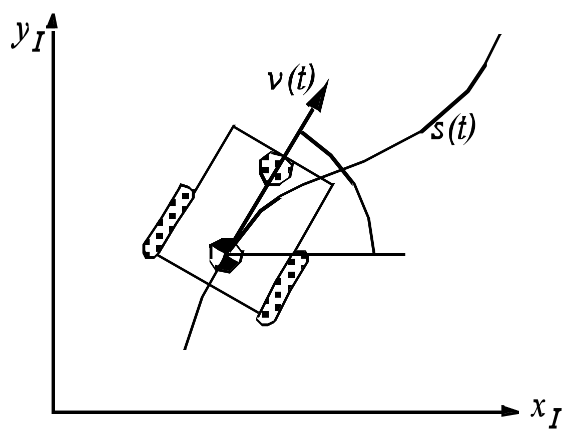
Robot Control Basics

Mobile robot kinematics

- Differential drive mobile robot
- Two wheels, with diameter r, point P centered
- Between two wheels is the origin of the robot frame
- Each wheel is a distance l from the center



Some terminology

- Effector (legs, arms, wheels, fingers)
- Actuator – enables effector to execute motion (electric, hydraulic)

- Degree of freedom DOF - number of parameters describing the pose/configuration of the robot
- Rigid body 6 DOF, mobile robot 3 DOF

- Simplest case one actuator controls one DOF → all degrees of freedom are controllable

- We have derived kinematics equations of the robot

$$
\begin{aligned}
\dot{x} &= v\cos\theta \\
\dot{y} &= v\sin\theta \\
\dot{\theta} &= \omega
\end{aligned}
$$

Some terminology

- Alternative derivation (optional) is in terms of wheel constraints section (3.2.3 – 3.4.2)
- Example
- sliding constraint – each wheel can only roll in the plane of the wheel
- steering constraint – streerable wheels can be steered

- degree of maneuverability – number of degrees of freedom robot can directly control δ_M
- Car-like mobile robot – 3-DOF , two control inputs

$$\delta_M = \delta_m + \delta_s$$

- Differential drive robot $\delta_M = 2 + 0 = 2$

$$\begin{aligned} \dot{x} &= v \cos\theta \\ \dot{y} &= v \sin\theta \\ \dot{\theta} &= \omega \end{aligned}$$

Some terminology

• Degrees of Freedom DOF
• Differential number of Degrees of freedom (DOF in the velocity space) – DDOF
• DDOF is always equal to δ_m degree of mobility

• Car-like mobile robot – 3-DOF , two control inputs, two differential degrees of freedom

•If DOF = DDOF robot is holonomic, otherwise it is non-holonomic

• Differential drive robot – non-holonomic
• Omnidirectional drive – holonomic

Connection between DOF and actuators/effectors

- If there is an actuator for each DOF then each DOF is controllable
- If not all DOF are directly controllable the control problems are much harder (see later)

The number of controllable DOF determines how hard the control problem will be.

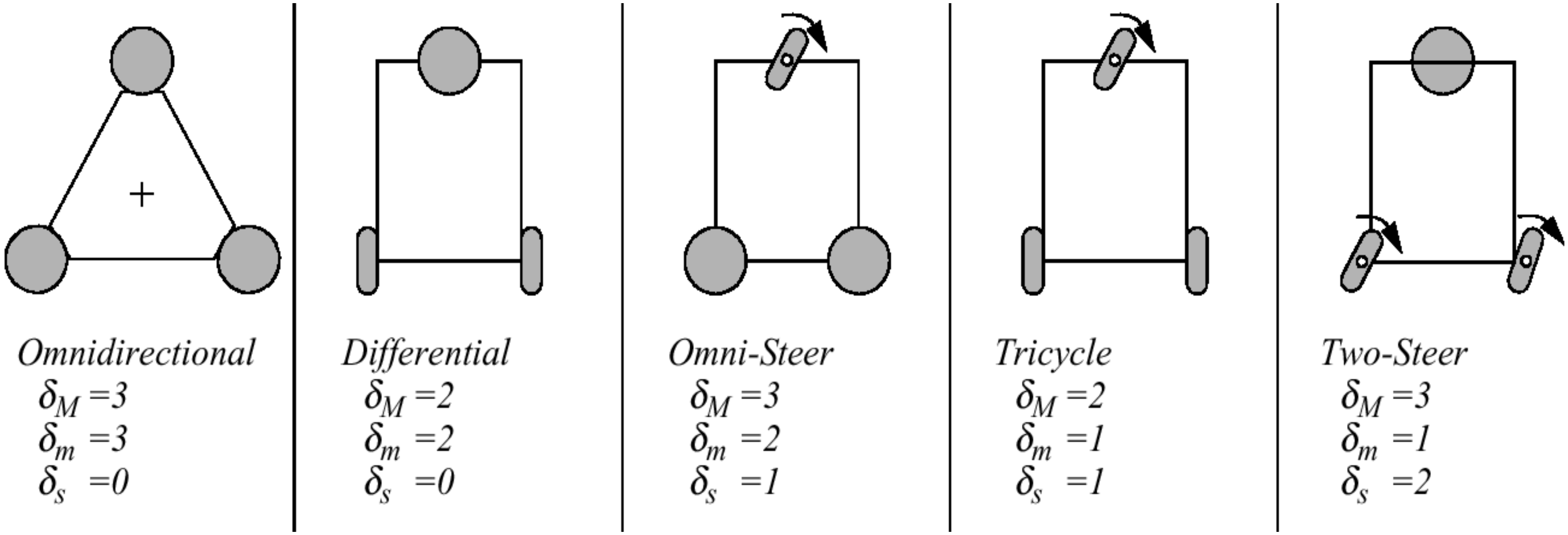
holonomic robots # of DOF is the same as # of controllable DOF's

nonholonomic robot # of DOF is bigger then # of controllable DOF's

redundant robot # of controllable DOF is larger then # of total DOF's

e.g. Human Arm 6 DOF's – position and orientation of the Fingertip in 3D space – 7 actuators - 3 shoulder, 1 elbow, 3 wrist

Five Basic Types of Three-Wheel Configurations



Previously

- Kinematics models of kinematic chain, arm and mobile robot
- Relationship between the position of the end-effector and joint angles (manipulator), pose of the mobile robot and angular and linear velocities
- Control and Planning - How to do the right thing ?

