Analytical Techniques for Performance Analysis of Multi-copy Routing Schemes in Delay Tolerant Networks

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Abstract—Delay tolerant networks (DTNs) are a class of networks that experience frequent and long-duration partitions due to sparse distribution of nodes. The topological impairments experienced within a DTN pose unique challenges for designing effective DTN multicasting protocols. In this paper, we examine multi-copy routing schemes for DTNs. We provide analysis of multi-copy routing schemes by deriving analytical results for important performance metrics such as message delay, message delivery ratio, and buffer occupancy. We use three different analytical methods for our analysis: recursive method, ordinary differential equations, and phase-type distribution. Through extensive simulation study, we show that our analytical results for performance metrics are accurate.

I. INTRODUCTION

There has been tremendous recent interest in mobilityassisted routing protocols. The need for such protocols arises from mobile and wireless applications that must operate successfully even when the network is partitioned or disconnected most of the time. Such networks are generally referred to as Delay Tolerant Networks (DTNs). Under DTNs, frequent network partitions and large delays are common, and it is difficult to maintain continuous communication paths open. These conditions can arise for a number of reasons, such as resource constraints, as in the case of mobile sensor networks [8, 30], geographical constraints, as in the case of interplanetary networks [3], mobility constraints as in vehicular networks [6,7], hostile environments such as a battlefield, etc. One common characteristics of these networks are that there may never exist an intact end-to-end path from source to destination and that mobility is used as a means for message delivery.

As DTN protocols evolve there is a growing need for accurate performance modeling and evaluation. The metrics of interest for DTN protocol designers are similar to traditional network performance evaluation. For instance, under mobilityassisted routing message delay is an important performance metric, not only because it is a major concern for many applications, but also because of its effect on other performance metrics, such as message delivery ratio and buffer occupancy. This is especially true when there is a message expiration time associated with messages. Consequently, message delay has been the main focus of much theoretical work in this field [10, 20, 22–24, 29]. Most DTN applications are also sensitive to other performance measures such as the message delivery ratio and buffer occupancy.

The topic of this paper is to describe multiple techniques for DTN routing analysis. In terms of protocols we study the performance of multi-copy mobility-assisted routing schemes through Direct Transmission [23] and multi-copy (*L*-copy) routing schemes [24]. We focus on multi-copy routing because of the flexible and tunable nature of the associated protocols.

Due to the special nature of DTN routing a fundamental characteristic for performance modeling is how often nodes come into contact with each other. In practice this means how often nodes are in radio range. The time between two consecutive contacts is referred to as node inter-contact time. For reasons presented in Section 2 we can assume that node inter-arrival times are exponentially distributed. Our techniques can therefore be used to model a wide variety of mobility scenarios.

We provide performance analysis of common routing schemes for the following three performance metrics: *message delay*, *delivery ratio*, and *buffer occupancy*. One novel aspect of our analysis is that we include message *time constraint* as a factor. We also assume that nodes do not have any prior knowledge or oracles regarding node mobility or connectivity information.

One of our goals is to provide the analyst with a suite of usable analytical tools. To this end we present three different methods: a recursive scheme, using ordinary differential equations (ODE), and a phase-type distribution approach. Each of these methods offers a different way of analyzing routing schemes for all three performance metrics. The recursive scheme provides an easy-to-compute method for calculating performance metrics. However, the performance analysis results are not in closed-form. The ODE method gives closed-form expressions through a fluid limit model of state transitions of the system. Results obtained using the ODE method are scalable with respect to the number of nodes in the system. Approximations may be necessary for some metrics, and modeling some discrete values in continuous domain may introduces errors in the calculation. The phasetype distribution is used to model the system behavior as a

whole in a uniform manner. The phase-type model, however, may not be as scalable as ODE method when the number of nodes in the system increases. Through extensive simulation studies we show that our analytical results for performance metrics are in most cases highly accurate.

Section II goes over the related work. Section III gives an overview of the Direct Transmission and multi-copy routing schemes. Section IV presents the performance analysis of different mobility-assisted routing schemes. Section V presents experimental results. Finally, Section VI concludes the paper.

II. BACKGROUND AND RELATED WORK

Routing schemes for traditional mobile ad hoc networks (MANETs) assume that nodes are well connected most of the time. Generally, proactive schemes, where nodes try to keep up to date routing information [17], or reactive schemes, where nodes find routing paths on demand [14, 18], are used to achieve message delivery. Both schemes assume that there exists an end-to-end path from source to destination at the time of message transfer. However, such assumptions do not hold true when the mobile network is sparse and is intermittently connected. In such systems network partitions and large delays are common. Under these conditions, traditional MANET routing algorithms fail to work well, as proactive schemes do not converge, while reactive schemes fail to find a path to the destination.

Routing methods for such sparse mobile networks use a different paradigm for message delivery; these schemes utilize node mobility by having nodes carry messages, rather than transmitting them over a path [13, 19, 23, 24, 30]. Under such mobility-assisted routing protocols [25], nodes forward messages only when they encounter the appropriate relay or the destination node. Due to this dependence on mobility, understanding mobility characteristics such as inter-arrival times of mobile nodes within each other or at a static location plays an important role in the design and analysis of routing algorithms under this paradigm.

