

Untyped Lambda Calculus with Extensions.

$t ::= x \mid \lambda x.t \mid (t \ t) \mid \text{true} \mid \text{false} \mid \text{if } t \ t \ t \mid \sim t \mid t = t$
 $\mid \langle \# \rangle \mid t + t \mid t - t \mid t * t \mid t < t \mid t > t$
 $\mid \text{nil} \mid \text{cons } t \ t \mid \text{isnil } t \mid \text{head } t \mid \text{tail } t$
 $\mid \text{fix } t$

$v ::= \lambda x.t \mid \text{true} \mid \text{false} \mid \langle \# \rangle \mid \text{nil} \mid \text{cons } t \ t$

Evaluation Rules

$$\mathbf{E-App1} \frac{t_1 \rightarrow t'_1}{(t_1 \ t_2) \rightarrow (t'_1 \ t_2)}$$

$$\mathbf{E-App2} \frac{t_2 \rightarrow t'_2}{(v \ t_2) \rightarrow (v \ t'_2)}$$

$$\mathbf{E-App-Abs} \frac{}{((\lambda x.t) \ v) \rightarrow (t[x \mapsto v])}$$

$$\mathbf{E-If} \frac{t_1 \rightarrow t'_1}{\text{if } t_1 \ t_2 \ t_3 \rightarrow \text{if } t'_1 \ t_2 \ t_3}$$

$$\mathbf{E-If-true} \frac{}{\text{if true } t_2 \ t_3 \rightarrow t_2}$$

$$\mathbf{E-If-false} \frac{}{\text{if false } t_2 \ t_3 \rightarrow t_3}$$

$$\mathbf{E-Neg1} \frac{t \rightarrow t'}{\sim t \rightarrow \sim t'}$$

$$\mathbf{E-Neg-T} \frac{}{\sim \text{true} \rightarrow \text{false}}$$

$$\mathbf{E-Neg-F} \frac{}{\sim \text{false} \rightarrow \text{true}}$$

$$\mathbf{E-Eq1} \frac{t_1 \rightarrow t'_1}{t_1 = t_2 \rightarrow t'_1 = t_2}$$

$$\mathbf{E-Eq2} \frac{t_2 \rightarrow t'_2}{v = t_2 \rightarrow v = t'_2}$$

$$\mathbf{E-Eq} \frac{\text{ans: are } v_1, v_2 \text{ } \alpha\text{-equivalent?}}{v_1 = v_2 \rightarrow \text{ans}}$$

$$\mathbf{E-Add1} \frac{t_1 \rightarrow t'_1}{t_1 + t_2 \rightarrow t'_1 + t_2}$$

$$\mathbf{E-Add2} \frac{t_2 \rightarrow t'_2}{v + t_2 \rightarrow v + t'_2}$$

$$\mathbf{E-Add} \frac{}{v_1 + v_2 \rightarrow (\text{perform addition})}$$

$$\mathbf{E-GT1} \frac{t_1 \rightarrow t'_1}{t_1 > t_2 \rightarrow t'_1 > t_2}$$

$$\mathbf{E-GT2} \frac{t_2 \rightarrow t'_2}{v > t_2 \rightarrow v > t'_2}$$

$$\mathbf{E-GT} \frac{}{v_1 > v_2 \rightarrow (\text{perform relation check})}$$

E-Mul/1/2, E-Sub/1/2, E-LT/1/2 follow the same patterns as E-Add/1/2, E-GT/1/2.

$$\mathbf{E-isnil1} \frac{t \rightarrow t'}{\text{isnil } t \rightarrow \text{isnil } t'}$$

$$\mathbf{E-isnil-nil} \frac{}{\text{isnil nil} \rightarrow \text{true}}$$

$$\mathbf{E-isnil-cons} \frac{}{\text{isnil } (\text{cons } t_1 \ t_2) \rightarrow \text{false}}$$

$$\mathbf{E-head1} \frac{t \rightarrow t'}{\text{head } t \rightarrow \text{head } t'}$$

$$\mathbf{E-head} \frac{}{\text{head } (\text{cons } t_1 \ t_2) \rightarrow t_1}$$

$$\mathbf{E-tail1} \frac{t \rightarrow t'}{\text{tail } t \rightarrow \text{tail } t'}$$

$$\mathbf{E-tail} \frac{}{\text{tail } (\text{cons } t_1 \ t_2) \rightarrow t_2}$$

$$\mathbf{E-Fix} \frac{}{\text{fix } (\lambda x.t) \rightarrow t[x \mapsto \text{fix } (\lambda x.t)]}$$