1. Open loop control
2. Feedback control
3. Potential field based methods (feedback control)

Paths and Trajectories

- In general – control problem – need to generate set of control commands to accomplish the task
- In an open loop setting there are two components

1. Geometric Path Generation
2. Trajectory generation (time indexed path)
3. Trajectory tracking

- Example omni-directional robot – can control all degress of freedom independently

Trajectories

- Smooth 1D trajectories – scalar functions of time
- Polynomials of higher orders
- Piecewise linear segments and polynomial blends
- Blackboard

- Interpolation of orientation in 3D
- Rigid Body Pose and Motion
- Varying coordinate frames

Example trajectory generation

- Given end points in either work space or joint angle space
- Generate joint angle trajectory between start and end position
- Specify start and end position (or additional constraints – spatial (obstacles) or temporal (time of completion)
- How to generate velocities and accelerations to follow the trajectory
- Example blackboard: use cubic polynomials to generate the trajectory

Trajectory generation

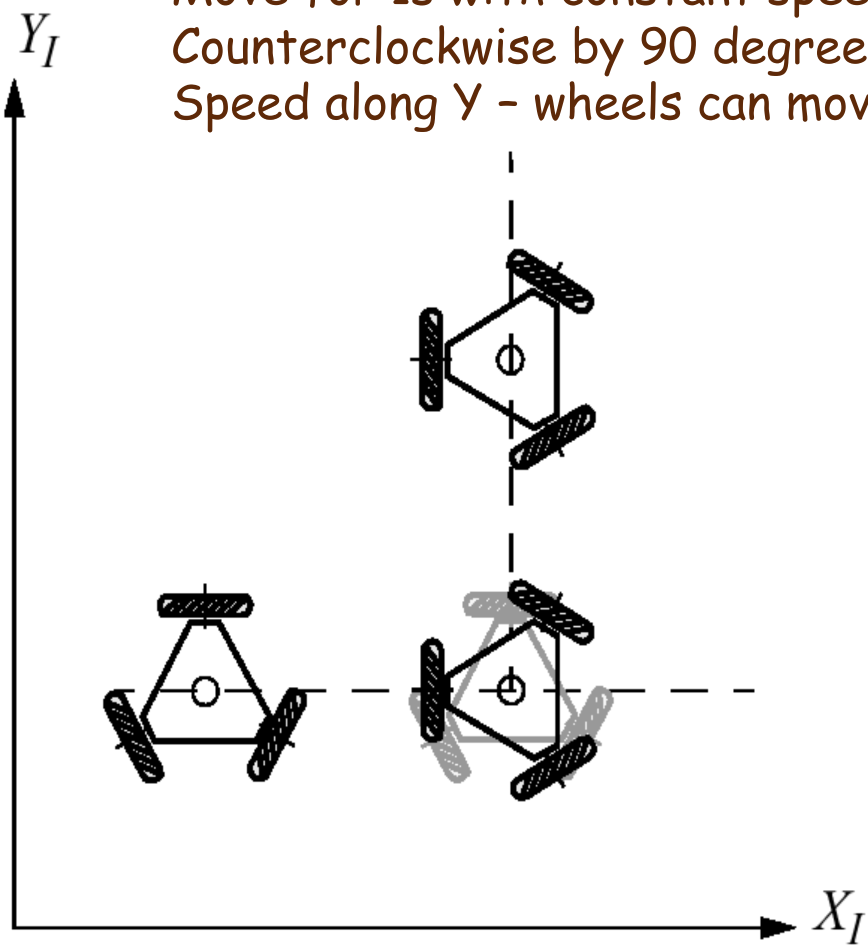
- Alternatives linear paths with parabolic blends (splines) – the via points are not actually reached
- Previous example – paths computed in joint space – the path in the workspace depends on the kinematics of the manipulator
- Another scenario – compute paths in the workspace – specify at each instance of time the pose (R,T) the end effector robot should be at – interpolate between poses
- Problems with workspace and singularities

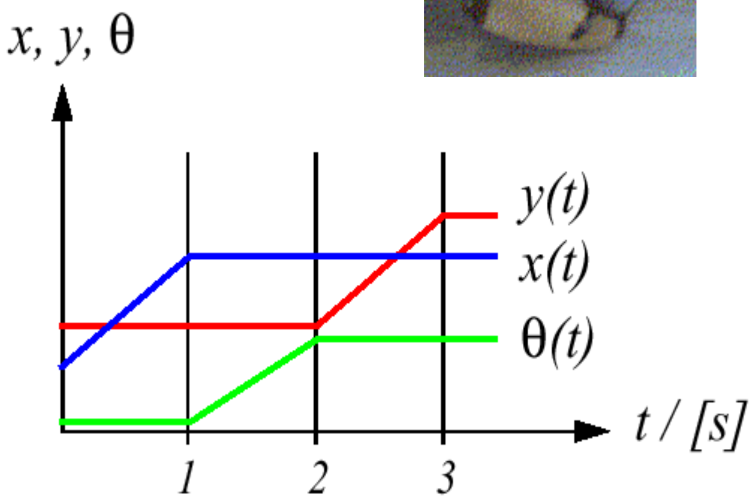
  make sure all intermediate points are reachable