Delay tolerant networks (DTNs) are characterized by frequent network partitions and large message delays [3,9,13]. Because of frequent network partitions in the DTN environment many traditional routing techniques for Mobile Ad Hoc Networks (MANETs) will not work properly. This fact has led to recent interest in developing new approaches for routing in a DTN environment. The basic routing paradigm for effective routing in DTNs is to use the *Store-Carry-Forward* approach, where intermediate nodes keep the messages until new links come up in the path to the destination.

One general class of proposed DTN routing algorithms assumes some level of knowledge regarding node mobility and connectivity. For instance, Jain et. al. formulates the DTN routing in terms of a directed multi-graph, where more than one edge may exist between a pair of nodes [13]. Such multiple edges exist because there may be more than one distinct physical connections or different network links may only be available at different time intervals. By using different levels of information regarding connectivity and/or mobility, routing decisions can be made at individual nodes. Other approaches include using special nodes for routing assistance, including proposals for *message ferries* [30] and *throwboxes*[28].

Although the knowledge about node connectivity is useful for making routing decisions, such information may not be available to the nodes in the network. Further, it may not be possible to utilize specialized nodes for routing assistance. Under such network conditions, nodes can only deliver messages by opportunistically utilizing contacts that become available due to node mobility, requiring different routing approaches for effective message delivery. Our work analyzes performance in this unassisted and non-deterministic mobility pattern scenario

For our mobility pattern of interest, recent DTN routing approaches concentrate on trading off message complexity versus increasing the likelihood of message delivery. To limit the number of messages *single-copy* routing schemes allow only one copy of the message at a time to be present in the network [23]. *Direct Transmission* is the simplest form of single-copy routing, where each source node keeps its messages until it comes into direct contact with the respective destination nodes. Under this scheme only one message transfer is made per delivered message, incurring minimal message passing. However, in intermittently connected networks, such an approach may produce low delivery ratios and has an unbounded delivery delay [11].

One way to improve the performance of a single-copy approach is to have multiple copies of the same message within the network. One policy to implement a multi-copy scheme is to use flooding. One example is Epidemic Routing [27]. In Epidemic Routing when a pair of nodes come into contact the nodes exchange any missing packets. Given enough storage space and bandwidth, Epidemic routing can be used to reliably disseminate data across the network. However, due to its large overhead, a flooding scheme such as Epidemic Routing may not be applicable under circumstances where storage and power supplies are limited.

To address overhead problems caused by flooding, different forms of controlled flooding have been proposed, including message expiration times, limiting the number of hops a message can travel, and using active and passive "curing" techniques [12, 20]. Controlling the number of copies spread for a message is another effective approach for controlled flooding, for which Spray and Wait routing scheme is described in [24]. In this method, a total of L copies of a message are initially spread to other "relay" nodes. If the destination is not found in this phase, each of the nodes carrying a copy of the message will perform direct transmission. This is known as the L-copy scheme, and is the focus of our performance analysis study.

There has been a considerable amount of theoretical work in the performance analysis of routing schemes for DTNs [10, 20, 23, 24, 29]. Most of the work focuses on the message delay, as it is an important performance metric, not only as an application level requirement, but also as a factor affecting other metrics. The *L*-copy routing scheme is described in [24], where under the assumption that the node inter-contact times are exponentially distributed an analysis of expected message delay is given. However in many cases we are also interested other elements such as message delivery ratio (MDR) and buffer occupancies. Further, time constraints such as message expiration times often need to be considered. This is because such constraints either occur as an application level requirement, or as a routing policy [12]. In this paper we incorporate time constraints in our analysis of performance metrics. We note that for the calculation of message delay without any time constraint, a recursive method is used [24]. We extend this method for other performance metrics when message expiration times are present.

The ODE method was introduced in the context of DTNs in [20] to provide a fluid limit for applicable Markovian models. A more detailed use of ODEs is provided for the Epidemic routing approach in [29] for different performance metrics. In our work we use similar methods for the study of multicopy routing schemes to obtain closed-form expressions for our performance metrics. We also use the phase-type (PH) distribution as another method for our study, which we have not seen in other performance analysis studies for DTNs, as it provides a unified approach to the analysis of performance metrics.

The exponentiality of inter-contact times has been assumed in our study, as in most of previous performance analysis studies studies [10, 20, 23, 24, 26, 29]. The exponentiality of intercontact times have been discussed in recent literature [1, 10] epoch-based mobility models under DTN settings. Although [4] provided empirical evidence that the inter-contact times observed in MANET traces are power-law distributed, recent studies show that inter-contact times in real-world traces show power-law distribution upto certain time, and show exponential tail afterwards [15]. Our analysis is applicable to scenarios where the inter-contact time is known to be exponential, or can be reasonably approximated as exponential.

III. OVERVIEW OF DIRECT TRANSMISSION AND MULTI-COPY ROUTING PROTOCOLS

In this section we describe the Direct Transmission and multi-copy routing protocols. Direct Transmission is perhaps the most basic DTN routing scheme. The sender simply waits until it comes into contact with the destination to deliver a message. The advantage of this scheme is low overhead and simplified message delivery semantics. Direct Transmission is a form of single-copy routing, where only on message copy can exist in the system at any time. Other single-copy routing schemes include Randomized Routing, Probabilistic Routing, etc. [23].

