Characteristics of Common Mobility Models for Opportunistic Networks

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ABSTRACT

Understanding the characteristics of synthetic mobility models is important for the design and analysis of routing schemes for mobile ad hoc networks (MANETs). This is especially true for mobile opportunistic networks where node mobility is utilized to achieve message delivery. In this paper, we study the properties of common mobility models. Specifically, we analytically show that inter-contact times of mobile nodes can be closely approximated as exponentially distributed in Random Waypoint and Random Direction mobility models under typical opportunistic network settings. Analytical and experimental results are presented.

Categories and Subject Descriptors

C.2.1 [**Network Architecture and Design**]: Wireless Communication

General Terms

Performance

Keywords

mobility models, opportunistic networks

1. INTRODUCTION

The understanding of mobility characteristics plays an important role for the design and analysis of routing schemes for mobile ad hoc networks. This is especially true for mobile opportunistic networks where node mobility is utilized to achieve message delivery. Routing schemes for traditional mobile ad hoc networks (MANETs) assume that nodes are well connected most of the time. In such systems network partitions and large delays are common. Under these conditions, traditional MANET routing algorithms fail to work well, as proactive schemes do not converge, while reactive schemes fail to find a path to the destination.

Routing methods for sparse mobile networks use a different paradigm for message delivery; these schemes utilize node mobility by having nodes carry messages, waiting for an opportunity to transfer messages to the destination or the next relay rather than transmitting them over a path [9, 13]. Under such opportunistic network (ON) routing protocols [14], nodes forward messages only when they encounter the appropriate relay or the destination node. Due to this dependence on mobility, understanding mobility characteristics such as inter-contact times of mobile nodes within each other or at a static location plays an important role in the design and analysis of routing algorithms under this paradigm. Understanding such characteristics of mobility models not only helps us to relate experimental results to analytical models, but also enables us to choose appropriate mobility models in simulation studies when the mobility characteristics of real world applications are known.

In this paper, we study the mobility characteristics of commonly used mobility models. Specifically, assuming opportunistic network scenarios we show that inter-contact times can be closely approximated as exponentially distributed under Random Waypoint and Random Direction mobility models. We give analytical results for the inter-contact rate and the contact time when nodes come into contact with each other. We show through simulation studies that our analytical results for both mobility characteristics are accurate.

The rest of the paper is organized as follows. Section 2 goes over the related work. Section 3 discusses the stochastic mobility properties of the Random Waypoint and Random Direction mobility models. Section 4 presents experimental results. Finally, Section 5 concludes the paper.

2. RELATED WORK

Mobility models play an important role in the simulation study of mobile networks. Two common mobility models are the Random Waypoint (RWP) and Random Direction (RD). Issues such as non-uniform node distribution and speed decay have been addressed for the RWP model [7, 15]. Other mobility models are proposed by different groups [3, 5, 14]. RWP and RD mobility models are currently widely used in network simulations and are the focus of our study.

The stochastic properties of the Random Waypoint mobility model have been extensively studied [6, 7, 11]. Similar studies are also available for the Random Direction model [2, 12]. Most of these studies focus on node distributions, epoch lengths, and movement directions, and are a foundation for further analysis of node movement characteristics under the mobility models. Results for the expected inter-contact times in RWP and RD models are given in [14] in the

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context of mobility-assisted routing. Similar results are presented in [8] for the analysis of message delay under sparse networks. In our study, we focus on the rate and the distribution of inter-contact times under RWP and RD models. We show that that inter-contact times in RWP and RD models can be closely approximated by an exponential distribution and provide analytical and experimental results. Unlike this earlier work, we show that the inter-contact times can be approximated as exponentially distributed. This helps to simplify the analysis of routing schemes under RWP or RD models, and to relate experimental results to analytical models. It also enables us to use these two mobility models for simulation when the inter-contact times are known to be exponentially distributed.

