Image Formation

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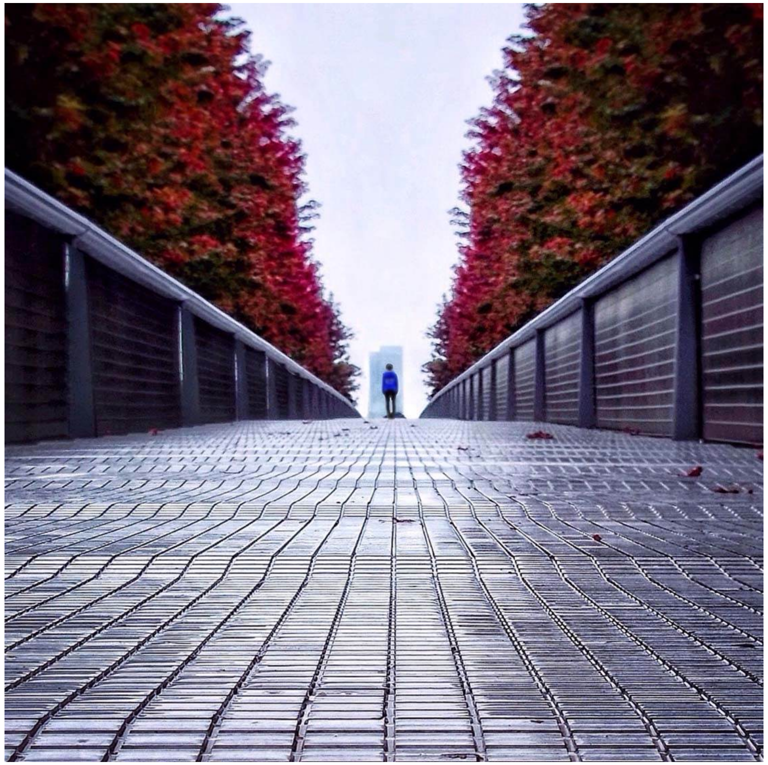
- pinhole camera
- perspective projection
- coordinate frames (world, camera, sensor)
- thin lens model, field of view, depth of field
- radial distortion

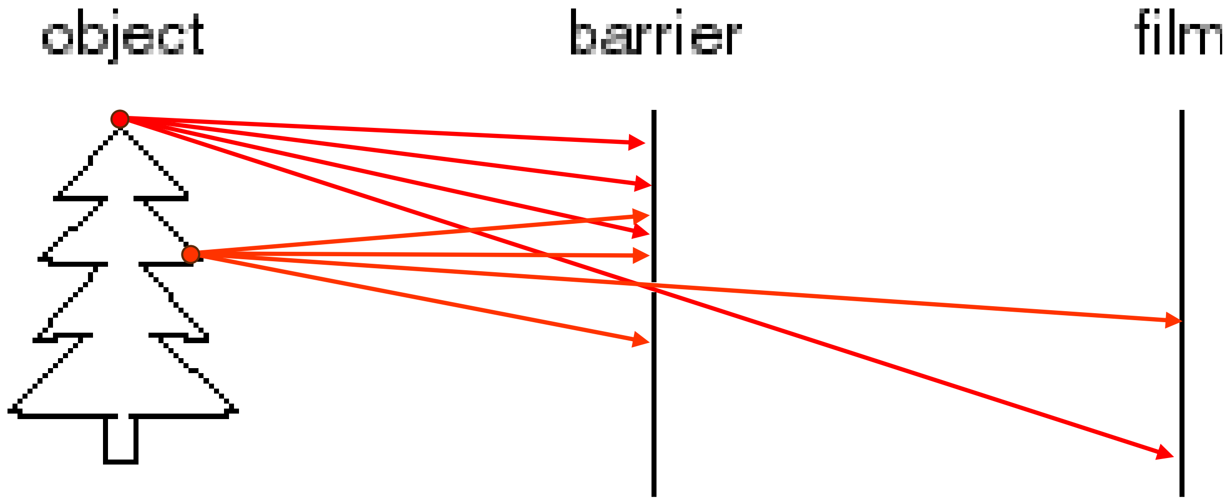
Image Formation

- If the object is our lens the refracted light causes the images
- How to integrate the information from all the rays being reflected from the single point on the surface ?
- Depending in their angle of incidence, some are more refracted then others – refracted rays all meet at the point – basic principles of lenses
- Also light from different surface points may hit the same lens point but they are refracted differently - Kepler's retinal theory


photopills.com



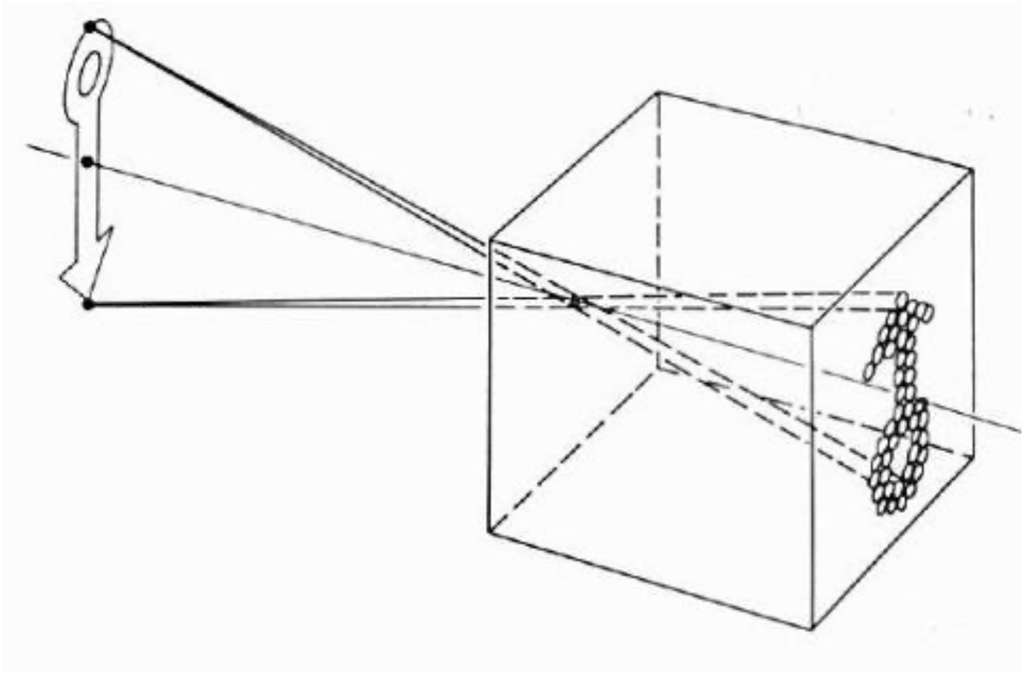
Pinhole camera



- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening is known as the **aperture**

Slide by Steve Seitz

Pinhole camera model

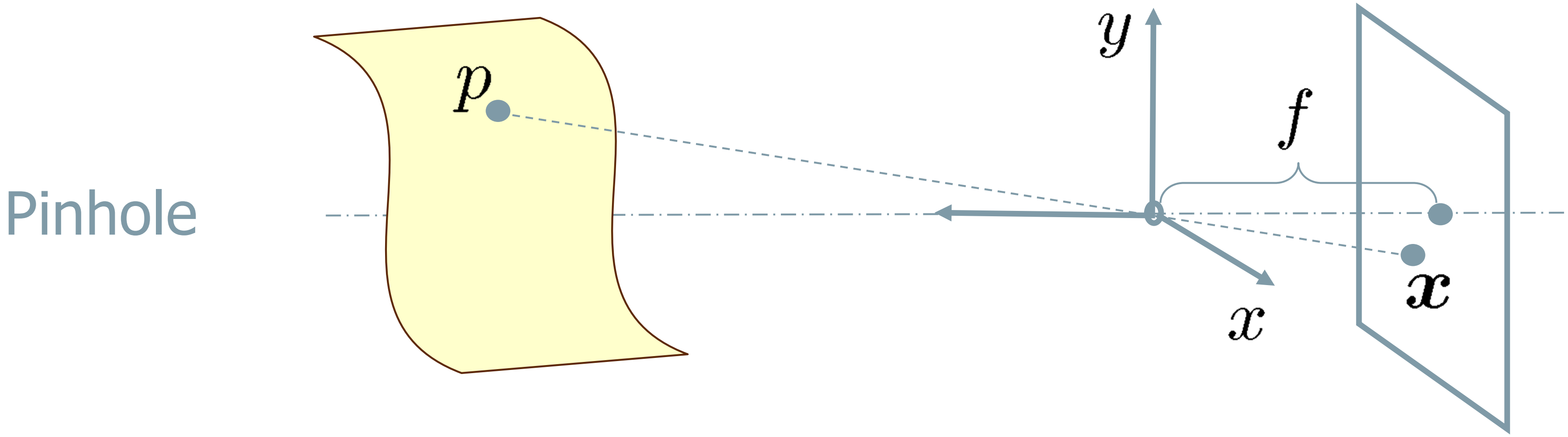


- Pinhole model:
  - Captures **pencil of rays** – all rays through a single point
  - The point is called **Center of Projection (focal point)**
  - The image is formed on the **Image Plane**