Generally, single-copy schemes are more efficient in terms of reducing traffic overhead. However, message delivery ratios are normally lower while delivery delays are high. Direct Transmission has the upper bound for message delay for any non-adversarial mobility-assisted routing scheme [25]. Although not applicable for many scenarios due to large delays and low delivery ratios, Direct Transmission can be used as a base case for performance analysis and comparison.



Fig. 1. State Transition Diagram for the Number of Messages in the System (Source Spray and Wait)

One way to improve the performance of single-copy schemes is to use multiple copies of the same message within the network. Each copy can take a different path, thereby increasing the likelihood of delivery as well as decreasing message delay. At one end of the multi-copy spectrum is message flooding. Flooding methods such as Epidemic routing [27] achieve very low delay among routing schemes for mobility-assisted routing. As expected, Epidemic routing has large overhead in terms of message transmission and buffer occupancy. For this reason, various protocols have been proposed to limit the number of message transmissions. Typical techniques include using Time-To-Live or message expiration times.

Another basic approach to limit the number of message copies is known as multi-copy routing [10, 24]. There have been several DTN-oriented multi-copy routing protocols proposed in the literature. For instance, two different forms of multi-copy routing are described in [24]: Source Spray and Wait and Binary Spray and Wait. Both schemes consist of a spray phase, in which message copies are spread to other relay nodes, and wait phase, in which nodes perform Direct Transmission to delivery a message copy to the destination. In Source Spray and Wait scheme, the source distributes L-1copies to the first L-1 nodes (relays) it encounters, and keeps one for itself. The source and relays then perform direct transmission. In the Binary Spray-and-Wait scheme, the source keeps $\lfloor L/2 \rfloor$ and hands over $\lfloor L/2 \rfloor$ copies to the first relay it meets. This process continues in this manner until the number of message copies drops to one at the source and relay nodes, and then then nodes perform Direct Transmission. It is shown in [24] that the Binary Spray and Wait scheme is optimal among Spray and Wait routing schemes, in the sense that it gives the lowest expected delay.

Our work analyzes the performance characteristics of the Source Spray and Wait multi-copy scheme [24]. We do not focus on Binary Spray and Wait despite its optimal performance due to complexities in the analysis. Such complexities arise mainly because of the need to keep track of the source and relay nodes that can spread message copies to potential relays at each step of the spray phase. State transition diagram for Source Spray and Wait scheme is shown in Figure 1, where each state represents the number of copies of the message.

In this study, we focus on multi-copy routing schemes under time constraints. Such time constraints may exit either as part

Table 1: COMMON NOTATIONS

N	Number of nodes in the system
L	Maximum number message copies to be spread
γ	Nodal inter-contact rate
λ	Message generation rate of a node
E[R]	Expected message delivery ratio
E[D]	Expected delay of delivered messages
E[Q]	Expected buffer queue occupancy
f(t)	PDF of message delivery
F(t)	CDF of message delivery
T_x	Message expiration time
T_L	Expected time when L nodes have message copy
I(t)	No. nodes that have received a message copy at time t
J(t)	No. nodes that are carrying a message copy at time t

of the routing scheme or as an application requirement, and are implemented using message expiration times that are set when a message is generated.

Table III gives a list of commonly used notations used in the performance analysis of Direct Transmission and multi-copy routing schemes in this paper.

IV. ANALYSIS OF DIRECT TRANSMISSION AND MULTI-COPY ROUTING SCHEMES

Given that the inter-contact times of mobile nodes are exponentially distributed, we analyze fundamental performance metrics for mobility-assisted routing schemes, including *Direct Transmission* and *Spray and Wait*. The performance metrics that we are interested in are *Message Delivery Ratio* (MDR), *Delay of Delivered Messages*, and *Buffer Occupancy*.

In our model, we assume inter-contact times of each pair of nodes are distributed with a rate of γ . Each node generates a message at a rate of λ and randomly selects a node other than itself as the destination. The message generation rate λ is independent of the number of nodes in the system. Messages are assumed to have a message expiration time, T_x , beyond which they will be dropped.

A. Analysis of Direct Transmission Routing Scheme

1) Message Delivery Ratio: Provided that node intercontact times are exponentially distributed with a rate of γ , for a message entering in the queue at time 0 the probability that the message is delivered before it is expired can be given in the form of CDF as follows:

$$E[R] = 1 - e^{-\gamma T_x} \tag{1}$$

where T_x is the message expiration time. Here we assume that no messages are dropped due to buffer overflow.

2) Delay of Delivered Messages: Given message expiration time T_x , messages get delivered if the destination is reached within T_x , or it will be dropped. From an application point of view, we are only interested in the expected time that the delivered messages spend in the buffer queue before it gets delivered, i.e., the delay of delivered messages.

Since we assume that the inter-contact times of nodes are exponentially distributed with a rate of γ , the probability of

a message being delivered to the destination at time t after it enters the queue can be given by

$$f(t) = \gamma e^{-\gamma t}$$

As we are only interested in delivered messages, the probability function given above becomes a conditional probability for the messages that are delivered:

$$f_d(t) = \frac{f(t)}{P(t < T_x)} = \frac{\gamma e^{-\gamma t}}{1 - e^{-\gamma T_x}}$$

where $P(t < T_x)$ denotes the probability that the destination is reached before T_x , which is given by the CDF of f(t).