Real world mobility traces have also been used for the study of mobility characteristics for opportunistic networks. Results on node inter-contact times based on human mobility traces are presented in [4], providing empirical evidences that the inter-contact times are distributed according to power law upto moderate time scale. This is in disagreement with the common exponentiality assumptions. However, recent work by Karagiannis et al [10] presented evidence showing that although the inter-contact time follows power law distribution upto certain time, it shows exponential decay afterwards. Therefore, we believe that our results can be applied to study certain real world application scenarios.

3. STOCHASTIC PROPERTIES OF MOBILITY MODELS

In this section, we study the statistical properties of node encounters, focusing on node inter-contact times and contact times using two commonly used, epoch-based mobility models: Random Direction and Random Waypoint. Within node inter-contact times, we look at inter-contact times between two mobile nodes and inter-contact times of a mobile to a static location.

For practical purposes, we consider a two-dimensional system space $\mathcal{A}$ of size A as a square area of width a or a circular region with radius a. Nodes pause for a random amount of time T_p at the end of an epoch, randomly chosen with the expected value of $\bar{T}_{pause}$. We use $\bar{L}$ to denote average epoch distance, and use $\bar{T}$ to denote average epoch duration. The movement speed v is uniformly and randomly chosen from $[v_{min}, v_{max}]$, where $0 < v_{min} < v_{max} < \infty$.

Although movement direction may change during an epoch, as in the case of Random Direction with reflection, we assume the speed remains the same in an epoch. The average node speed, $\bar{v}$, is defined as

$$\bar{v} = \frac{\bar{L}}{\bar{T}}$$

Nodes are assumed to have circular radio range with radius r. For the case of sparse mobile networks, we assume that $r \ll a$ and that $r \ll \bar{L}$.

From the viewpoint of a mobility model, node movements consist of interleaving periods of movements and pauses. From an application's point of view, node N sees the movements of another node M in terms of the time that M spends in its radio range, which we call as *contact time*, and the *inter-contact time* between two contacts. The contact time is defined as the time elapsed from a node's entry into another node's radio range until its consequent exit. The inter-contact time is defined as the time passed since previous exit

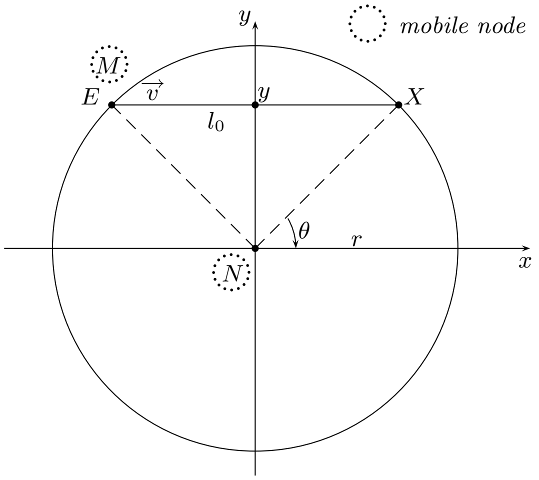


Figure 1: Calculation of Contact Time

until next entry into the radio range. Below we examine these two factors.

3.1 Contact Times

In terms of movement behavior upon entering the radio range of another node, there are no fundamental differences between RWP and RD mobility models, especially when $r \ll \bar{L}$. Hence, we do not discriminate between the two models in our analysis of contact times.

Figure 1 depicts the scenario where node M is moving into the radio range of node N at velocity $\overrightarrow{v}$. We take the position of node N as the center of the coordinate system and the direction of M's velocity $\overrightarrow{v}$ to be the direction of x-axis. We first examine the case where the direction of $\overrightarrow{v}$ does not change while M covers the distance, l_0, from E to X.

