3D Reconstruction from Scene Knowledge

Multiple-View Reconstruction from Scene Knowledge

**SYMOMETRY & MULTIPLE-VIEW GEOMETRY**
- Fundamental types of symmetry
- Equivalent views
- Symmetry based reconstruction

**MULTIPLE-VIEW MULTIPLE-OBJECT ALIGNMENT:**
- Scale alignment: adjacent cells in a single view
- Scale alignment: same cell in multiple views

**APPLICATIONS & Experiments:**
- Building 3D geometric models
- Symmetry extraction, detection, and matching
- Camera calibration

*Scene knowledge and symmetry*

Symmetry is ubiquitous in man-made or natural environments

*Scene knowledge and symmetry*

Parallelism (vanishing point)  Orthogonality

Translational invariance  Rotation and reflection
Wrong assumptions

- Ames room illusion
- Necker’s cube illusion

SYMOMETRY & MULTIPLE-VIEW GEOMETRY

- Why does an image of a symmetric object give away its structure?
- Why does an image of a symmetric object give away its pose?
- What else can we get from an image of a symmetric object?

Equivalent views from rotational symmetry

Equivalent views from reflectional symmetry
Equivalent views from translational symmetry

Recovery using rectangular structure

* Recovery of the camera displacement from a planar rectangular structure
* Rectangle – reflectional symmetry
* Square – rotational and reflectional symmetric
* Special cases of symmetric objects

Camera pose recovery

- Assume partially calibrated camera
  \[ K = \begin{bmatrix} f & 0 & n_x \\ 0 & f & n_y \\ 0 & 0 & 1 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]

- Explicitly parametrize the homography \( x \sim HX \)
  \[ \lambda x = \begin{bmatrix} f_{x1} & f_{x2} & f_{x3} \\ f_{y1} & f_{y2} & f_{y3} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = H \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} \]

- Explicitly parametrize the unknown structure
  \[ S = \begin{bmatrix} 0 & 0 & n_x & n_y \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \]

Homography Estimation

- Decouple known and unknown structure
  \[ S = S_0 S_2 = \begin{bmatrix} r_{10} & 0 & 0 \\ 0 & r_{20} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \lambda x \end{bmatrix} \]

- Estimate the unknown homography \( H_{13} \)
  \[ H_{13} = \begin{bmatrix} s_{10} f_{12} f_{13} \\ s_{20} f_{22} f_{23} \\ s_{30} f_{32} f_{33} \end{bmatrix} = \begin{bmatrix} h_{13} & h_{12} & h_{11} \\ h_{23} & h_{22} & h_{21} \\ h_{33} & h_{32} & h_{31} \end{bmatrix} \]
Homography Factorization

- Exploiting orthogonality constraints
  \[ r_1^T r_2 = 0, \|r_1\| = \|r_2\| = 1 \]

- Directly estimate the focal length
  \[ f = \sqrt{\frac{h_{11} h_{22} + h_{22} h_{33} + h_{33} h_{11}}{h_{11} + h_{22} + h_{33}}} \]

- remaining parameters and final pose

Example

- With shared segment the pose can be reconciled and we obtain single consistent pose recovery up to scale and error \( \sim 3 \) degrees

Example for multiview matching

- Recovery of the camera displacement from a planar structure
Definition. A set of 3-D features $S$ is called a symmetric structure if there exists a non-trivial subgroup $G$ of $E(3)$ that acts on it such that for every $g$ in $G$, the map

$$g \in G : S \rightarrow S$$

is an (isometric) automorphism of $S$. We say the structure $S$ has a group symmetry $G$.

$$X = [x, y, z, 1]^T \in \mathbb{R}^4, \quad x = [x, y, z]^T \in \mathbb{R}^3$$

$$g_0 = \begin{bmatrix} R_0 & T_0 \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^{4 \times 4}, \quad R_0 = [R, 0] \in \mathbb{R}^{3 \times 3}$$

$$x \sim R_0 X, \quad g(x) \sim R_0 g_0 X$$

Detailed treatment in chapter 10 (some examples next)
Alignment of multiple symmetric structures

\[
\alpha = 0.7322 \quad \theta(N_1, N_2) = 90.36^\circ
\]
Alignment of multiple symmetric images

\[
\begin{bmatrix}
    x_i - \bar{x} \\
    y_i - \bar{y}
\end{bmatrix} = \alpha \begin{bmatrix}
    u_i - \bar{u} \\
    v_i - \bar{v}
\end{bmatrix}
\]

\(i = 1, 2, 3, 4\)

\(d_2 = \alpha\)

\(g_2 = (R_2, \alpha T_2)\)

\(g_2 \rightarrow g_2 g_1^{-1}\)

APPLICATION: Symmetry detection and matching

Extract, detect, match symmetric objects across images (over an arbitrary motion), and recover the camera poses.
Segmentation & polygon fitting

- Color-based segmentation (mean shift)
- Polygon fitting.

Symmetry verification & pose recovery

- Symmetry verification (rectangles)
- Single-view pose recovery

Symmetry-based matching

- Only one set of camera poses is consistent with all correctly detected symmetry objects

Pose and 3D recovery

Accuracy (aspect ratios)

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Multiple-view matching and recovery

APPLICATION: Building 3D geometric models

Building 3D geometric models (camera poses)

Building 3D geometric models (rendering)
APPLICATION: Calibration from symmetry

Calibrated homography $H = R + \frac{1}{2}t^tN^t$

Uncalibrated homography

$H = \kappa (R + \frac{1}{2}t^tN^t)\kappa^{-1}$

$H(\kappa T) = \kappa T'$

(vanishing point)

$S = \kappa^{-1}K^{-1}$

$KZ_{1,2}^T \kappa (\kappa T') = 0,$

$KZ_{2,3}^T \kappa (\kappa T') = 0,$

$KZ_{3,1}^T \kappa (\kappa T') = 0.$

SUMMARY

Multiple-view 3-D reconstruction in presence of symmetry

- Symmetry based algorithms are accurate, robust, and simple.
- Methods are baseline independent and object centered.
- Alignment and matching can and should take place in 3D space.
- Camera self-calibration and calibration are simplified and linear.

Related applications

- Using symmetry to overcome occlusion.
- Reconstruction and rendering with non-symmetric area.
- Large scale 3D map building of man-made environments.

Calibration with a rig is also simplified: we only need to know that there is sufficient symmetries, not necessarily the 3D coordinates of points on the rig.