Therefore, the expected delay of a delivered message, E[D], can be written as

$$E[D] = \int_0^{T_x} tf_d(t)dt$$

= $\frac{\gamma}{1 - e^{-\gamma T_x}} \int_0^{T_x} te^{-\gamma t}dt$
= $\frac{1}{\gamma} - \frac{e^{-\gamma T_x}}{1 - e^{-\gamma T_x}} T_x$ (2)

This result gives us the expected delay of a message with expiration time T_x when the inter-contact rate of the destination is γ . Since relation given in (2) is used frequently later, we define $\Phi(\gamma, t_x)$ as a function a of inter-contact rate and message expiration time as follows:

$$\Phi(\gamma, t_x) = \frac{1}{\gamma} - \frac{e^{-\gamma t_x}}{1 - e^{-\gamma t_x}} t_x$$

It can be shown that the value of $\Phi(\gamma, t_x)$ is upper bounded by $min\{t_x, 1/\gamma\}$, and approaches $1/\gamma$ when $t_x \to \infty$.

3) Buffer Occupancy: Given the message generation rate, λ , and inter-contact rate of nodes, γ , we can find the number of nodes in the buffer using a queueing system model with vacations, where the message arrival rate is λ and vacation time is distributed exponentially with a rate of γ . Since the message transfer time is very small compared to inter-contact times, we can take the service rate as infinite for simplification. We assume that no messages are dropped due to buffer overflow.

First, we find the expected time that a message spends in the buffer, whether it is delivered or dropped due to message expiration. We already obtained the expected delay of delivered messages, E[D], above in (2). The time, $E_x[T]$, that an expired message spends in buffer is simply T_x . Therefore, the expected time, E[T], that a message spends in the buffer is given as follows:

$$E[T] = (1 - e^{-\gamma T_x})E[D] + e^{-\gamma T_x}E_x[T]$$

$$= \frac{1 - e^{-\gamma T_x}}{\gamma} - T_x e^{-\gamma T_x} + T_x e^{-\gamma T_x}$$

$$= \frac{1 - e^{-\gamma T_x}}{\gamma}$$

Given message generation rate of λ , we can give the expected number of messages in the buffer queue, E[Q], as

follows by using Little's Law:

$$E[Q] = \lambda * E[T]$$

= $\frac{\lambda}{\gamma} (1 - e^{-\gamma T_x})$

B. Analysis of L-copy Routing Scheme: Recursion Method

In this section, we use a recursive method to compute the performance metrics. In this scheme, at each state i, we consider the rate at which the system leaves state i and the probabilities of the system moving to the next state i+1 or the delivery state D. For the former case, we call the respective function for state i+1 in a recursive manner. The depth of the recursion is L.

The main advantage of this scheme is that it is straightforward analysis of the system state transition. Also, the calculation of the metrics is relatively easy. The main disadvantage is that it does not give a closed-form expression for any of the performance metrics.

1) Message Delivery Ratio: To obtain the expected message delivery ratio, let us first consider the case when all L copies have been spread a without reaching the destination. Let us further assume that remaining message expiration time is t_x . Since node movements are independent and the intercontact times are exponentially distributed, the rate at which the destination meets any one of the L nodes is $L * \gamma$. Given the inter-contact rate and expiration time, we get the expected delay after reaching state L is $1 - e^{-L\gamma t_x}$, as defined in (1).

At state *i*, where i < L, the system leaves the state at a rate of $(N-1)\gamma$. As shown in Figure 1, under the condition that the system moves either to state i + 1 or to state *D* within t_x , which occurs with a probability of $1 - e^{-(N-1)\gamma t_x}$, the probability of moving to state i + 1 is (N - i - 1)/(N - 1), whereas the probability, P_D , of moving to state *D* is i/(N-1). In either case, the expected time that system stays at state *i*, ED, is given as $ED = \Phi((N-1)\gamma, t_x)$. Based on the analysis above, the expected message delivery ratio in *L*-copy routing scheme can be given as a recursive set of equation as follows, where $E[R] \equiv ER_x(1, T_x)$:

$$ER_{x}(i, t_{x}) = F((N-1)\gamma, t_{x}) \left[\frac{i}{N-1} + \frac{N-i-1}{N-1} ER_{x}(i+1, t_{x} - ED) \right], \quad i \in [1, L-1]$$
$$ER_{x}(i, t_{x}) = F(L\gamma, t_{x}), \quad i = L$$
(3)

where $F(\gamma, t_x) = 1 - e^{-\gamma t_x}$, and $ED = \Phi((N-1)\gamma, t_x)$.

2) Message Delay: From the analysis above, we know that the expected message delivery ratio at state L is given as $1 - e^{-L\gamma t_x}$, where t_x denotes the time remaining before message expiration. Based on Equation (2), the expected delay at this state is given as $\Phi(L\gamma, t_x)$. Considering that the time passed since message generation when system enters state Lis $T_x - t_x$, we obtain that the expected message delay is $(1 - e^{-L\gamma t_x})(T_x - t_x + \Phi(L\gamma, t_x))$.

