Pyramid**

Idea

when we convolve and down-sample some information will get lost i.e. if we reverse the process we cannot get the original image back. Example:

Blurred image of Lower resolution

(Original image - upsampled blurred image)

they are not the same - fine details are lost
Laplacian pyramid

- Store the fine differences which get lost
- Each level of the Laplacian pyramid difference between two consecutive levels of gaussian pyramids

Schematic for construction/reconstruction

Image → blur/down2 → Blurred1 → up2/blur → add → Recon

up2/blur

→ subtract → Fine1

Laplacian pyramids

- Laplacian pyramids – each level holds the additional information which is needed for better resolution
- We can obtain same image by convolving the image with difference of Gaussians filter or appropriate width
- Reflects the similarity between difference of Gaussians DoG and Laplacian operator introduced in the edge detection stage
Finding faces

- Faces "look like" templates (at least when they're frontal).
- General strategy:
  - search image windows at a range of scales
  - Correct for illumination
  - Present corrected window to classifier

Issues
- How corrected?
- What features?
- What classifier?
- What about lateral views?

Naive Bayes

- (Important: naive not necessarily perjorative)
- Find faces by vector quantizing image patches, then computing a histogram of patch types within a face
- Histogram doesn't work when there are too many features
  - features are the patch types
  - assume they're independent and cross fingers
  - reduction in degrees of freedom
  - very effective for face finders
  - why? probably because the examples that would present real problems aren't frequent.

Many face finders on the face detection home page
http://home.t-online.de/home/Robert.Frischholz/face.htm

Chamfer matching

- Given:
  - binary image, B, of edge and local feature locations
  - binary "edge template", T, of shape we want to match
- Let D be an array in registration with B such that D(i,j) is the distance to the nearest "1" in B.
  - this array is called the distance transform of B
- Goal: Find placement of T in D that minimizes the sum, M, of the distance transform multiplied by the pixel values in T
  - if T is an exact match to B at location (i,j) then M(i,j) = 0
  - but if the edges in B are slightly displaced from their ideal locations in T, we still get a good match using the distance transform technique

Computing the distance transform

- Brute force, exact algorithm, is to scan B and find, for each "0", its closest "1" using the Euclidean distance.
- expensive in time, and difficult to implement

Binary Image and its distance transform – suitable for medial axis representations

Distance transform example

\[ D(i,j) = \min[D(i,j), D(i,j+1)+1, D(i+1,j-1)+1, D(i+1,j)+1, D(i+1,j+1)+1] \]

Chamfer matching

- Chamfer matching is convolution of a binary edge template with the distance transform
- for any placement of the template over the image, it sums up the distance transform values for all pixels that are "1's" (edges) in the template
- if, at some position in the image, all of the edges in the template coincide with edges in the image (which are the points at which the distance transform is zero), then we have a perfect match with a match score of 0.
Example

Match score is \( \sum_{i=1}^{n} \sum_{j=1}^{n} T(i, j)D(i + k, j + l) \)

Chamfer Matching

From Shape Context and Chamfer Matching
In Cluttered Scenes. A. Thayananthan, B. Stenger, P. Tarr and R. Cippola

Hausdorff distance matching

- Let M be an nxn binary template and N an nxn binary image we want to compare to that template
- \( H(M, N) = \max(h(M, N), h(N, M)) \) where
  \[
  h(A, B) = \max_{a \in A} \min_{b \in B} |a - b|
  \]
- \( || \) is a distance function like the Euclidean distance function
- \( h(A, B) \) is called the directed Hausdorff distance.
- ranks each point in A based on closeness to a point in B
- most mis-matched point is measure of match
- if \( h(A, B) = e \), then all points in A must be within distance \( e \) of B.
- generally, \( h(A, B) \neq h(B, A) \)
- easy to compute Hausdorff distances from distance transform
Shape Context

- From Shape Matching and Object Recognition using Shape Context, by Belongie, Malik, Puzicha, IEEE PAMI (24), 2002

Summary of 2-D object matching/recognition

- Binary vision systems
  - segmentation by thresholding and connected component analysis
  - object modeling using statistical techniques
    - means and variances of global object features such as area, perimeter, etc.
    - recognition using statistical recognition techniques
    - k-nearest neighbors
    - Bayesian recognition
  - Drawbacks
    - touching objects
    - occluded objects
    - weak segmentation techniques

- Grey level vision systems
  - (optional) segmentation by edge detection
  - object modeling by templates
    - grey level region templates
    - edge templates (binary)
  - recognition using correlation
    - brute force image correlation
    - speedup methods
    - Hough transform methods
    - Chamfer matching
  - Drawbacks
    - computational complexity
    - to support rotations and scaling of templates