To calculate l_0, we introduce two auxiliary segments NX and NE as shown in the figure. Due to symmetry, we only consider NX and denote the angle between x-axis and segment NX as θ. We also use y to denote the intersection of EX and y-axis. It is easy to see that

$$l_0 = \|EX\| = 2\sqrt{r^2 - y^2} = 2r\cos\theta.$$

Since the intersection point y can occur anywhere in the range of $(-r, r)$, the expected distance covered within the circle, $\bar{l}_0$, can be given as follows:

$$\begin{aligned} \bar{l}_0 &= \frac{1}{2r}\int_{-r}^{r} 2r\cos\theta dy \\ &= \frac{1}{2r}\int_{-r}^{r} 2\sqrt{r^2 - y^2}dy \\ &= \frac{\pi r}{2} \end{aligned} \tag{1}$$

Here we assumed that the PDF of a mobile node crossing the y-axis at a point y is uniform in the range $(-r, r)$. This is reasonable as we assume that $r \ll a$. Therefore the expected contact time, T_C, is given by

$$\bar{T}_C = \frac{\bar{l}_0}{\bar{v}} = \frac{\pi r}{2\bar{v}} \tag{2}$$

The result above is obtained under the assumption that

$\overrightarrow{v}$ does not change. More detailed analysis that considers possible epoch change is given in [1].

3.2 Inter-contact Times in Random Waypoint Model

We first show that after reaching stationary distribution, the contact times of mobile nodes at a static location can be closely approximated as exponentially distributed, and show that result also holds true for inter-contact times under our assumptions regarding the network. For the simplicity of expressions, some probabilities may appear in unnormalized forms. For the same reason, we ignore T_{pause} in our analysis. To include the effect of the pause time, speed v of a single node can be replaced with $v' = v * \bar{T}/(\bar{T} + \bar{T}_{pause})$ for the adjusted speed.

Due to ergodicity properties of node movements in RWP, we only have to consider the mobility of a single node. Given that the PDF $f(x, y)$ denotes the probability of mobile node M being found at position (x, y), the probability, P_a, of M going through the radio range of N in an epoch is approximated as

$$P_a = 2r\bar{L}f(x, y) \tag{3}$$

This is due to the fact that M can cross anywhere in the segment $(-r, r)$ of length $2r$ in the y-axis, as shown in Figure 1, at any phase of an epoch. Here we used $f(x, y)$ to approximate the PDF of node distribution within the radio range, which is reasonable under the assumption that $r \ll a$. Although two consecutive epochs are not independent due to the overlap of end-points, we can view epochs in RWP model as independent as shown in [6] due to ergodicity. Let N_{hit} denote the time that M arrives at N, and let $P(N_{hit} > n)$ denote probability that M has not encountered N till after n epochs, then

$$P(N_{hit} > n) = \left(1 - 2r\bar{L}f(x, y)\right)^n \tag{4}$$

This result is given in [14].

Let us now consider the same problem in time scale. Given the probability of contact, P_a, in an epoch, let λ denote the rate of contact as follows:

$$\lambda = \frac{P_a}{\bar{T}} = \frac{2r\bar{L}f(x, y)}{\bar{T}} = 2r\bar{v}f(x, y)$$

Then the relation given in (4) can be written as

$$P(T_{hit} > n\bar{T}) = \left(1 - \lambda\bar{T}\right)^n \tag{5}$$

where T_{hit} denotes the time that M comes into contact with N, and $P(T_{hit} > n\bar{T})$ denotes that probability that M has not come into contact with N by time T_{hit}.

Since the node movement in one epoch continues in the same direction till the end of an epoch, we cannot extend the results above for arbitrarily small time intervals as the independence assumption does not hold within an epoch. However, when the average epoch time, $\bar{T}$, is much smaller than the expected inter-contact time, $\bar{T}_a$, the geometric distribution given in (5) can be closely approximated with the following exponential distribution in continuous time domain:

$$P(T_{hit} > t) = e^{-\lambda t} \tag{6}$$

which gives the CDF, $F(t)$, as

$$F(t) = 1 - e^{-\lambda t}$$

where $\lambda = 2r\bar{v}f(x, y)$.

To see this, we consider the ratio of $\bar{T}$ to $\bar{T}_a$ in a square area of size $a \times a$. Without loss of generality, we assume that $f(x, y) = 1/a^2$, and get

$$\frac{\bar{T}}{\bar{T}_a} = \frac{\bar{L}/\bar{v}}{1/(2r\bar{v}f(x, y))} = \frac{2\bar{L}r}{a^2} \approx \frac{r}{a}$$

Since it is assumed that $r \ll a$, the assumption that $\bar{T} \ll \bar{T}_a$ also holds. Similar results can be obtained for circular or rectangular areas.