  the joint rates are attainable

Path / Trajectory Considerations: Omnidirectional Drive

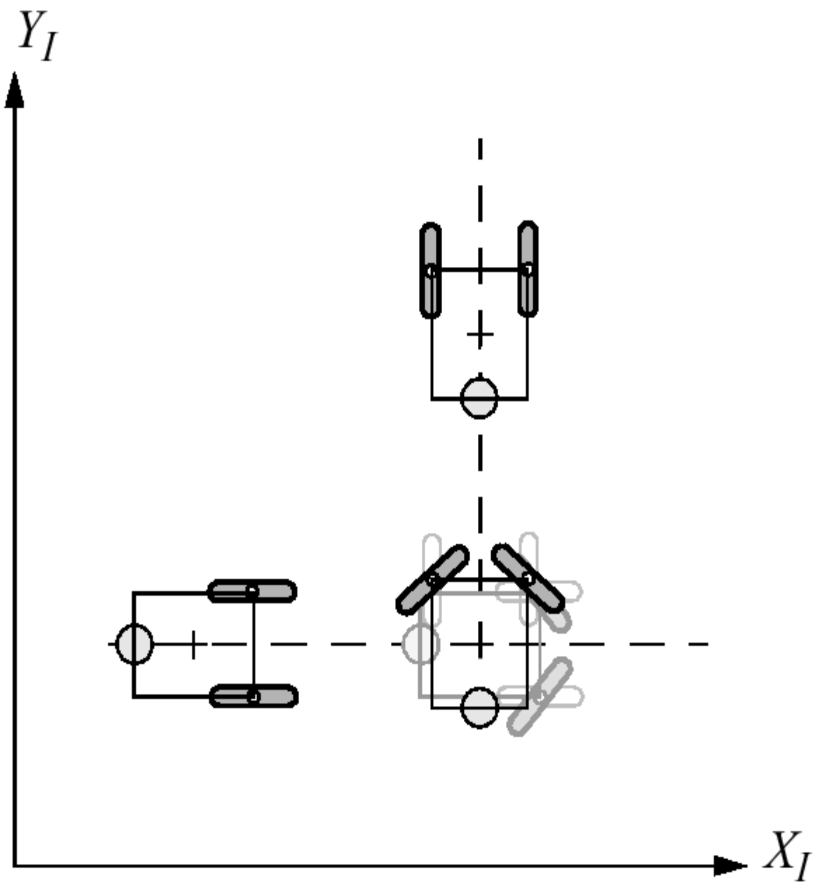
Move for 1s with constant speed along X, change orientation Counterclockwise by 90 degrees in 1s, move for 1s with constant Speed along Y – wheels can move and roll – omni-directional drive

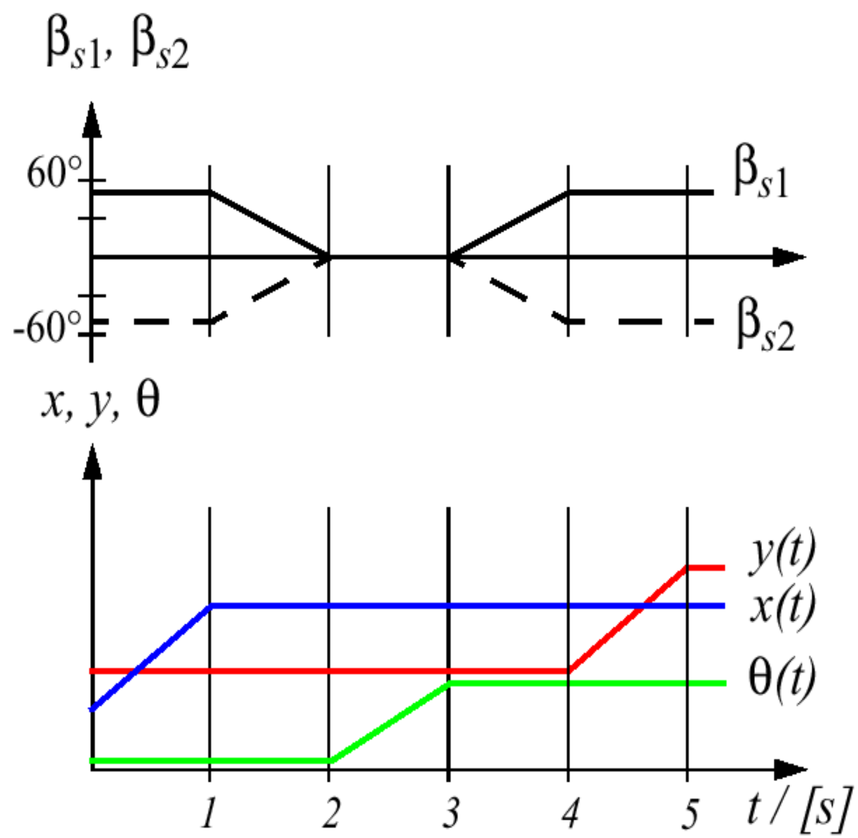




Path / Trajectory Considerations: Two-Steer

Move for 1s with constant speed along X, rotate steered wheels by -50/50 degrees; change orientation counterclockwise by 90 degrees in 1s, move for 1s with constant speed along Y





Beyond Basic kinematics

- So far we have considered only trajectories in time and space – no velocities
- When handling more dynamic scenarios velocities