Let us now consider the case when the system enters state i, where i < L, with message expiration t_x . With probability $1-P_D$, which is given above, the system enters state i+1, with a new expiration time of $t_x - ED_x$, incurring further delay, and with probability P_D system moves to state D. The probability

that either of these two events happening is $1 - e^{-(N-1)\gamma t_x}$. Based on the analysis above, the expected message delay in *L*-copy routing scheme can be given as follows, where $E[D] \equiv ED_x(1, T_x)/E[R]$:

$$ED_{x}(i,t_{x}) = F((N-1)\gamma,t_{x})\left[\frac{i}{N-1}(T_{x}-t_{x}+ED)+\frac{N-i-1}{N-1}ED(i+1,t_{x}-ED)\right], \quad i \in [1,L-1]$$
$$ED_{x}(i,t_{x}) = (1-e^{-L\gamma t_{x}})[T_{x}-t_{x}+\Phi(L\gamma,t_{x})], \quad i = I_{4}4$$

where $ED = \Phi((N-1)\gamma, t_x)$. The expected message delivery ratio, E[R], is included in the calculation as we are interested in the delay of delivered messages.

3) Buffer Occupancy: When the system is at state L with expiration time t_x , the number of message in the system is given as $L(\lambda/\gamma)(1-e^{\gamma t_x})$, following (3). Note that the rate is taken as γ , instead of $L\gamma$, since each node with a copy of the message keep the copy until it meets the destination or until the message is expired.

At state *i*, where i < L, the source gives out a copy, either to the destination, or to another node that has not received a copy of the message, at a rate of $(N - i)\gamma$. The probability of delivering it to the destination is 1/(N-i), and the probability of delivering to another relay is (N - i - 1)/(N - i). Based on the analysis, the expected buffer occupancy at each node can be expressed as follows, where $E[Q] \equiv EQ_x(1, T_x)$:

$$EQ_x(i,t_x) = \frac{1}{N-i} \lambda \cdot i \cdot ED_i + \frac{N-i-1}{N-i} EQ_x(i+1,t_x-ED_i), \quad i \in [1,L-1]$$
$$EQ_x(i,t_x) = \frac{\lambda}{\gamma} \cdot L \cdot F(\gamma,t_x), \quad i = L$$
(5)

where $F(\gamma, t_x) = 1 - e^{-\gamma t_x}$, and $ED_i = \Phi((N - i - 1)\gamma, t_x)$.

In the calculation of buffer occupancy above, we equated the buffer occupancy at each node with the total buffer occupancy of the system caused by the message generation at a single node, which is given by the set of equations in (5). The reason is as follows. Assuming the total expected buffer occupancy in the system (including the source) caused by the message generation at a single node is Q, the expected total buffer occupancy of all the nodes of the system is given by $N \times Q$, as there are N nodes in the system. Since each of the N nodes in the system is equally likely to share the total buffer occupancy of the system, the expected buffer occupancy at each node is again given by Q. The same reasoning is applied in the calculation of buffer occupancy in the analytical methods that follow.

C. Analysis of L-copy Routing Scheme: ODE Method

In this section, we give the performance analysis of the *L*-copy routing scheme using ordinary differential equations (ODEs). Observing the similarities between infectious diseases and Epidemic routing, [21] used the ODE models following infectious disease-spread model used in [5]. In this model, two processes are present: "infection process" where nodes spread copies of the message, and "recovery process" in which nodes delete message after successful delivery of the message to the destination.

Applicability of ODEs in solving the performance metrics under Source Spray and Wait routing scheme is driven by the observation that the rate at which the system leaves state idepends on the value of i. By treating the i as a continuous variable, a fluid limit of the Markovian model can be provided using ODE model. The main advantage of this model is the scalability of analysis and the ability of the model to provide closed-form solutions for simple cases. For more complex scenarios, one normally has to resort to numerical computation for desired results.

1) Message Delivery Ratio: Assuming a message is generated at the source, we let I(t) denote the number of "infected nodes", including the source, that have a copy of the message at time t, where t is calculated from message generation time. The following relation can be given:

$$\frac{dI}{dt} = \gamma (N - I)$$

where N is the number of nodes in the system. The function I(t) is abbreviated I in the equation above, and we follow this convention in other differential equations that follow.

Solving the equation above with the initial condition I(0) = 1 yields

$$I(t) = N - (N - 1)e^{-\gamma t}$$
 (6)

In *L*-copy scheme the equation above is only valid for $I \leq L$. Before we consider the scenario further, we find the expected time, T_L , at which system reaches state *L* by using Equation (6):

$$T_L = \frac{\ln(N-1) - \ln(N-L)}{\gamma}$$

With this we can define I(t) for $t \ge 0$ as follows:

$$I(t) = \begin{cases} N - (N-1)e^{-\gamma t}, & t < T_L \\ L & t \ge T_L \end{cases}$$

Now let F(t) denote the cumulative probability of message delivery at time t, we have the following equation [29]:

$$\frac{dF}{dt} = \gamma I(1-F)$$

Solving the equation above using Equation (6) and initial condition F(0) = 0, we have

$$F(t) = 1 - e^{N - Nt\gamma - 1 - e^{-t\gamma}(N-1)}$$

We can directly obtain the expected message delivery ratio, F_L , at time T_L as follows:

$$F_{L} = F(T_{L})$$

$$= 1 - e^{N - NT_{L}\gamma - 1 - (N-1)e^{-T_{L}\gamma}}$$

$$= 1 - e^{L - 1 - N \ln \frac{N-1}{N-L}}$$

$$= 1 - e^{L-1} \left(\frac{N-L}{N-1}\right)^{N}$$

After the system reaches state L we can give the following expression for F(t):

$$\frac{dF}{dt} = \gamma L(1-F), \quad t \ge T_L$$

Solving the equation above with the initial condition $F(T_L) = F_L$, we have

$$F(t) = 1 - (1 - F_L)e^{-L\gamma(t - T_L)} = 1 - e^{L - 1 - L\gamma t} \left(\frac{N - L}{N - 1}\right)^{N - L}, \quad t \ge T_L$$