Before we discuss the inter-contact times when both nodes are moving according to the RWP model, we first show that the relative movement angle of the two mobile nodes is uniformly distributed.

THEOREM 1. *Let A and B be two mobile nodes in a circular region moving according to the Random Waypoint mobility model, then the relative movement angle, $\theta_{v_{AB}}$, between their velocities is uniformly distributed. That is:*

$$f_{\Theta_{V_{AB}}}(\theta_{v_{AB}}) = \frac{1}{2\pi}, \quad \theta_{v_{AB}} \in [0, 2\pi)$$

where $f_{\Theta_{V_{AB}}}(\theta_{v_{AB}})$ is the PDF of relative movement angle distribution.

PROOF. Refer to [1] for the proof. □

We can see that independent of the PDF of node distribution, $f_R(r)$, and the PDF of movement angle distribution, $f_{\Theta_V}(\theta_v)$, the uniformity of relative movement angles holds in a circular region due to rotational symmetry.

Now let us discuss the inter-contact time among mobile nodes. Let $f(x, y)$ denote the stationary node distribution PDF at location (x, y), and let $\tilde{v}$ denote the expected relative speed of the mobile nodes, then the PDF of node distribution for node N at location (x, y) is given by $f(x, y)$. And, following (3), the probability, $P_a(x, y)$, of node M meeting node N at $X(x, y)$ in one epoch is given by

$$P_a(x, y) = 2r\bar{L}f^2(x, y)$$

where $\tilde{v}$ is the relative speed of M and N. The unconditional probability, P_U, for area $\mathcal{A}$ is given by

$$P_U = \iint 2r\bar{L}f^2(x, y)dxdy = \lambda\bar{T}$$

where $\lambda = 2\rho r\tilde{v}/A$, in which $\rho = A\iint f^2(x, y)dxdy$.

Following similar arguments for the static location case, the approximated CDF of inter-contact times is approximated as follows:

$$F(t) = 1 - e^{-\lambda t} \tag{7}$$

where $\lambda = 2\rho r\tilde{v}/A$.

In RWP model, the value of ρ depends on the shape of $\mathcal{A}$, and can be calculated analytically or numerically when $f(x, y)$ is known. For the square and the circular areas, approximate values of ρ can be calculated as 1.3683 [8][1] and 1.4155 (see [1]), respectively.

The average speed of a single node under RWP model is given as below [15]:

$$\bar{v} = \frac{\bar{L}}{\bar{T}} = \frac{v_{max} - v_{min}}{\ln(v_{max}/v_{min})}$$

[1]This value is calculated in [8] based on analytical results from [11]. However, note that numerically calculating according to analytical results from [7] gives 1.3805.

This is due to the fact that the time a node spends at speed v is inversely proportional to v in RWP model. The relative speed of two mobile nodes is given by

$$\tilde{v}_{rwp} = \frac{1}{2\pi(v_{max}-v_{min})^2}\int_{v_{min}}^{v_{max}}\int_{v_{min}}^{v_{max}}\int_0^{2\pi} \frac{\sqrt{(v_2\sin\theta)^2+(v_1-v_2\cos\theta)^2}}{v_1 v_2} d\theta dv_2 dv_1 \quad (8)$$

where θ is the relative angle between $\overrightarrow{v}_1$ and $\overrightarrow{v}_2$, measured counter-clockwise from the direction of $\overrightarrow{v}_1$. As shown in Theorem 1, we take that θ is uniformly distributed. Under the simplifying condition that $v_{min} = v_{max} = \bar{v}$ or $v_{min}/v_{max} \approx 1$, the relative speed can be approximated as follows:

$$\tilde{v} = \frac{\bar{v}}{2\pi}\int_0^{2\pi}\sqrt{\sin^2\theta + (1-\cos\theta)^2}d\theta = \frac{4\bar{v}}{\pi} \quad (9)$$

where θ is the angle between two movement directions. Inter-contact times under Random Direction model is given in [1].