  Become important, we need to design trajectory profiles which can be nicely followed

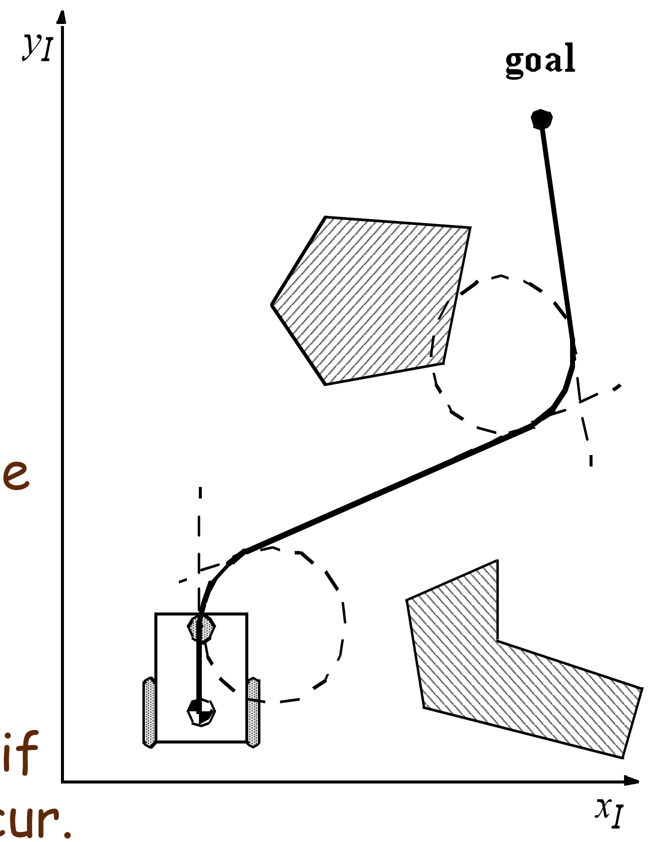
- Two main control approaches

  Open Loop Control

  Feedback Control

Motion Control: Open Loop Control

- Trajectory (path) divided in motion segments of clearly defined shape:
  - straight lines and segments of a circle.
- Control problem:
  - pre-compute a smooth trajectory based on line and circle segments
- Disadvantages:
  - It is not at all an easy task to pre-compute a feasible trajectory
  - limitations and constraints of the robots velocities and accelerations
  - does not adapt or correct the trajectory if dynamical changes of the environment occur.
  - The resulting trajectories are usually not smooth

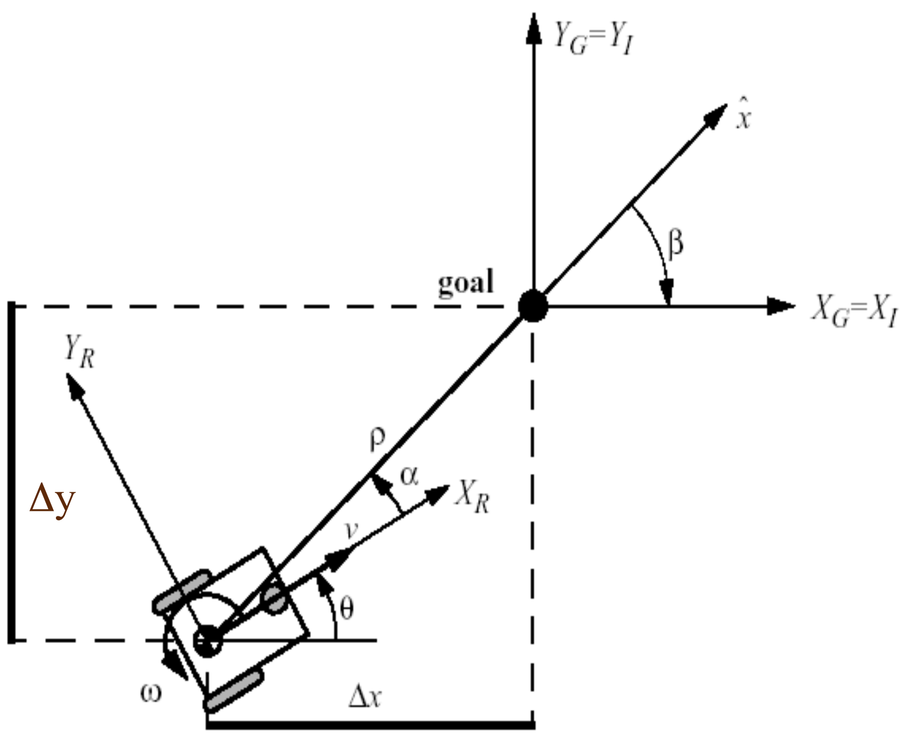


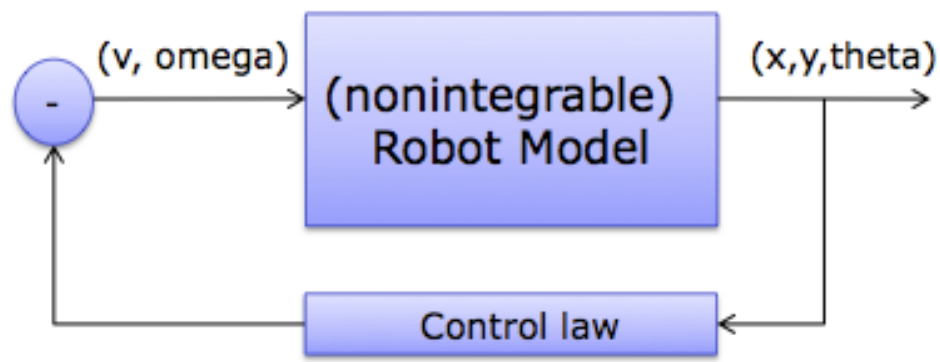
Motion Control: Open Loop

- Problem: given initial and final configuration of the robot, compute the path as a sequence of predefined motion segments – segments of straight line and circle (Dubins car, Reeds-Shep car)