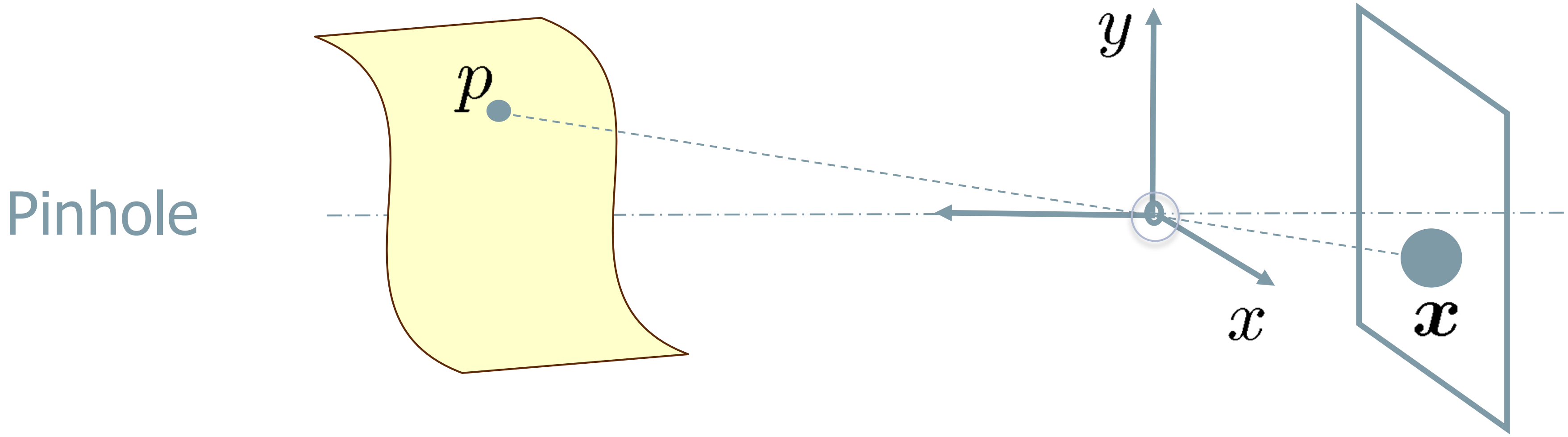
Slide by Steve Seitz

Limitations of Pinhole Camera Model

Ideal pinhole - one point small amount of light passes



Finite pinhole – one point generates - region - blurry



Camera obscura Images

- DIY Idealized pinhole, how to make your room pinhole camera

Source: https://www.diyphotography.net/build-room-sized-camera-obscura-timelapse/

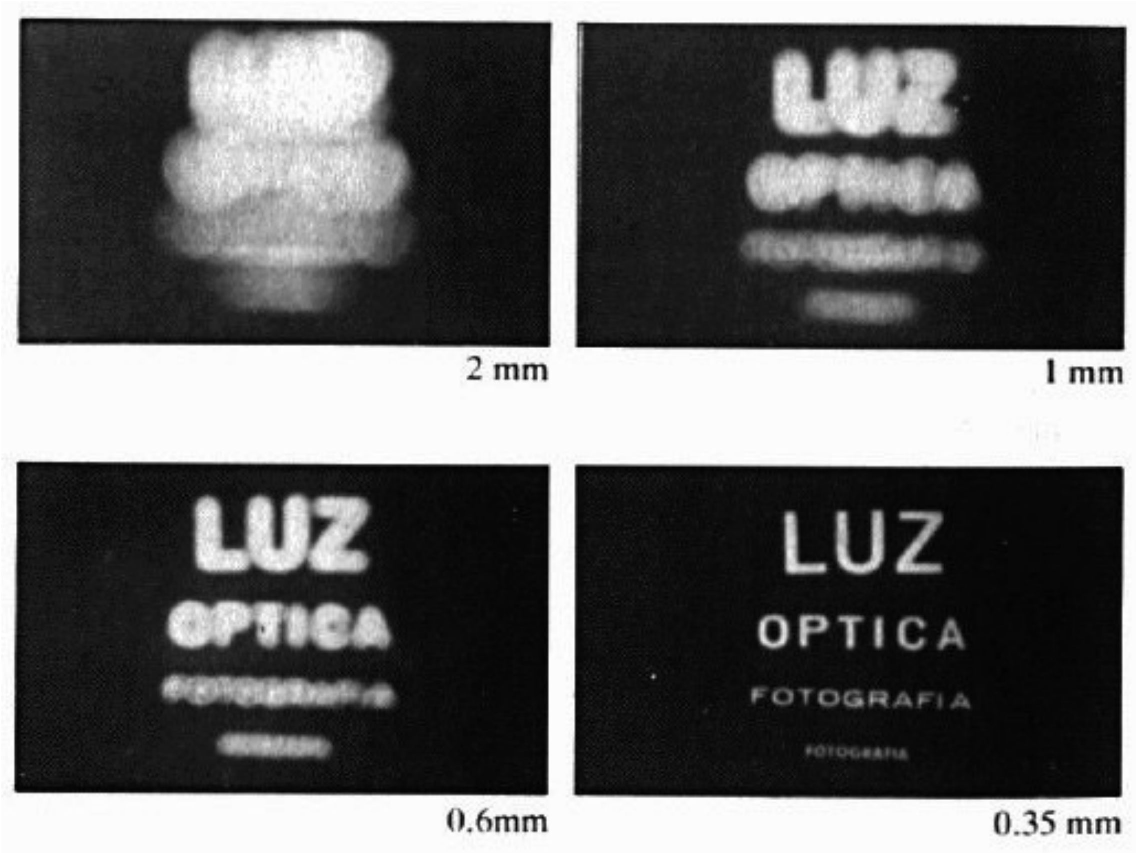
Home-made pinhole camera

Why so blurry?