Combining the results above, we can give the CDF of message delivery, F(t), as follows:

$$F(t) = \begin{cases} 1 - e^{N - Nt\gamma - 1 - e^{-t\gamma}(N-1)}, & t < T_L \\ 1 - e^{L - 1 - L\gamma t} \left(\frac{N - L}{N-1}\right)^{N - L} & t \ge T_L \end{cases}$$
(7)

Therefore, using (7) above the expected MDR, E[R], is given as

$$E[R] = F(T_x) \tag{8}$$

2) Message Delay: We use f(t) to denote the PDF of message delivery, where f(t) = F'(t). To find the delay of delivered messages, we first obtain the conditional PDF, $f_d(t)$, as follows:

$$f_d(t) = f(t|t \leqslant T_x) = \frac{f(t)}{F(T_x)}$$

With this result the expected delay of delivered messages, E[D], under message expiration time T_x is given as follows:

$$E[D] = \int_{0}^{T_{x}} tf_{d}(t)dt$$

= $\frac{1}{F(T_{x})} \int_{0}^{T_{x}} tf(t)dt$
= $T_{x} - \frac{1}{F(T_{x})} \int_{0}^{T_{x}} F(t)dt$ (9)

3) Buffer Occupancy: In our analysis so far for the delivery ratio and message delay we did not consider the fact that an infected nodes that has a copy of the message can "recover" by deleting the message from its buffer, as this did not affect these two metrics. This recovery process has to be considered for buffer occupancy as buffer space for a message is freed after the message is delivered. We assume that an "infected" relay node deletes a message copy if it delivers the message or the node is informed by the destination that the message has been received. We also assume that nodes will keep a record of delivered messages, and will not be re-infected by the source.

Let R(t) and J(t) denote the number of recovered nodes and the number of infected nodes at time t, respectively. We can give the following relation for J(t):

$$\frac{dJ}{dt} = \gamma (N - J - R)e^{-\gamma t} - \gamma J, \quad t \leqslant T_L$$
(10)

The factor $e^{-\gamma t}$ is introduced to reflect the fact that the source only spreads copies only if it has not delivered the message to the destination. When $1 \ll N$ and $L \ll N$, however, $e^{-\gamma t}$ can be approximated as 1, since $t \leqslant T_L$ and $T_L \ll 1/\gamma$. So we can simplify the equation above and the following set of equations can be give for J(t) for t > 0. The results we obtain here for buffer occupancy can be viewed as a close upper bound for



Fig. 2. Total number of infected nodes (I(t)), number of currently infected nodes (J(t)), and number of recovered nodes (R(t))

the actual values. We note that if $L \ll N$ does not hold, the deviation due to our approximation will be larger.

$$\frac{dJ}{dt} = \begin{cases} \gamma(N - J - R) - \gamma J & t < T_{H} \\ -\gamma R & t \ge T_{H} \end{cases}$$

Similarly, the following can be given for R(t):

$$\frac{dR}{dt} = \gamma J \quad 0 \leqslant t$$

Solving the equations above with the initial conditions J(0) = 1 and R(0) = 0, we obtain the following:

$$J(t) = \begin{cases} e^{-\gamma t} (1 - \gamma t + \gamma t N) & t < T_L \\ J_L e^{-\gamma (t - T_L)} & t \ge T_L \end{cases}$$

and

$$R(t) = \begin{cases} N - e^{-\gamma t} (N - \gamma t + \gamma t N) & t < T_L \\ L - J_L e^{-\gamma (t - T_L)} & t \ge T_L \end{cases}$$

where

$$J_L = J(T_L) = \frac{N - L}{N - 1} + (N - L) \ln \frac{N - 1}{N - L}$$

We can see that relation I(t) = J(t) + R(t) holds for t > 0. A sample graph depicting the relationship of these three functions is shown in Figure 2. In this this graph, the total number of nodes that have been "infected" at time t is given by I(t). As can be seen, once I(t) reaches L at time T_L , it becomes constant as there will be no more copies spread. The number of nodes that are *currently* infected at time t is given by function J(t), whereas the total number of "recovered" nodes is given by R(t). For buffer occupancy, we are only interested in J(t) as it gives the number of nodes that are carrying the message at time t.