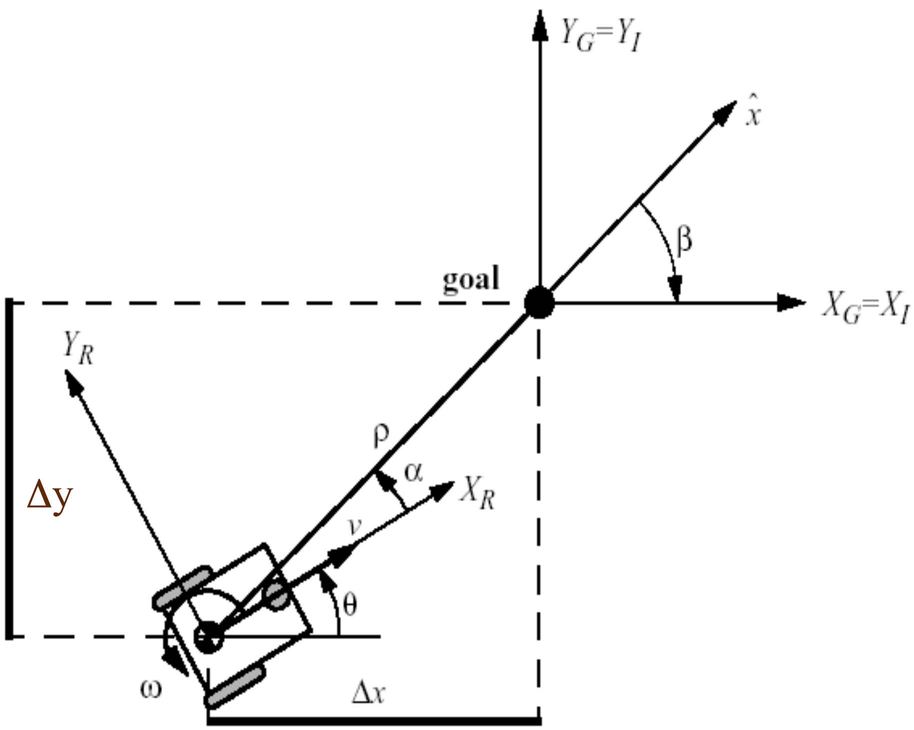
Feedback control

- More suitable alternative
- Use state feedback controller
- At each instance of time compute a control law
- Given the current error between current and desired position





Kinematic Position Control



The kinematic of a differential drive mobile robot described in the initial frame $\{x_I, y_I, \theta\}$ is given by,

$$
{}^{I}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}
$$

relating the linear velocities in the direction of the x_I and y_I of the initial frame.

Let α denote the angle between the x_R axis of the robots reference frame and the vector connecting the center of the axle of the wheels with the final position.

Motion Control (kinematic control)

- The objective of a kinematic controller is to follow a trajectory described by its position and/or velocity profiles as function of time.
- Motion control is not straight forward because mobile robots are non-holonomic systems.
- However, it has been studied by various research groups and some adequate solutions for (kinematic) motion control of a mobile robot system are available.
- Most controllers are not considering the dynamics of the system

Motion Control - Streering to a point

- Consider our robot

$$\dot{x} = v\cos\theta$$
$$\dot{y} = v\sin\theta$$
$$\dot{\theta} = \omega$$

- To steer the robot to desired position $[x_g, y_g]$

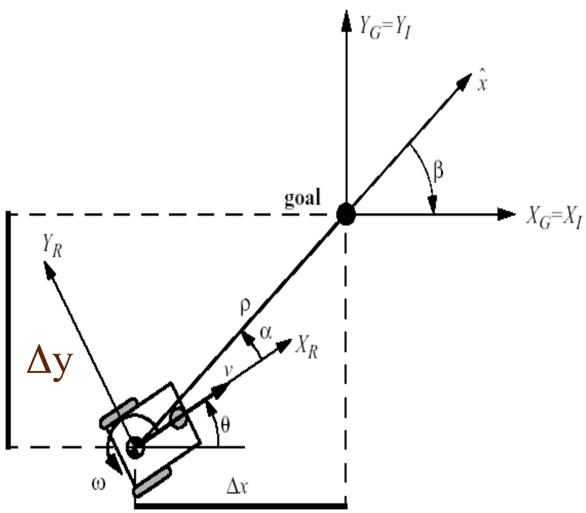
$$v = K_v\sqrt{(x - x_g)^2 + (y - y_g)^2}$$