The average relative mobile speed, v_{rd}, under RD model is given as:

$$\tilde{v}_{rd} = \frac{1}{2\pi(v_{max}-v_{min})^2}\int_{v_{min}}^{v_{max}}\int_{v_{min}}^{v_{max}}\int_0^{2\pi} \sqrt{(v_2\sin\theta)^2+(v_1-v_2\cos\theta)^2}d\theta dv_2 dv_1 \quad (10)$$

where θ is the relative angle between $\overrightarrow{v}_1$ and $\overrightarrow{v}_2$, measured counter-clockwise from the direction of $\overrightarrow{v}_1$.

Note that our result for inter-contact rate for RWP is different from the result for meeting times given in [14], where the difference of inter-contact rates among mobile nodes under RWP and RD models is attributed to relative speed on the account that the relative movement angle is not uniform under RWP model. With Theorem 1 we see that the relative movement angle can also be considered uniform under RWP, and that the difference in inter-contact rates under these two models is a combined result of non-uniform node distribution under RWP as explained by factor ρ and the differences in the calculation of the relative movement speed for RWP and RD models, as shown by Equations (8) and (10), respectively.

3.3 Observed Rate of Node Arrivals

In the analysis above, we assumed that when a mobile node enters within the range of another node it is immediately sensed. In practical application scenarios, however, nodes announce their presence by broadcasting *HELLO* or *beacon* messages at regular intervals, and a node may not be sensed if it finishes crossing the radio circle of another node before it sends a *HELLO* message. Other factors such as transmission errors, channel contention, etc., can also cause nodes to miss *HELLO* messages. Under such circumstances, the observed inter-contact rate will be different than the inter-contact rates discussed earlier in this section. Here we focus on the effect of *HELLO* interval on the observed rate of inter-contact.

We first define the expected distance covered by the mobile node within two hello messages as *HELLO Distance*, L_H, as $L_H = T_H * \bar{v}$, where T_H is the *HELLO* interval and $\bar{v}$ is the average speed ($\tilde{v}$ is used when both nodes are mobile). For practical purposes we assume that $0 < L_H < 2r$.

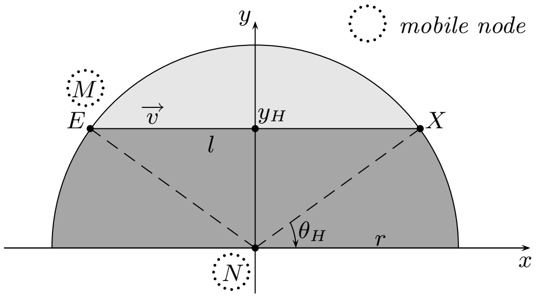
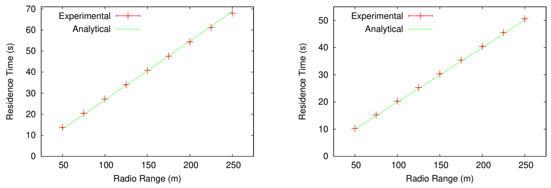


Figure 2: Calculation of Observed Arrival Rate



(a) Static Nodes (b) Mobile Nodes

Figure 3: Contact Times

THEOREM 2. *Let λ denote the theoretical inter-contact rate, the observed inter-contact rate, λ', is given as*

$$\lambda' = \lambda(\sin\theta_H + \frac{\pi - 2\theta_H - \sin 2\theta_H}{4\cos\theta_H}) \quad (11)$$

where $\theta_H = arccos(L_H/2r)$.

PROOF. Refer to [1] for the proof. ☐

As expected, we notice that observed rate of inter-contact decrease as we increase the *HELLO* interval.

4. EXPERIMENTAL RESULTS

In this section, we present experimental results for mobility characteristics. The goal of our experiments is to verify the correctness of analytical results regarding contact times and inter-contact times.

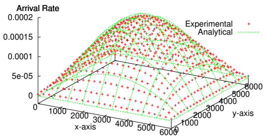
4.1 Experimental Settings

Most of our experiments use the *ns-2* network simulator extended with our own code. We also use our custom simulator for experiments in circular areas, as well as for detailed measurements of the relative speed and movement angles.