http://www.debevec.org/Pinhole/

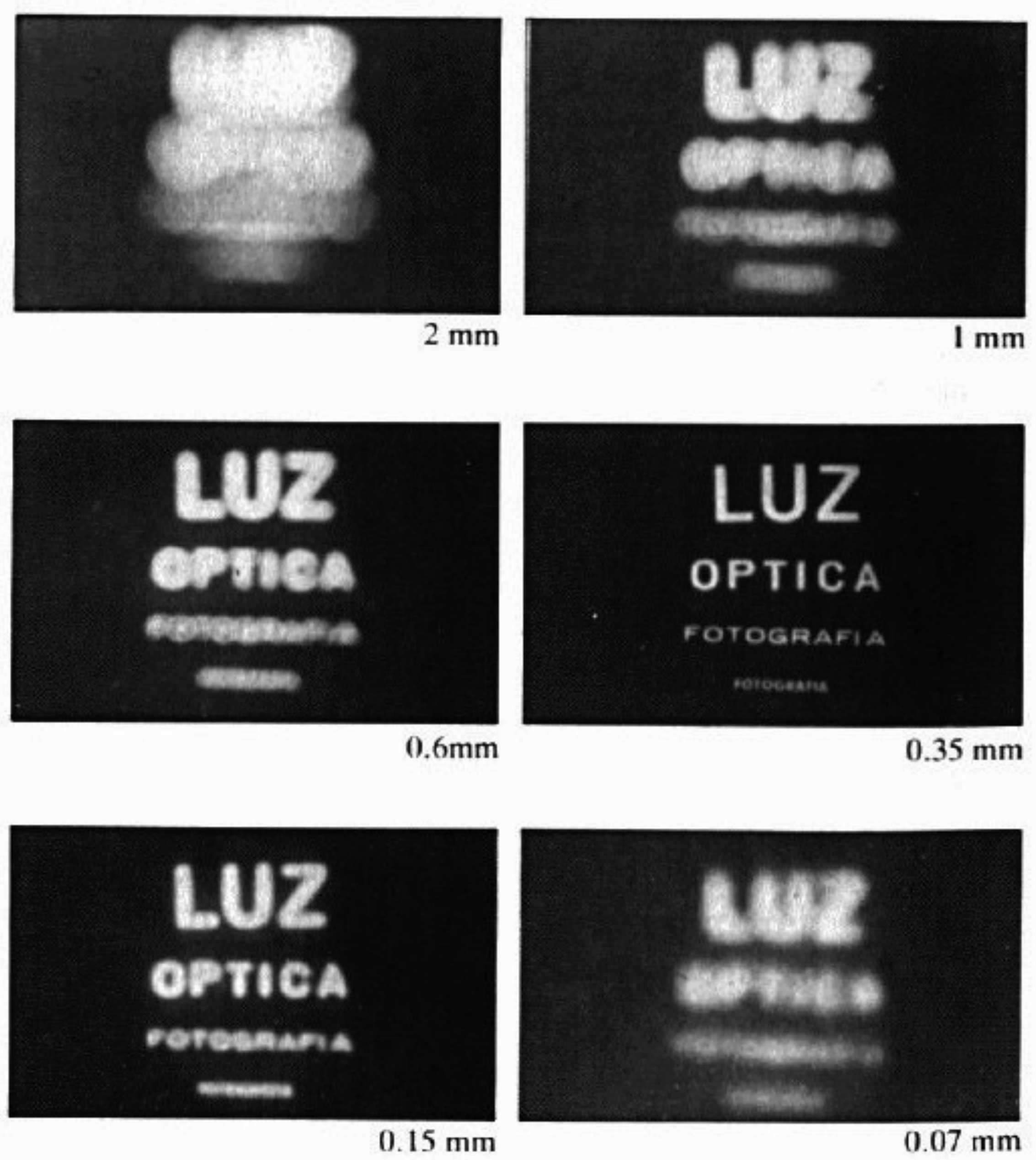
Slide by A. Efros

Shrinking the aperture

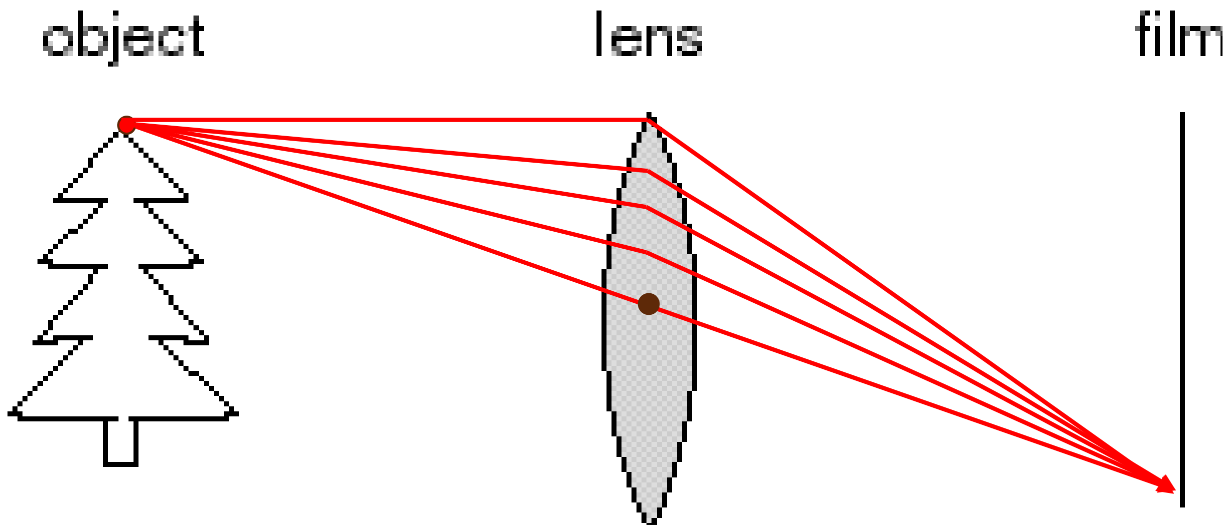


- Why not make the aperture as small as possible?
  - Less light gets through
  - Diffraction effects…

Shrinking the aperture



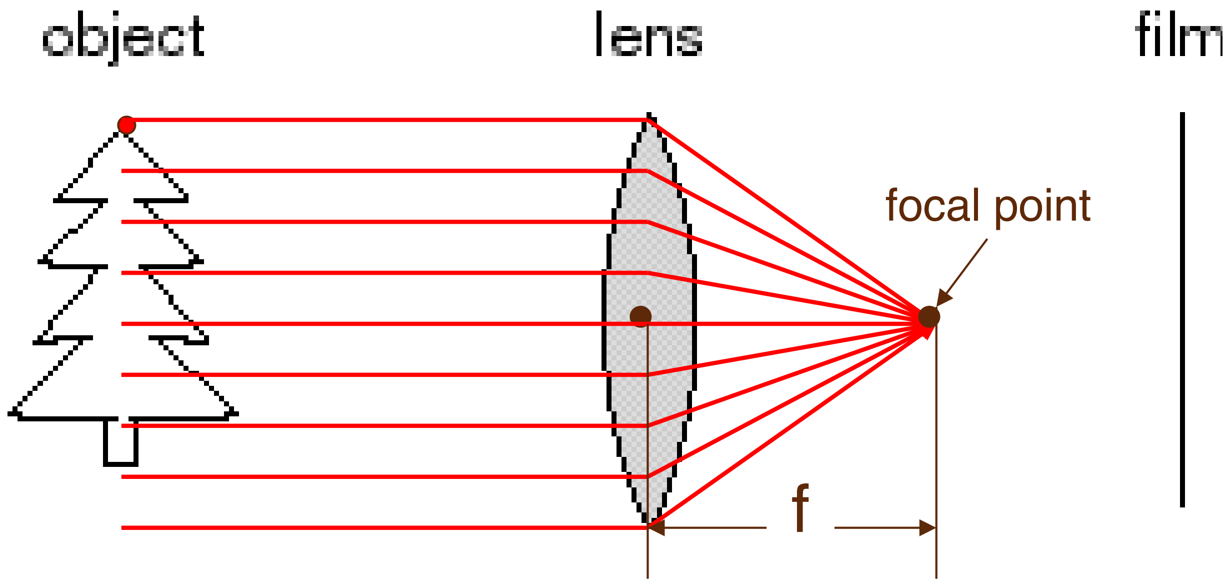
Adding a lens



- A lens focuses light onto the film
  - Thin lens model:
    - Rays passing through the center are not deviated (pinhole projection model still holds)
    - Each point in the image is illuminated by a cone of light

Slide by Steve Seitz

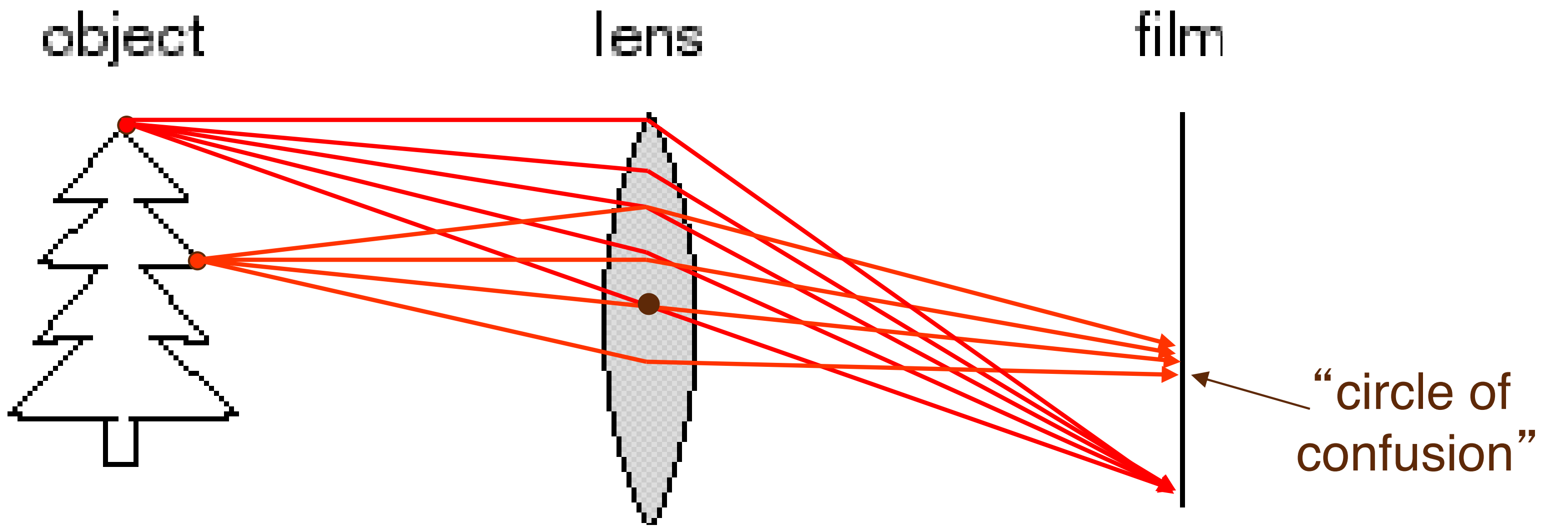
Adding a lens



- A lens focuses light onto the film
  - Thin lens model:
    - Rays passing through the center are not deviated (pinhole projection model still holds)
    - All parallel rays converge to one point on a plane located at the *focal length f – some function of radius of the lens and refraction index*