$$\omega = \tan^{-1}\frac{y_g - y}{x_g - x}$$

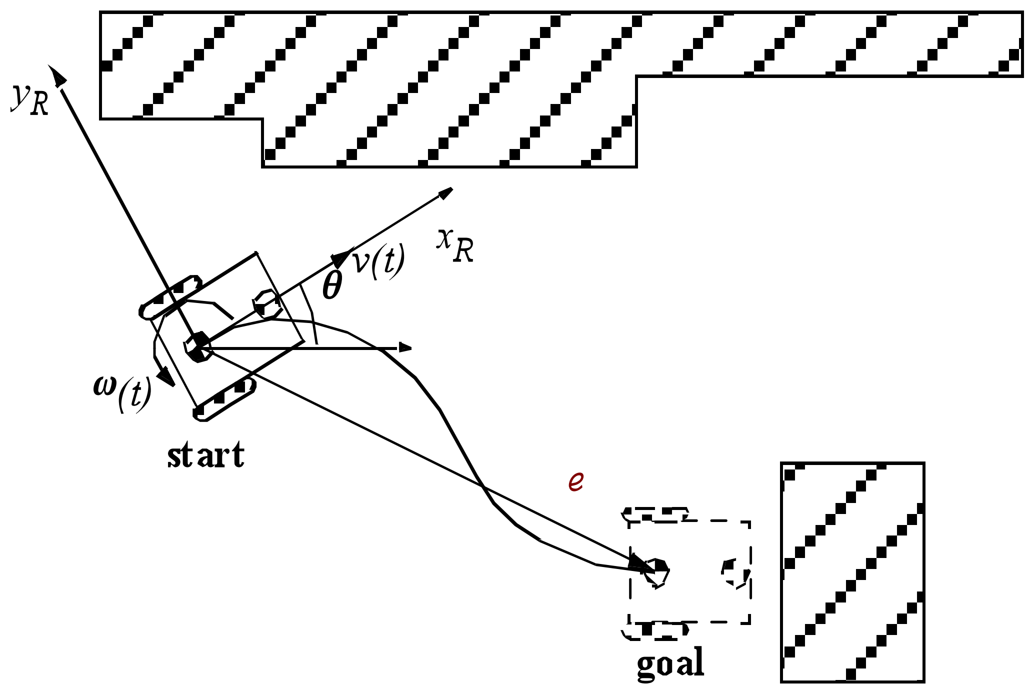
Motion Control: Steering to a pose

- Set intermediate positions lying on the requested path.
- Given a goal how to compute the control commands for
- linear and angular velocities to reach the desired configuration

- Problem statement
- Given arbitrary position and orientation of the robot $[x, y, \theta]$
  how to reach desired goal orientation and position $[x_g, y_g, \theta_g]$



Motion Control: Feedback Control, Problem Statement



- Find a control matrix *K*, if exists

$$K = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{bmatrix}$$

- with $k_{ij}=k(t,e)$
- such that the control of $v(t)$ and $\omega(t)$

$$\begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} = K \cdot e = K \cdot {}^{R}\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

- drives the error e to zero. $\lim_{t \to \infty} e(t) = 0$

Motion Control:

Kinematic Position Control

• The kinematic of a differential drive mobile robot described in the initial frame $\{x_I, y_I, \theta\}$ is given by,

$$^{I}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

where and are the linear velocities in the direction of the x_I and y_I of the initial frame.

Let α denote the angle between the x_R axis of the robots reference frame and the vector connecting the center of the axle of the wheels with the final position.

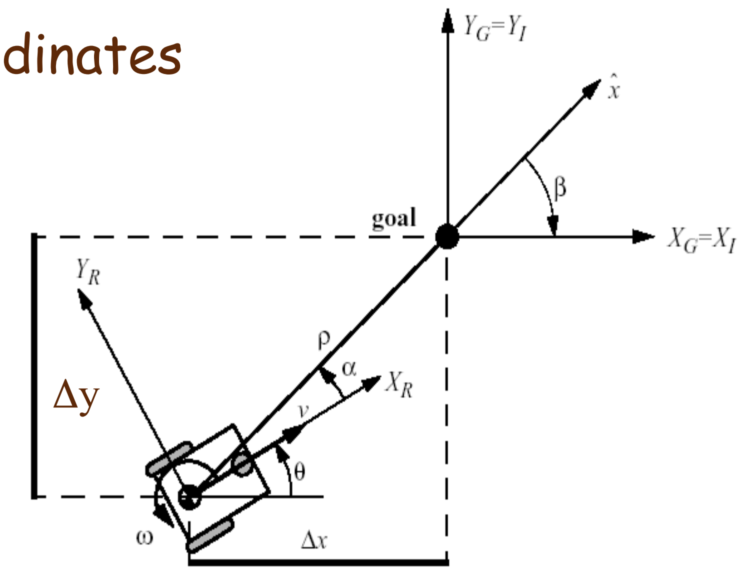
Kinematic Position Control: Coordinates Transformation

Coordinates transformation into polar coordinates with its origin at goal position:

$$\rho = \sqrt{\Delta x^2 + \Delta y^2}$$

$$\alpha = -\theta + \operatorname{atan2}(\Delta y, \Delta x)$$

$$\beta = -\theta - \alpha$$



System description, in the new polar coordinates

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -\cos\alpha & 0 \\ \dfrac{\sin\alpha}{\rho} & -1 \\ -\dfrac{\sin\alpha}{\rho} & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

For α from $I_1 = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 \\ -\dfrac{\sin\alpha}{\rho} & 1 \\ \dfrac{\sin\alpha}{\rho} & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

for $I_2 = (-\pi, -\pi/2] \cup (\pi/2, \pi]$

Kinematic Position Control: Remarks

- The coordinates transformation is not defined at $x = y = 0$; as in such a point the determinant of the Jacobian matrix of the transformation is not defined, i.e. it is unbounded
- For $\alpha \in I_1$ the forward direction of the robot points toward the goal, for $\alpha \in I_2$ it is the backward direction.
- By properly defining the forward direction of the robot at its initial configuration, it is always possible to have $\alpha \in I_1$ at $t=0$. However this does not mean that α remains in I_1 for all time t.