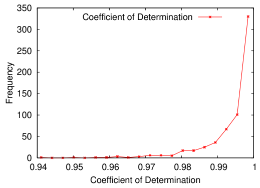
The default settings for *ns-2* simulations are as follows. Each simulation run has 40 nodes moving according to the specified mobility model in a $6000m \times 6000m$ square area. By default, nodes have a radio range of $250m$. Minimum and maximum speeds, v_{min} and v_{max}, are $3m/s$ and $10m/s$, respectively. The *HELLO* interval is set to 3 seconds.

4.2 Mobility Characteristics

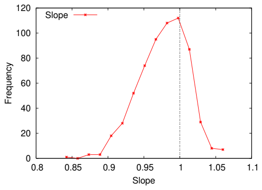
Below we discuss the simulation results for contact times and inter-contact times for static locations as well as for cases where both nodes are mobile. For this purpose, we place static nodes on a 25×25 grid, where neighboring nodes are $250m$ apart, for a total of 625 static nodes.



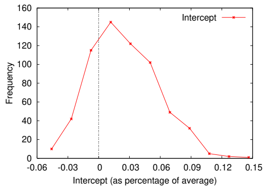
(a) Inter-contact Rate at Static Locations



(b) Coefficient of Determination

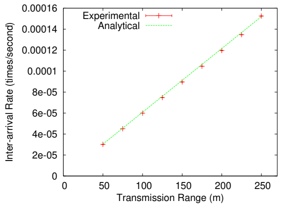


(c) Slope

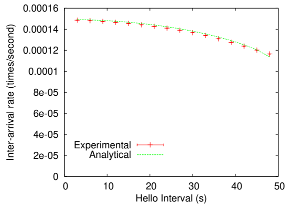


(d) Intercept

Figure 4: Linear Regression Analysis for the Exponentiality of Inter-contact Times



(a) Transmission range



(b) Hello interval

Figure 5: Effects of Transmission Range and Hello Interval on Inter-contact Rate

4.2.1 *Contact Times*

Figure 3 shows the experimental and analytical results for contact times at static locations and among mobile nodes under RWP model. Analytical results for these two metrics are calculated according to Equation (2). We can see that our analysis closely approximates the experimental results. We also obtain similar results for the RD model.

4.2.2 *Inter-contact Times*

Under RWP model, inter-contact rates at specific locations depend on the PDF of node distribution. Figure 4(a) shows the average inter-contact rates observed by 625 static nodes. For each static location, we run similar linear regression analysis as described above for mobile nodes. Overall statistics for regarding R^2 values, slopes, and intercepts are shown in Figures 4(b)–4(d). We can see that exponentiality of inter-contact times strongly holds at static locations. Results for inter-contact times when both nodes are mobile are given in [1].

The effects of transmission range and *HELLO* interval on the inter-contact rate is shown in Figure 5(a) and 5(b), respectively. These experimental results conform to analytical values that we obtain from (7) and (11).

Figure 6 shows the experimental and analytical results for inter-contact rates under varying speed in square and circular areas of the same size. As can be seen, Equation (7) with different ρ values accurately explains differences.

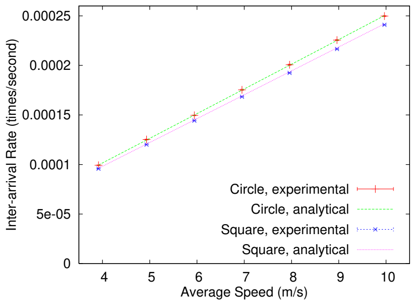


Figure 6: Inter-contact Rates in Square and Circular Areas under Varying Speed

5. CONCLUSIONS

In this paper, we studied the properties of two commonly-used mobility models: Random Waypoint and Random Direction. We showed analytically that the inter-contact times of nodes can be approximated by the exponential distribution in these models under opportunistic network settings. We also provided analytical results for contact time, inter-contact time, and the effect of *HELLO* intervals. Through extensive simulation study, we showed that our analytical results for mobility characteristics are accurate.

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