Slide by Steve Seitz

Adding a lens



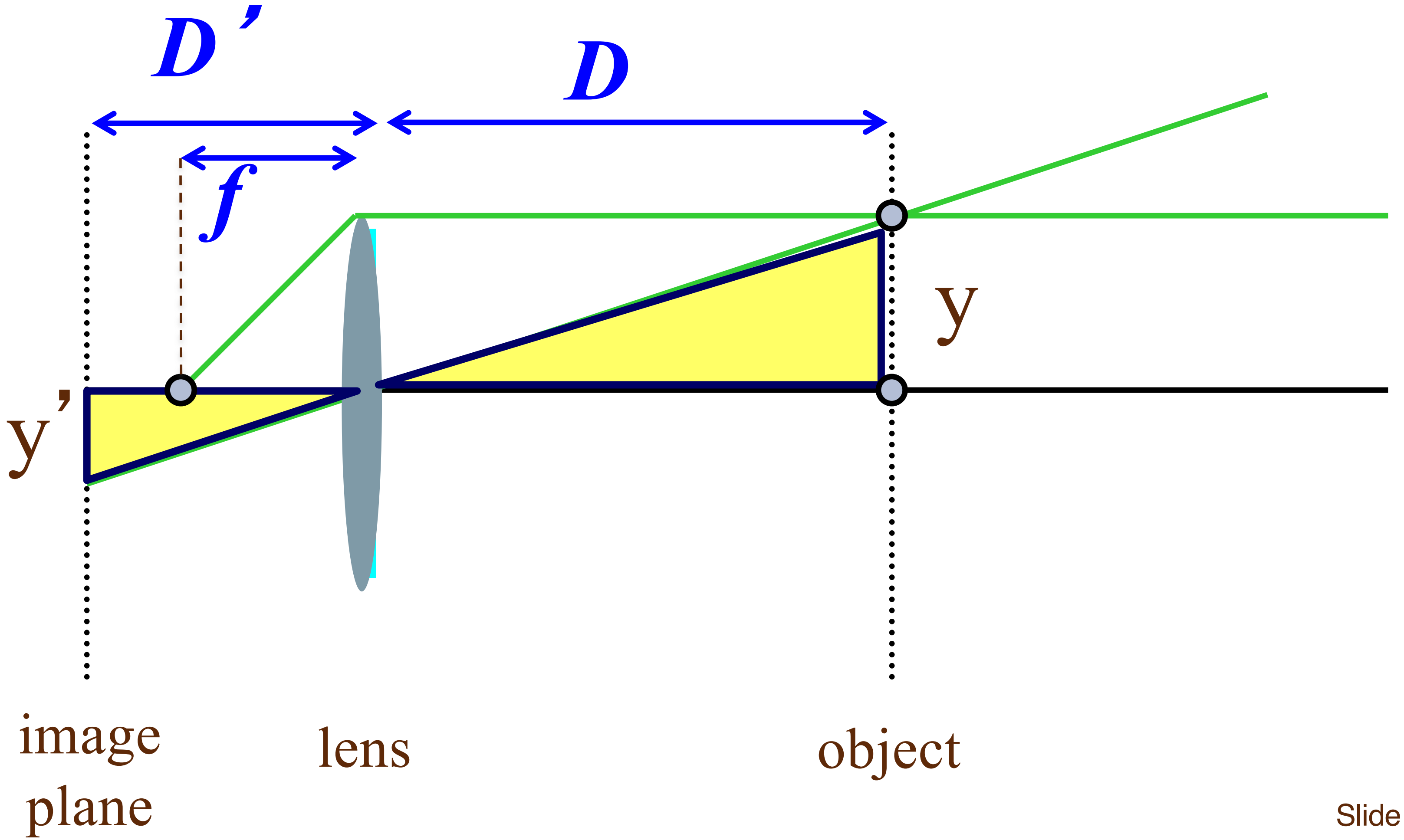
- A lens focuses light onto the film
  - There is a specific distance at which objects are "in focus"
    - other points project to a "circle of confusion" in the image

Slide by Steve Seitz

Thin lens formula

Similar triangles everywhere!

$$\frac{y'}{y} = \frac{D'}{D}$$



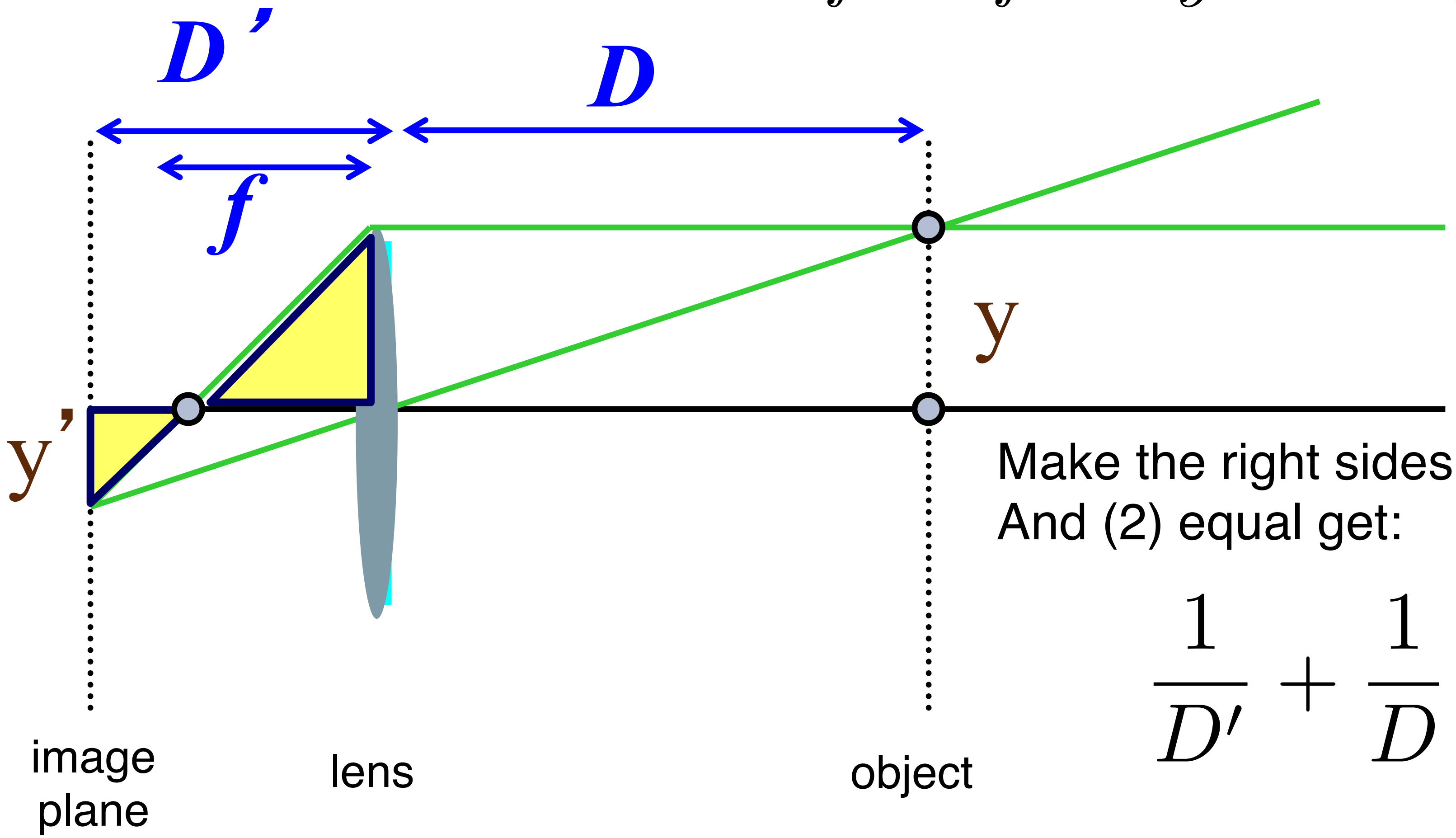
Slide by Frédo Durand

Thin lens formula

$$\frac{y'}{y} = \frac{D'}{D} \qquad (1)$$

Similar triangles everywhere!

$$\frac{y'}{D' - f} = \frac{y}{f} \rightarrow \frac{y}{y'} = \frac{D' - f}{f} \qquad (2)$$



Make the right sides on (1)
And (2) equal get:

