CS 463: Quiz 1: Untyped Lambda Calculus
Spring 2024 **CLOSED NOTES**

Name Answer Key
G#:

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1. **(7pts) Review the tiny language to the right.** Simplify each of the three given expression of the language to a value. You must show every single step, and you must fully name the steps used.

$\mathbf{t ::= x \mid \lambda x.t \mid (t\ t) \mid \mathbb{Z} \mid t + t}$
$\mathbf{v ::= \lambda x.t \mid \mathbb{Z}}$

E-App1 $\dfrac{t_1 \to t_1'}{(t_1\ t_2) \to (t_1'\ t_2)}$

E-App2 $\dfrac{t_2 \to t_2'}{(v\ t_2) \to (v\ t_2')}$

E-App-Abs $\dfrac{}{((\lambda x.t)\ v) \to t[x \mapsto v]}$

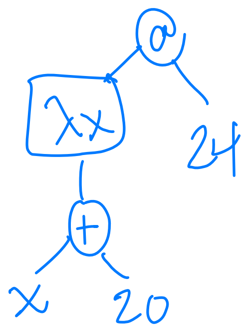
E-Add1 $\dfrac{t_1 \to t_1'}{(t_1 + t_2) \to (t_1' + t_2)}$

E-Add2 $\dfrac{t_2 \to t_2'}{(v + t_2) \to (v + t_2')}$

E-Add $\dfrac{v_1, v_2 \in \mathbb{Z}}{(v_1 + v_2) \to \text{<sum of } v_1, v_2\text{>}}$

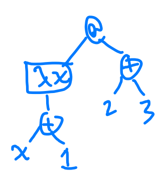
((λx . x+20) 24)

⇒ E-App-Abs
24 + 20
⇒ E-Add



((λx.x+1) (2 + 3))

⇒ E-App2 via E-Add
((λx.x+1) 5)
⇒ E-App-Abs
5 + 1
⇒ E-Add
6



(((λa. λb. b) 3) 4)

⇒ E-App1 via E-App-Abs
((λb.b) 4)
⇒ E-App-Abs
4

2. **(3pts)** Using the same language above (*no further extensions!*), we'll *encode* Booleans as before:
true = λa. λb. a **false** = λa. λb. b
Now, <u>define encodings of **not** and **and**</u>, similar to how we defined **nand** in the homework. Your answers should be functions that accept one (not) or two (and) arguments (arguments are assumed to be the true/false encodings above), and return the correct logical answer.

not =	λx. x false true
and =	λx. λy. x y x