When the message generation rate is λ , the expected buffer occupancy, E[Q], under message expiration time T_x is given as

$$E[Q] = \lambda \int_{0}^{T_{x}} J(t)dt$$

=
$$\begin{cases} \frac{\lambda}{\gamma} [N - e^{-\gamma T_{x}} (N - \gamma T_{x} + N\gamma T_{x})] & T_{x} < T_{L} \\ Q_{L} + \frac{\lambda}{\gamma} J_{L} \left(1 - e^{-\gamma (T_{x} - T_{L})}\right) & T_{x} \ge T_{L} \end{cases}$$
(11)

where

$$Q_L = \lambda \int_0^{T_L} J(t) dt$$

= $\frac{\lambda}{\gamma} [N - e^{-\gamma T_L} (N - \gamma T_L + N\gamma T_L)]$
= $\frac{\lambda}{\gamma} \left[\frac{N(L-1)}{N-1} - (N-L) \ln \frac{N-1}{N-L} \right]$

D. Analysis of L-copy Routing Scheme: Phase-type Method

As we can see from Figure 1 depicting the state transition diagram for L-copy routing, the message delivery process can be seen as a sequence of Poisson processes. Unlike hypoexponential distribution, the system can reach absorbing state D from any state non-absorbing state i, and can be viewed as finite-state continuous absorbing Markov chain for our analysis using phase-type (PH) distribution. The main advantage of the PH distribution is its ability to analyze the system in a unified and algorithmically tractable manner.

1) Message Delivery Ratio: The distribution of message delivery time can be represented with a random variable describing the time until the system reaches the absorbing state of the corresponding Markov process with one absorbing state (D). We note that the absorbing state D can be reached from any state $i \in 1, ..., L$. To model this we construct a Coxian (phase-type) distribution [16], PH(α, Θ), as follows:

$$\boldsymbol{\alpha} = (1, 0, \dots, 0),$$

$$\boldsymbol{\Theta} = \begin{bmatrix} -\gamma_1 & p_1\gamma_1 & 0 & \dots & 0 & 0 \\ 0 & -\gamma_2 & p_2\gamma_2 & \ddots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \ddots & -\gamma_{L-2} & p_{L-2}\gamma_{L-2} & 0 \\ 0 & 0 & \dots & 0 & 0 & -\gamma_L \end{bmatrix}$$

Here α is a $1 \times L$ row matrix denoting initial probability distribution, and Θ is a $L \times L$ transition matrix, where

$$\gamma_i = \gamma \cdot (N-1), \quad 1 \leq i < L$$

 $\gamma_L = \gamma \cdot L$

and

and

$$p_i = \frac{N - 1 - i}{N - 1}, \quad 1 \le i < L$$

Here, γ_i denotes the rate at which the system leaves state *i*, and p_i denotes the probability that the message is not delivered to the destination, either by the source or by a relay. The states $\{1, \dots, L\}$ are also called **phases**, and the dimension *L* of the matrix is called the **order** of the distribution PH(α, Θ).

With this, the CDF of message delay, F(t), can be given by the cumulative distribution of the time until absorption in state D as follows [2, 16]:

$$F(t) = 1 - \alpha e^{t\Theta} \mathbf{1} \tag{12}$$

where 1 is a column vector of size $N \times 1$, with all elements being one, and $e^{t\Theta}$ is the matrix exponential of $t\Theta$, which is given by

$$e^{t\Theta} = \sum_{n=0}^{\infty} \frac{t^n}{n!} \Theta^n$$

The corresponding PDF of message delivery, f(t), is given as follows:

$$f(t) = \boldsymbol{\alpha} e^{t\boldsymbol{\Theta}} \boldsymbol{\Theta}^0, \tag{13}$$

where $\Theta^0 = -\Theta \mathbf{1}$.

Using the results above, we can give the expected MDR, E[R], under message expiration time T_x as follows:

$$E[R] = F(T_x) = 1 - \alpha e^{T_x \Theta} \mathbf{1}$$
(14)

2) Message Delay: Similar to the expression derived in the ODE method, we can write the delay of delivered messages, E[D], as follows:

$$E[D] = \int_0^{T_x} t f_d(t) dt = T_x - \frac{1}{F(T_x)} \int_0^{T_x} F(t) dt \quad (15)$$

where F(t) is given in (12), and

$$f_d(t) = f(t|t \leqslant T_x) = \frac{f(t)}{F(T_x)}$$

in which f(t) is given in (13).

3) Buffer Occupancy: We first consider the cases where the number of copies in the system is smaller than L. For a phase-type distribution discussed above, the expected total time that the system spends in state j is $(-\Theta^{-1})_{ij}$ given that the initial state is *i*. Since the system under consideration always starts from state 1, the expected time that system spends in state *j*, T_j , is given as follows:

$$T_j = (-\Theta^{-1})_{1,j}, \quad 1 \le j < L$$

Let S_j denote the expected time when system enters state j, we have

$$S_j = \begin{cases} 0, & j = 1, \\ \sum_{i=1}^{j-1} T_i, & 1 < j \le I \end{cases}$$

and T_L , the time system reaches state L, is given as $T_L = S_L$. When $T_x < T_L$, the expected buffer occupancy, E[Q], can be given as

$$E[Q] = Q_j + \lambda j (T_x - S_j), \quad T_x < T_L$$

where j is the largest number that satisfies $S_j < T_x$, and Q_j is the expected buffer occupancy when the system enters state j:

$$Q_{j} = \begin{cases} 0, & j = 1, \\ \lambda \times \sum_{i=1}^{j-1} (i \times T_{i}), & 1 < j \leq L \end{cases}$$
(16)

When $T_x > T_L$ the analysis is the same with the ODE case, and we can give the expected buffer occupancy, E[Q], for any $T_x > 0$ as below following Equation (11):

$$E[Q] = \begin{cases} Q_j + \lambda j (T_x - S_j), & T_x < T_L \\ Q_L + \frac{\lambda}{\gamma} \cdot P_L \cdot L \left(1 - e^{-\gamma (T_x - T_L)} \right) & T_x \geqslant T_L \\ \end{cases}$$
(17)

where Q_L is calculated using Equation (16), and P_L denotes the probability that the system reaches state L from the initial state:

$$P_L = \prod_{i=1}^{L-1} \frac{N-i-1}{N-i} = \frac{N-L}{N-1}$$

V. Experimental Results

In this section, we present experimental results for mobility characteristics and performance analysis. The goal of our experiments is to verify the correctness of analytical results regarding the performance analysis of mobility-assisted rout-

A. Experimental Settings

ing schemes.