$$\frac{1}{D'} + \frac{1}{D} = \frac{1}{f}$$

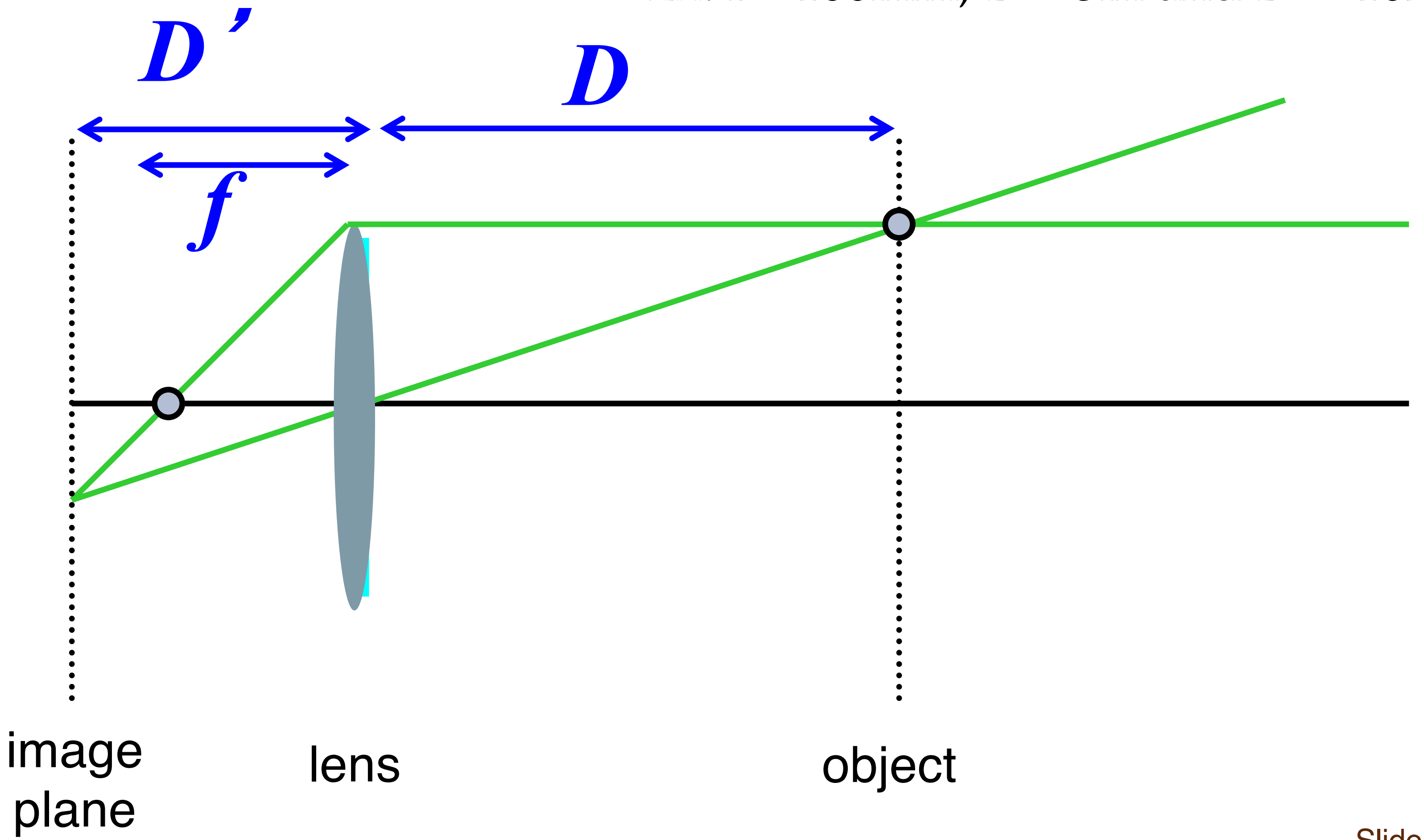
Slide by Frédo Durand

Thin lens formula

$$\frac{1}{D'} + \frac{1}{D} = \frac{1}{f}$$

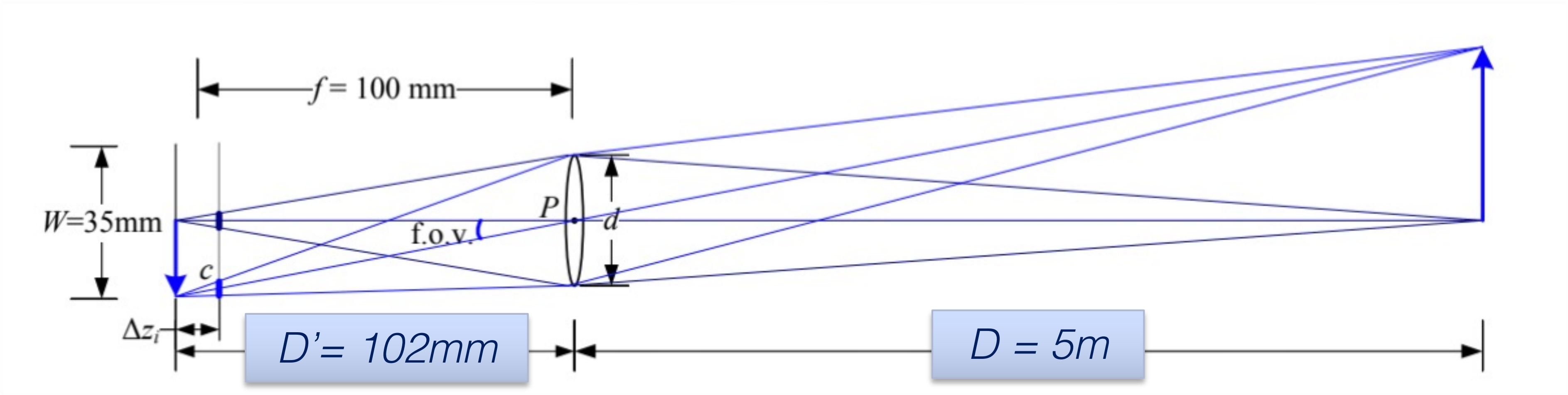
Any point satisfying the thin lens equation is in focus. If the focal plane D' is moved closer, circle of confusion.

Ex: f = 100mm, D = 5m and D' = 102mm



Slide by Frédo Durand

Thin Lens equation



$$\frac{1}{D'} + \frac{1}{D} = \frac{1}{f}$$

- Distance behind the lens at which points becomes in focus depends on the distance of the point from the lens
- In real camera lenses, there is a range of points which are brought into focus at the same distance

- If we let $D \rightarrow \infty$ such that all objects are at focus, we will move the image plane for $D' = f$
- we can think of pinhole as lens with focal length *f*

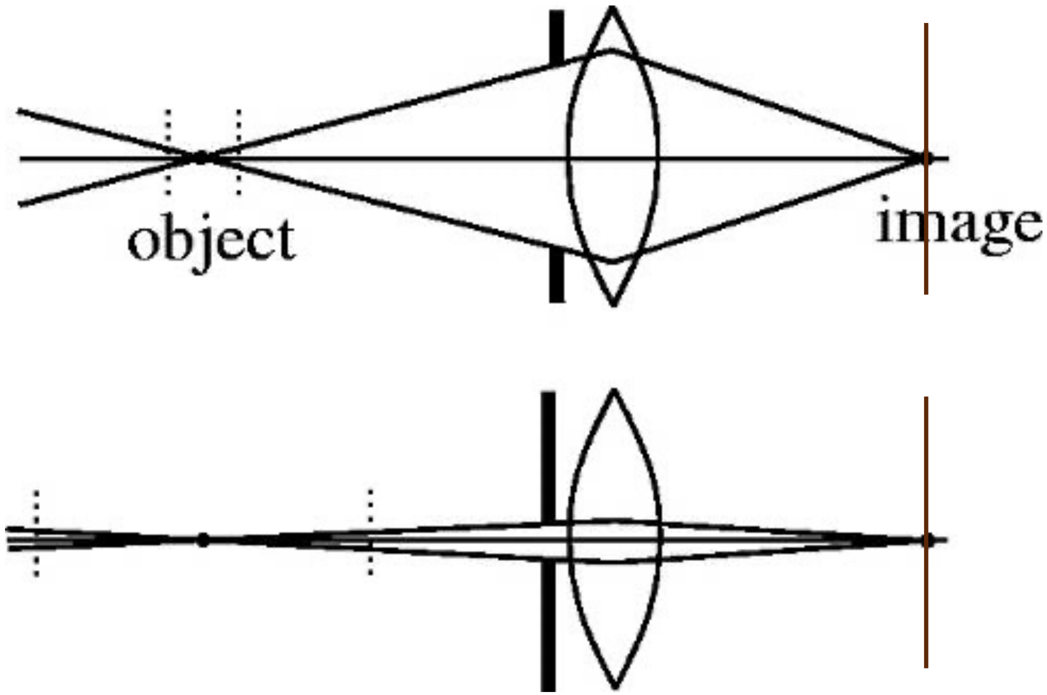
Image R. Szeliski

Depth of Field

http://www.cambridgeincolour.com/tutorials/depth-of-field.htm

Slide by A. Efros

How can we control the depth of field?



- Changing the aperture size affects depth of field
  - A smaller aperture increases the range in which the object is approximately in focus
  - But small aperture reduces amount of light – need to increase exposure