Kinematic Position Control: The Control Law

- It can be shown, that with

$$v = k_\rho \rho \qquad \omega = k_\alpha \alpha + k_\beta \beta$$

the feedback controlled system

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -k_\rho \rho \cos\alpha \\ k_\rho \sin\alpha - k_\alpha \alpha - k_\beta \beta \\ -k_\rho \sin\alpha \end{bmatrix}$$

- will drive the robot to $(\rho, \alpha, \beta) = (0,0,0)$
- The control signal *v* has always constant sign,
  - the direction of movement is kept positive or negative during movement
  - parking maneuver is performed always in the most natural way and without ever inverting its motion.

- Question: How to select the constant parameters
- to achieve that the error will go to zero

- Digression - review

Previously - Eigenvalues and Eigenvectors

For the previous example

$$\lambda_1 = -1, x_1 = [1, 1]^T \qquad \lambda_2 = -2, x_2 = [5, 2]^T$$

We will get special solutions to ODE $\dot{\mathbf{u}} = A\mathbf{u}$

$$A\mathbf{u} = e^{\lambda_1 t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \mathbf{u} = e^{\lambda_2 t} \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

Their linear combination is also a solution (due to the linearity of $\dot{\mathbf{u}} = A\mathbf{u}$)

$$\mathbf{u} = c_1 e^{\lambda_1 t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 e^{\lambda_1 t} \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

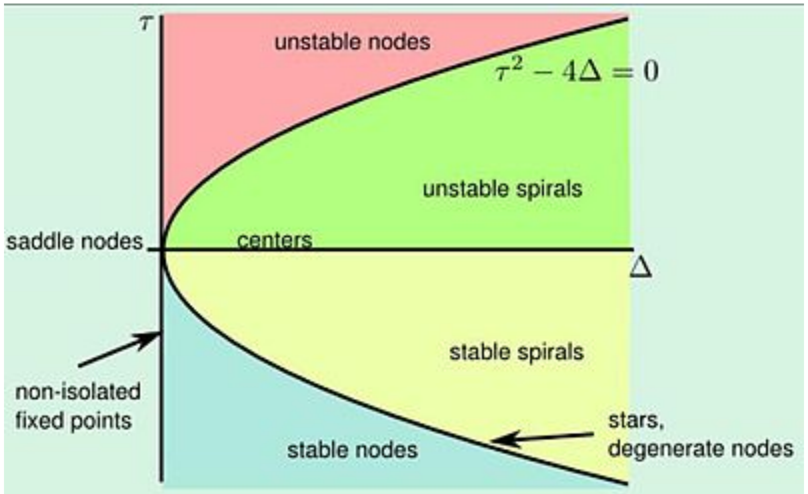
In the context of diff. equations – special meaning
Any solution can be expressed as linear combination
Individual solutions correspond to modes

Eigenvalues of linear system

- Given linear system of differential equations

$$\dot{\mathbf{x}} = A\mathbf{x}$$

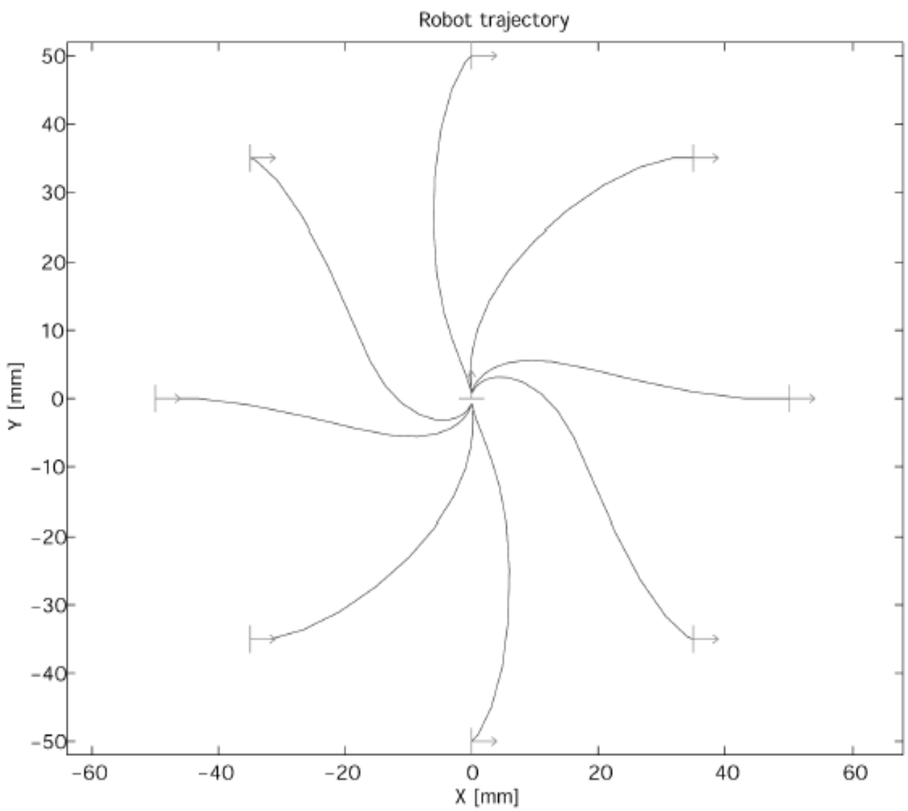
- For 2 dimesional system (A is 2 x 2), A has two eigenvalues
- Define $\Delta = \lambda_1 \lambda_2$ and $\tau = \lambda_1 + \lambda_2$