Our experiments mostly use the ns-2 network simulator extended with our own code. The default settings for ns-2simulations are as follows. Each simulation run has 40 nodes moving according to the specified mobility model in a $6000m \times 6000m$ square area. By default, nodes have a radio range of 250m. Nodes move according to the Random Waypoint mobility model. The minimum and maximum speeds, v_{min} and v_{max} , are 3m/s and 10m/s, respectively. We ran each experiment 29 times with random seeds. The duration of each simulation is 45000 seconds. Data points presented are plotted with 95% confidence intervals.

B. Performance Metrics

For our performance analysis we use Direct Transmission and Source Spray and Wait (SSW) schemes, and measure *message delay*, *message delivery ratio*, and *buffer occupancy* under different settings. We use three different values of Lunder SSW scheme: 2, 4, and 8.

We study the effects of message expiration times on performance by varying the message expiration time in the range of 0-13000 seconds. Messages will be dropped and will not be delivered if expired. We consider that such time constraints are realistic exist in many real life applications, either as an application level requirement, or as part of routing policy. We assume that no messages are dropped due to buffer shortage, and arrange our experiments accordingly.

C. Message Delivery Ratio

Results for message delivery ratios are shown in Figure 3. Sub-figures 3(a)-3(c) show the empirical and analytical results for message delivery ratios for the cases L=2, L=4, and L=8, respectively. Analytical results for MDR for recursion method, the ODE method, and the phase-type distribution are respectively obtained using Equations (3), (8), and (14). As expected, the message delivery ratio increases as L is increased. We can see that all the analytical results for message delivery ratios closely agree with the experimental results.



Fig. 3. Empirical and Analytical Results for Message Deliver Ratios under Different Multi-copy Routing Schemes

D. Message Delay

Figure 4 shows experimental and analytical results for message delay. Analytical results for recursion method, ODE, and PH-distributions are obtained from Equations (4), (9), and (15), respectively. As expected, the average message delay decreases as we increase the number of copies spread. For small expiration intervals, we see that the differences of delay of delivered messages among different schemes are small, but the variations grow as the expiration time increases. We note that since only the delivered messages are considered, the message delay should be considered along with delivery ratio to gain an understanding of the performance of different schemes.

E. Buffer Occupancy

Empirical and analytical results for buffer occupancies are shown in Figure 5. Analytical results for the buffer occupancy are obtained from Equations (5), (11), and (17), respectively. As expected, the buffer occupancy increases as we increase *L*. As opposed to the improved performance in message delivery ratio and delivery delay discussed above, this represents higher resource comsumption in buffer usage, as well as in the number of transmissions made for a message. This has important implications for the performance of for resource-constrained systems and massively partitioned networks.

We can see from the figure that analytical results from recursion and the phase-type distribution methods closely agree with the empirical results. Analytical values obtained from the ODE method deviates from the empirical values as L gets larger. These deviations might be introduced as a consequence of the simplification that we made in Equation (10), as well as the from the modeling of discrete buffer occupancy values in the continuous domain.

VI. CONCLUSIONS

In this paper, we studied the performance of multi-copy routing scheme for Delay tolerant networks (DTNs). DTNs are a class of networks that experience frequent and long-duration partitions due to sparse distribution of nodes. The topological impairments experienced within a DTN pose unique challenges for designing effective DTN routing protocols. For mobility-assisted DTN systems, routing schemes mainly focus



Fig. 4. Empirical and Analytical Results for Delay of Delivered Messages under Different Multi-copy Routing Schemes

on the trade-off between routing performance and resource usage. Therefore, the analysis of fundamental performance metrics is important in the design and analysis of DTN routing schemes.

We provided the analysis of multi-copy routing schemes by deriving analytical results for important performance metrics such as message delay, message delivery ratio, and buffer occupancy. We used three different analytical methods for our analysis: recursive method, ordinary differential equations (ODEs), and phase-type distribution. Each of these methods offers a different way of analyzing all three performance metrics. The recursive scheme provides an easy-to-compute method of calculating performance metrics. The obtained results, however, are not in closed-form. The ODE method provides closed-form results and is scalable with respect to the number of nodes in the system. Approximations may be necessary in some cases and continuous domain modeling in ODE may introduce errors when dealing with discrete values. Phase-type method is used to model the system behavior as a whole in a uniform manner. Understanding resourceperformance trade-offs through different performance metrics

is important for the design of routing protocols for DTNs, especially for resource constrained systems of small devices and massively partitioned networks. Through extensive simulation study, we showed that our analytical results for performance metrics are accurate. Future work includes the extension of our analytical methods for other DTN routing schemes, both for unicast and multicast communications.

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Fig. 5. Empirical and Analytical Results for Buffer Occupancy under Different Multi-copy Routing Schemes

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