Slide by A. Efros

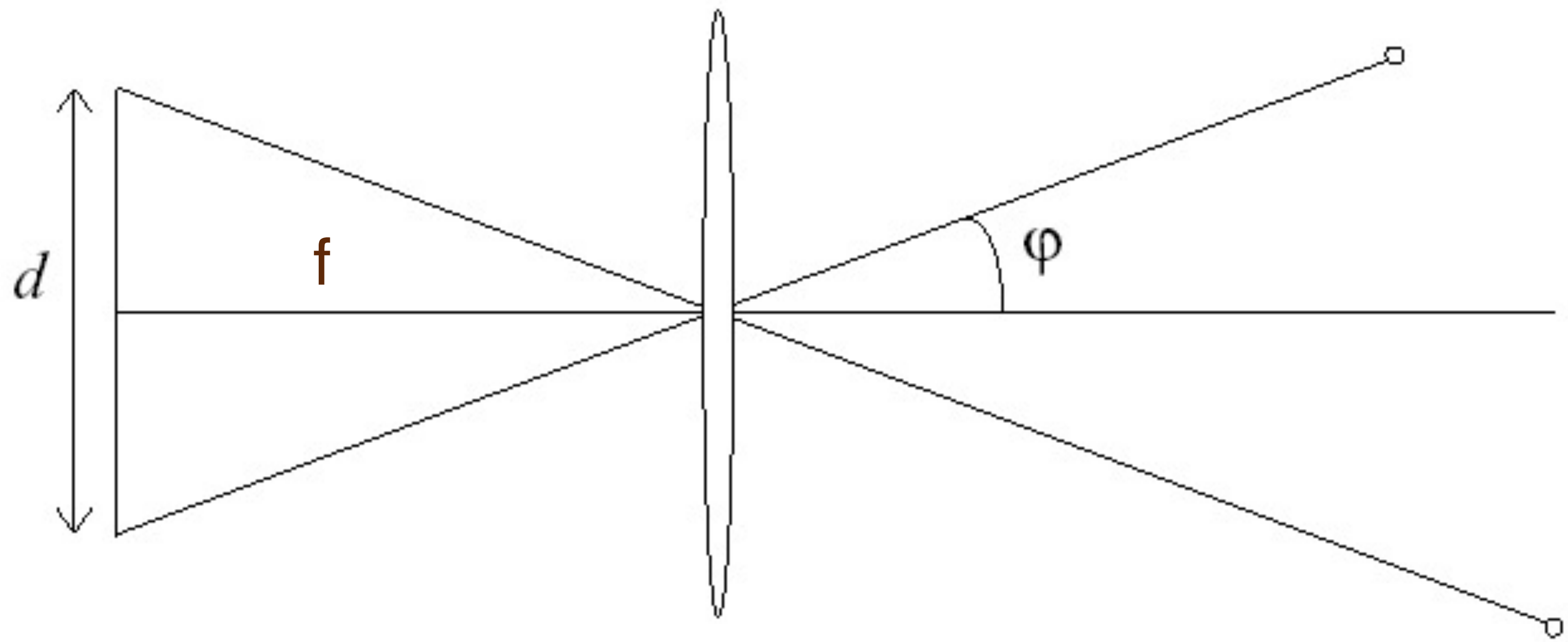
Varying the aperture

Large aperture = small DOF

Small aperture = large DOF

Slide by A. Efros

Field of View



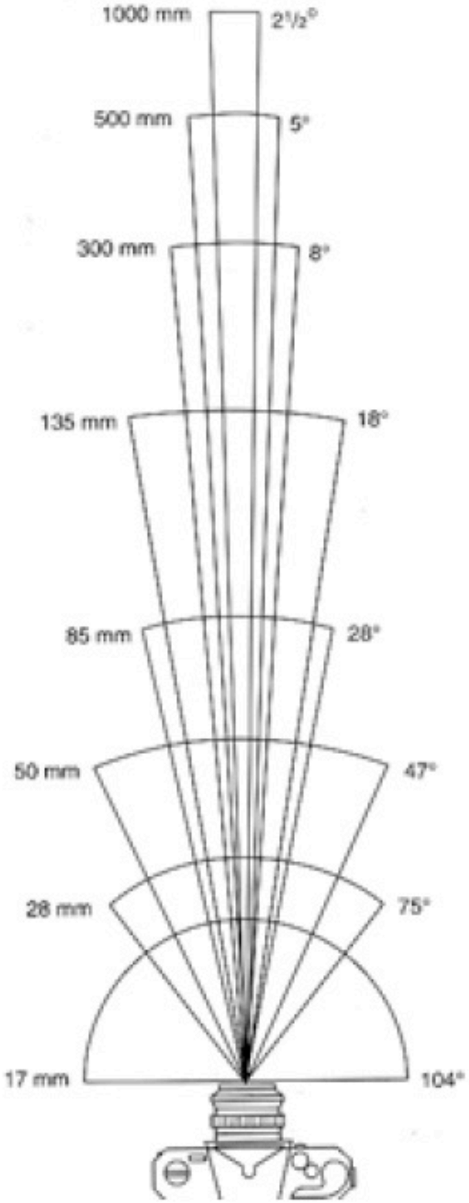
FOV depends on focal length and size of the camera retina

$$\varphi = \tan^{-1}\left(\frac{d}{2f}\right)$$

How to get a bigger FOV ?