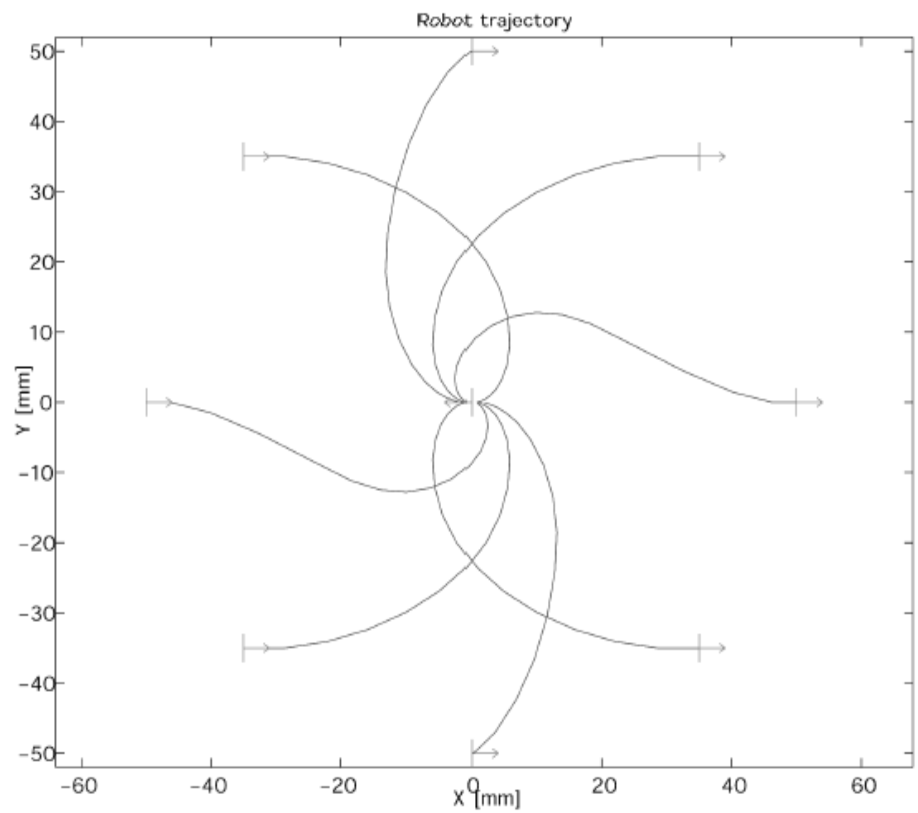


- if $\Delta < 0$ saddle node
-
- if $\Delta > 0$ we have two cases

  1. $\tau > 0$ eigenvalues positive
  2. $\tau < 0$ eigenvalues negative : stable nodes of the system

Kinematic Position Control: Resulting Path





- Digression
- Notes of system linearization

Linearization

- But our system is not linear, e.g. cannot be written in the form

$$\dot{\mathbf{x}} = A\mathbf{x}$$

Some terminology

- We have derived kinematics equations of the robot

$$\begin{aligned}\dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{\theta} &= \omega\end{aligned}$$

- Non-linear differential equation $\dot{x} = f(x,u)$
- In our case

$$\dot{x} = f_1(x,y,\theta,v,\omega)$$

$$\dot{y} = f_2(x,y,\theta,v,\omega)$$

$$\dot{\theta} = f_3(x,y,\theta,v,\omega)$$

Jacobian Matrix

- Suppose you have two dim function

$$f(\mathbf{x}) = \begin{bmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \end{bmatrix}$$

- Gradient operator $\nabla_{\mathbf{x}} = \begin{bmatrix} \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} & \cdots & \frac{\partial}{\partial x_n} \end{bmatrix}^T$

- Jacobian is defined as

$$\mathbf{F}_{\mathbf{x}} = \begin{bmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial}{\partial x_1} & \cdots & \frac{\partial}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \cdots & \frac{\partial f_2}{\partial x_n} \end{bmatrix}$$

- Linearization of a function

$$F(\mathbf{x}) = F(\mathbf{x}_0) + J_F(\mathbf{x}_0) d\mathbf{x}$$

Linearization

- But our system is not linear, e.g. cannot be written in the form

$$\dot{\mathbf{x}} = A\mathbf{x}$$

- Linearization of the system

$$\dot{\mathbf{x}} = J_F(\mathbf{x}_0)d\mathbf{x} + F(\mathbf{x}_0)$$

$$F(\mathbf{x}) = F(\mathbf{x}_0) + J_F(\mathbf{x}_0)d\mathbf{x}$$

Kinematic Position Control: Stability Issue

- Continuous linear time-invariant system is exponentially stable if and only of the system has eigenvalues (i.e. poles of input-to-output systems) with strictly negative real parts
- Exponential Stability is a form of asymptotic stability
- In practice the system will not "blow up" give unbounded output, when given an finite input and non-zero initial condition

Kinematic Position Control: Stability Issue

- It can further be shown, that the closed loop control system is locally exponentially stable if

$$k_\rho > 0 \;\; ; \;\; k_\beta < 0 \;\; ; \;\; k_\alpha - k_\rho > 0$$

- Proof: linearize around equilibrium
  for small x –> $\cos x = 1$, $\sin x = x$

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -k_\rho & 0 & 0 \\ 0 & -(k_\alpha - k_\rho) & -k_\beta \\ 0 & -k_\rho & 0 \end{bmatrix} \begin{bmatrix} \rho \\ \alpha \\ \beta \end{bmatrix} \qquad A = \begin{bmatrix} -k_\rho & 0 & 0 \\ 0 & -(k_\alpha - k_\rho) & -k_\beta \\ 0 & -k_\rho & 0 \end{bmatrix}$$

- and the characteristic polynomial of the matrix A of all roots have negative real parts.

$$(\lambda + k_\rho)(\lambda^2 + \lambda(k_\alpha - k_\rho) - k_\rho k_\beta)$$