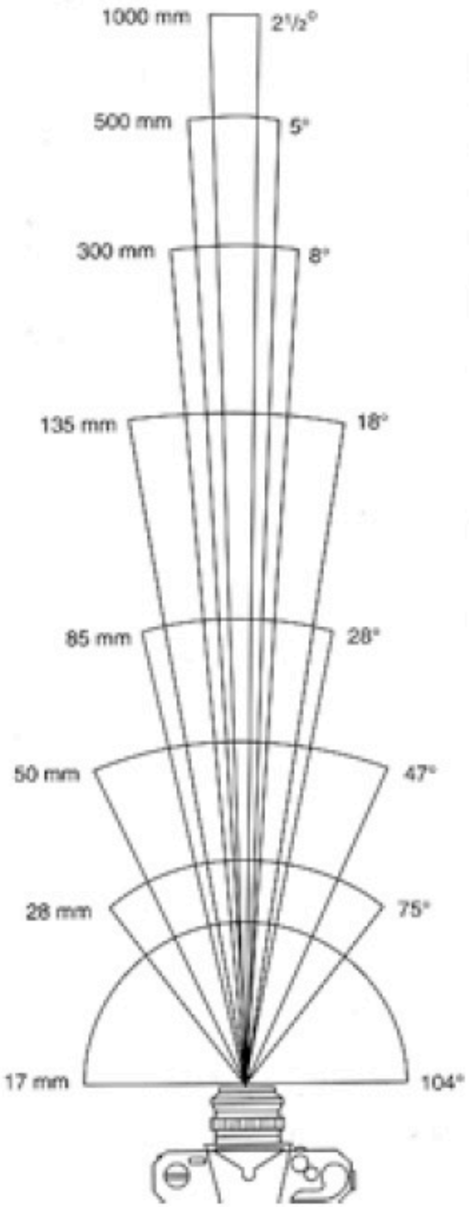
Slide by A. Efros

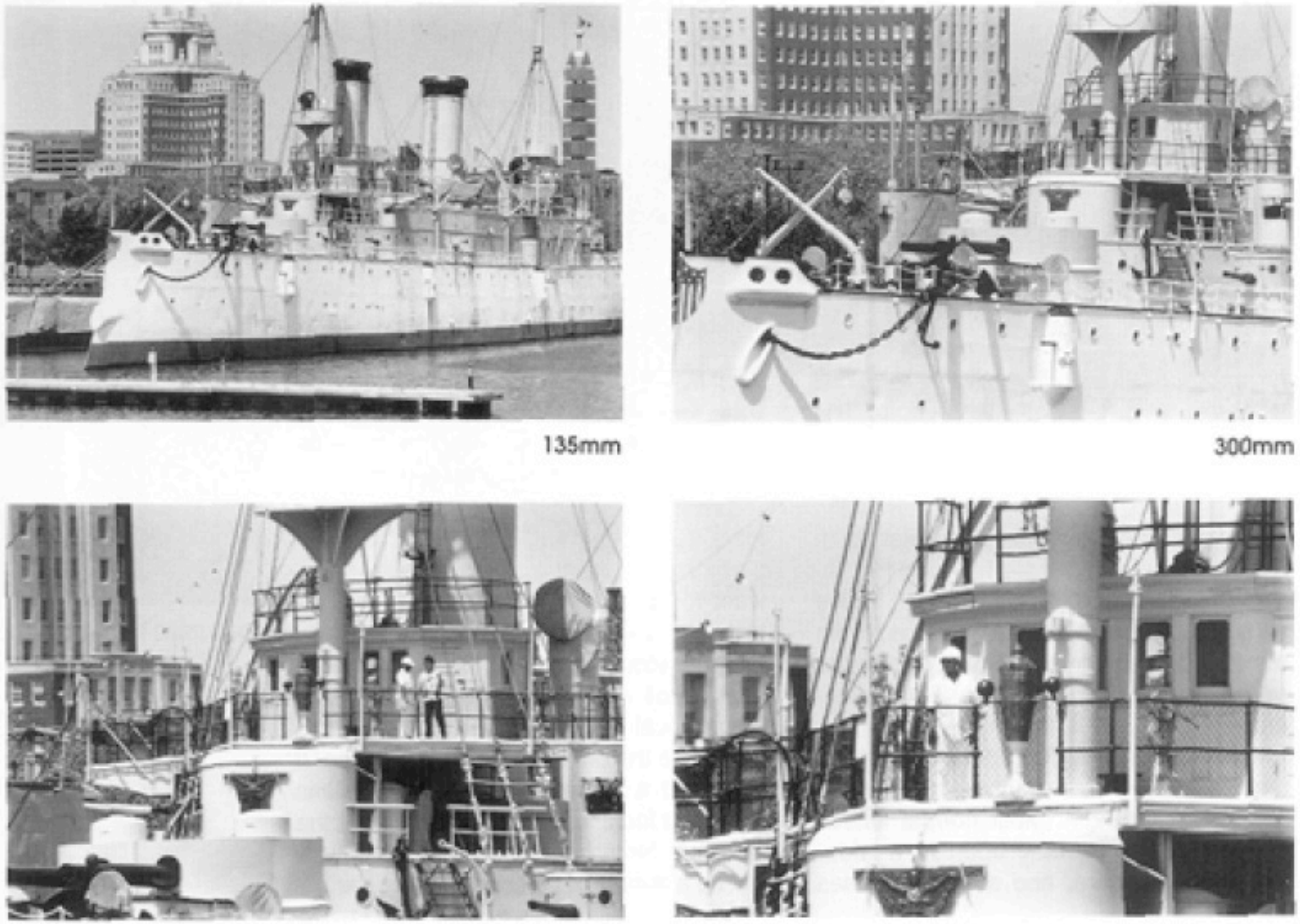
Field of View



Slide by A. Efros

Field of View

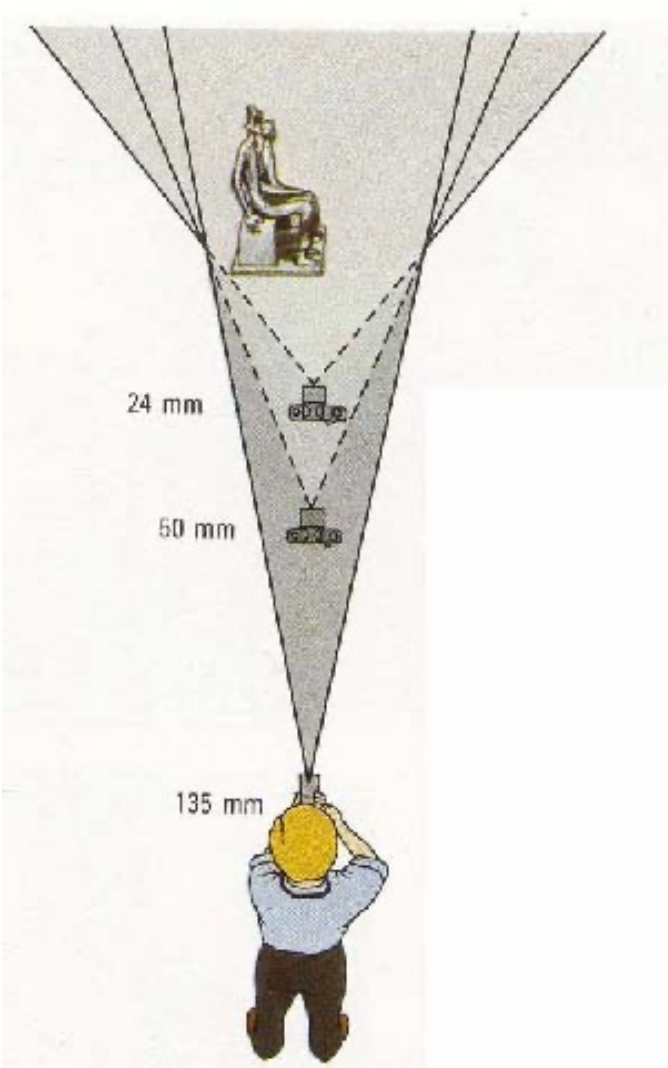




What does FOV depend on?

Slide by A. Efros

Field of View / Focal Length



Large FOV, small f
Camera close to car

Small FOV, large f
Camera far from the car

Sources: A. Efros, F. Durand

Same effect for faces

wide-angle

standard

telephoto

Source: F. Durand

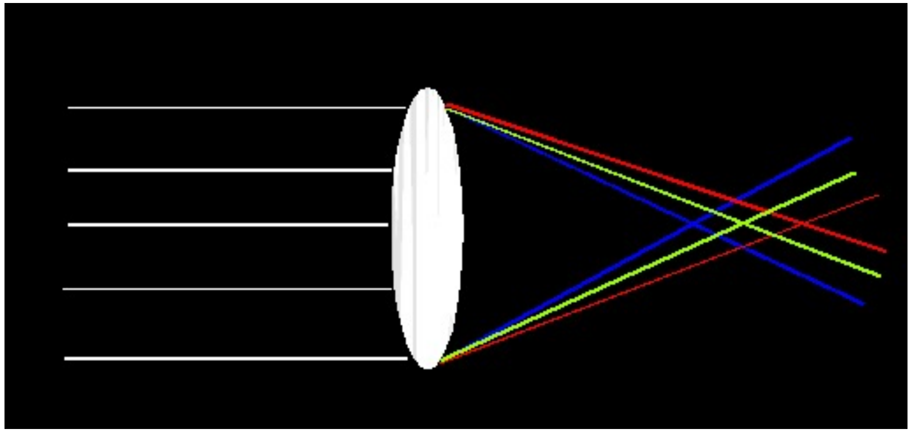
What is happening here ?

- Continuously adjusting the focal length while the camera moves away from (or towards) the subject
- “The Vertigo shot”

Examples of dolly zoom from movies (YouTube)

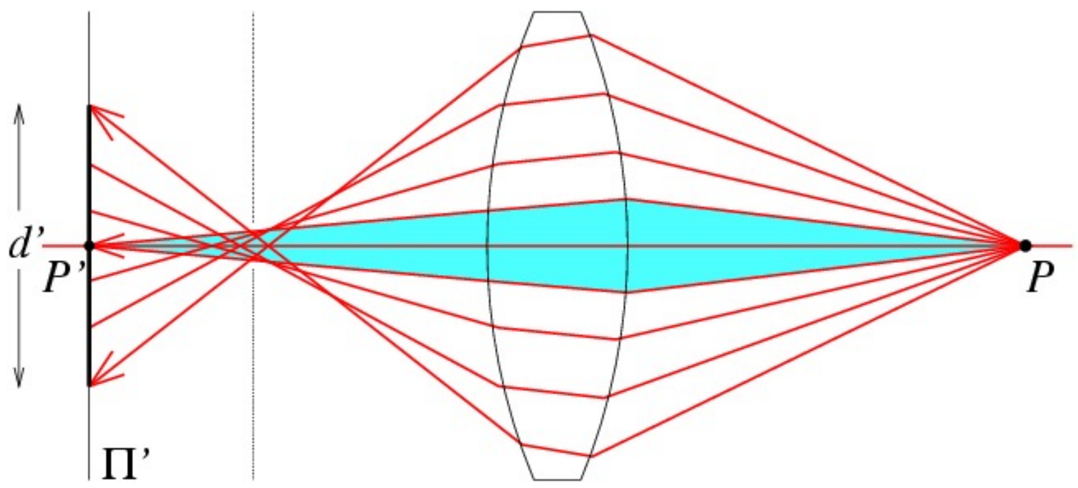
Real Lens Flaws: Chromatic Aberration

- Lens has different refractive indices for different wavelengths: causes color fringing
- Snell's law



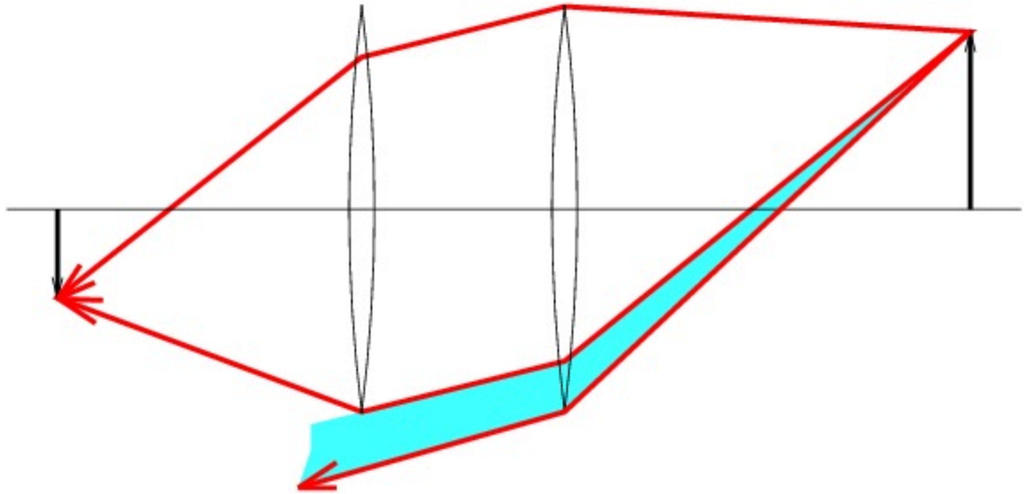
Lens flaws: Spherical aberration

- Spherical lenses don’t focus light perfectly
- Rays farther from the optical axis focus closer



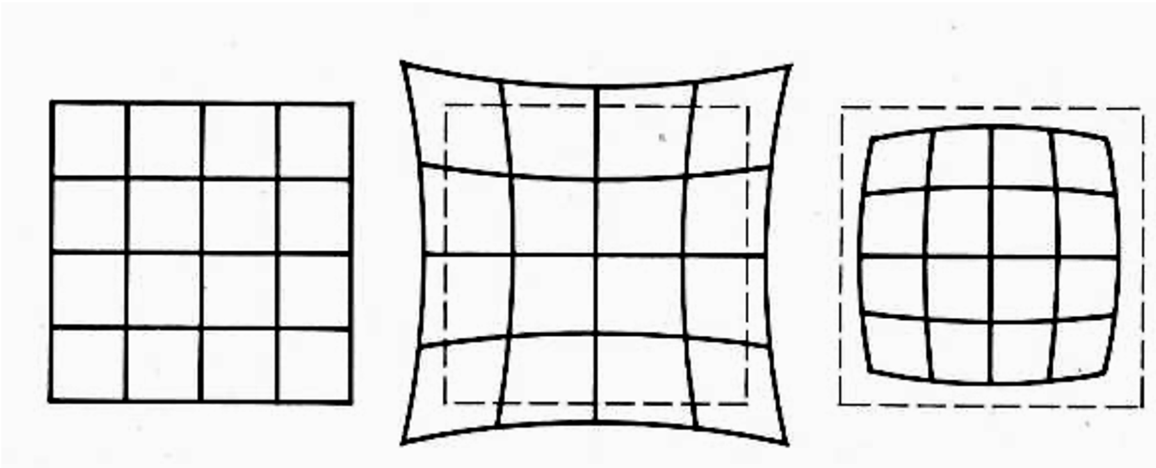
Lens flaws: Vignetting

- Reduction of brightness at the periphery



Radial Distortion

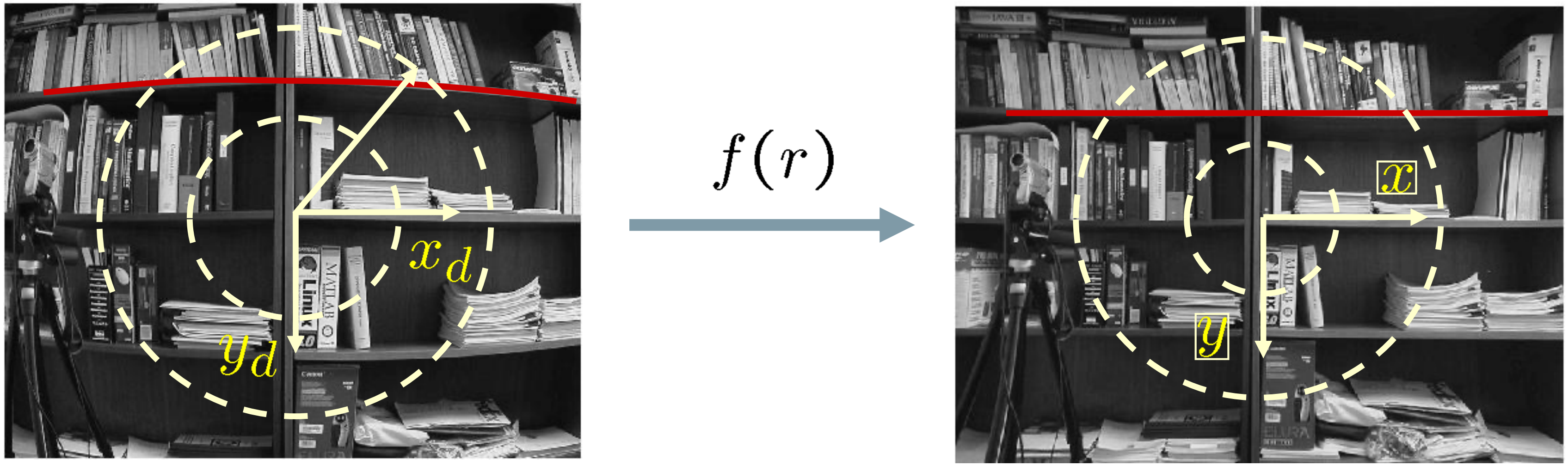
– Straight lines in the world are not straight in the image
– Deviations are most noticeable near the edge of the lens



No distortion Pin cushion Barrel

Radial Distortion

Nonlinear transformation along the radial direction



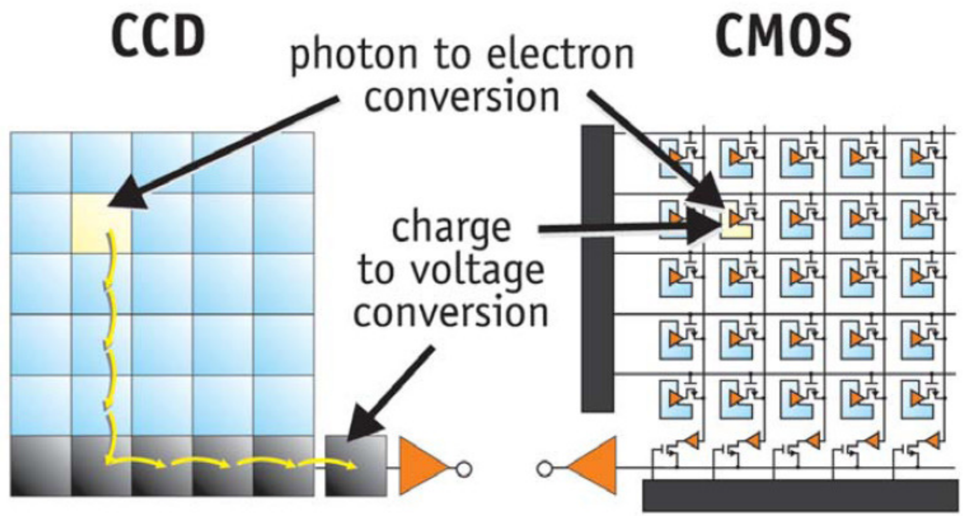
New coordinates

$$
\begin{aligned}
\boldsymbol{x} &= c + f(r)(\boldsymbol{x}_d - c), \qquad r = \|\boldsymbol{x}_d - c\| \\
f(r) &= 1 + a_1 r + a_2 r^2 + a_3 r^3 + a_4 r^4 + \cdots
\end{aligned}
$$

Distortion correction: make lines straight

Coordinates of distorted points

Digital camera



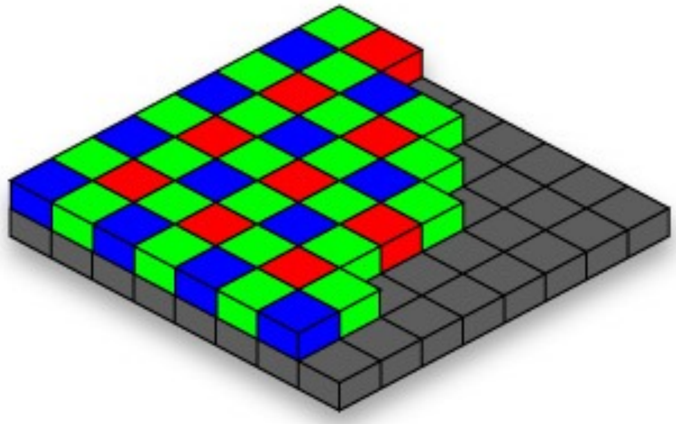
CCDs move photogenerated charge from pixel to pixel and convert it to voltage at an output node. CMOS imagers convert charge to voltage inside each pixel.

- A digital camera replaces film with a sensor array
  - Each cell in the array is light-sensitive diode that converts photons to electrons
  - Two common types
    - **Charge Coupled Device** (CCD)
    - **Complementary metal oxide semiconductor** (CMOS)
  - http://electronics.howstuffworks.com/digital-camera.htm

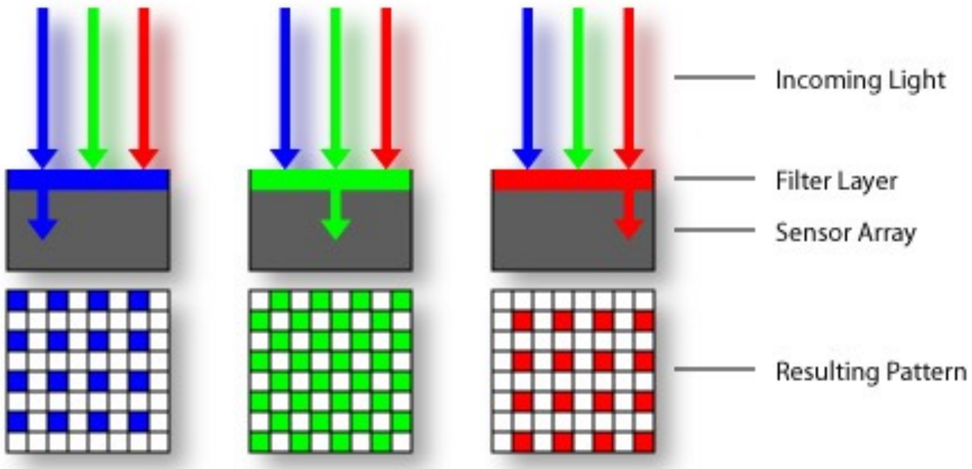
Slide by Steve Seitz

Color sensing in camera: Color filter array

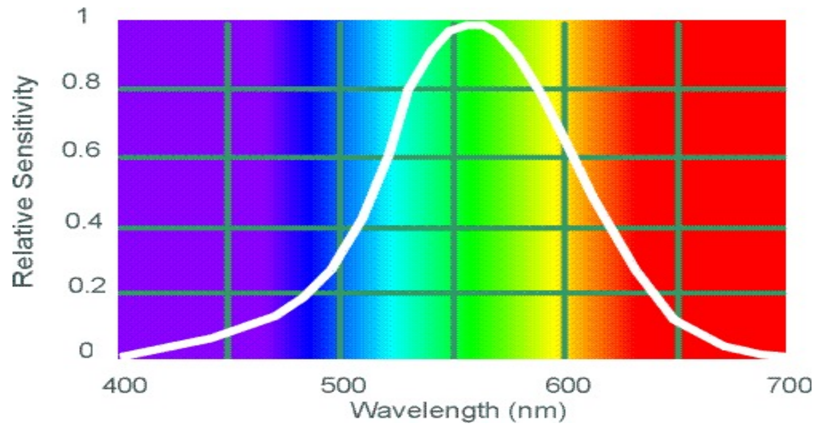
Bayer grid



Estimate missing components from neighboring values (demosaicing)



Why more green?



Human Luminance Sensitivity Function

Source: Steve Seitz

Bayer Grid