

1

a (+6)

$S \rightarrow T | \epsilon$
 $T \rightarrow TV | V$
 $V \rightarrow () | \{\} | [] | (T) | \{T\} | [T]$

other solutions are possible.

b

(+6)

S
 $\Rightarrow T$
 $\Rightarrow TV$
 $\Rightarrow VV$
 $\Rightarrow \{T\}V$
 $\Rightarrow \{V\}V$
 $\Rightarrow \{[]\}V$
 $\Rightarrow \{[]\}()$

S : accounts for the empty string.
 T : introduces s.t.e-by-sides.
 V : handles nested parts.

- (: skipped step.
 derivation doesn't need to be leftmost or rightmost.

2

a

(+3)

S
 $\Rightarrow S+N$
 $\Rightarrow N+N$
 $\Rightarrow 2+N$
 $\Rightarrow 2+3$

(+3)

```

    S
   / \
  S  +
 / \
S  2
|  |
2  3
    
```

b

(+3)

S
 $\Rightarrow S+N$
 $\Rightarrow S+N+N$
 $\Rightarrow N+N+N$
 $\Rightarrow 5+N+N$
 $\Rightarrow 5+8+N$
 $\Rightarrow 5+8+9$

(+3)

```

    S
   / | \
  S  +  N
 / | \
S  8  9
|  |
5  2
    
```

- 1,5 : derivation isn't leftmost

3

a

(+8: two parse trees)

```

    S
   / | \
  S  *  S-N-S
 / \
S  *
|  |
S  2-4
|  |
3  4
    
```

```

    S
   / | \
  S  *  S
 / \
S  *
|  |
S  2-4
|  |
3  5
    
```

b

(+4)

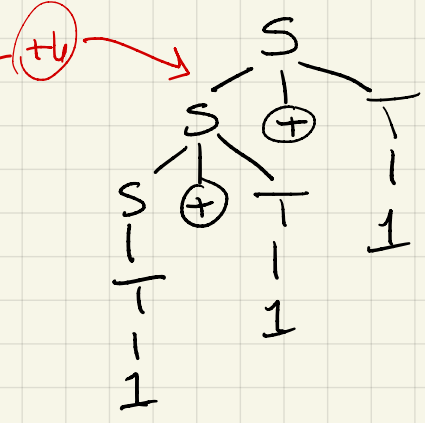
yes, it's ambiguous.
 We just drew two different parse trees of the same sentence.

4

Here, \oplus is left-associative. (+4)

Consider $1+1+1$:

- $\Rightarrow \underline{S} + T$
- $\Rightarrow \underline{S} + T + T$
- $\Rightarrow \underline{T} + T + T$
- $\Rightarrow \underline{1} + T + T$
- $\Rightarrow \underline{1} + \underline{1} + T$
- $\Rightarrow \underline{1} + \underline{1} + \underline{1}$



Notice the tree is "left-weighted".

5

a) $S \rightarrow l|o|m|f|g|ll|oo|mm|ff|gg|lsl|oso|msm|fsf|gsg$ (+8)

middle of odd-length sentences *middle of even-length ones* *onion-layers of matching terminals.*

b

- $\Rightarrow \underline{S}$
- $\Rightarrow lsl$
- $\Rightarrow loSof$
- $\Rightarrow lolol //$

*note: no ϵ possible.

6

a) $S \rightarrow S \text{ and } T \mid T$ (+5)

$T \rightarrow T \text{ or } U \mid U$

$U \rightarrow \text{true} \mid \text{false}$

alternative: right associative.

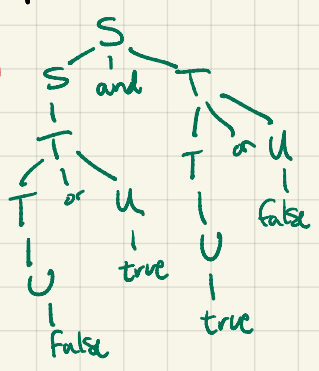
$S \rightarrow T \text{ and } S \mid T$

$T \rightarrow U \text{ or } T \mid U$

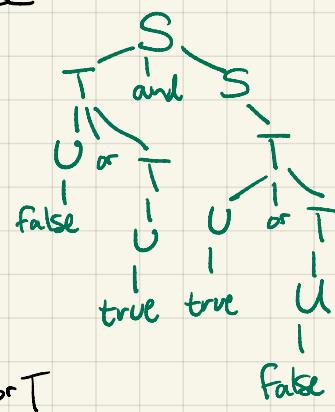
$U \rightarrow \text{true} \mid \text{false}$

b

- $\Rightarrow \underline{S}$
- $\Rightarrow \underline{S} \text{ and } T$
- $\Rightarrow \underline{T} \text{ and } T$
- $\Rightarrow \underline{T} \text{ or } U \text{ and } T$
- $\Rightarrow \underline{U} \text{ or } U \text{ and } T$
- $\Rightarrow \text{false or } \underline{U} \text{ and } T$
- $\Rightarrow \text{false or true and } \underline{T}$
- $\Rightarrow \text{false or true and } \underline{T} \text{ or } U$
- $\Rightarrow \text{false or true and } \underline{U} \text{ or } U$
- $\Rightarrow \text{false or true and } \underline{\text{true or } U}$
- $\Rightarrow \text{false or true and } \underline{\text{true or false}}$

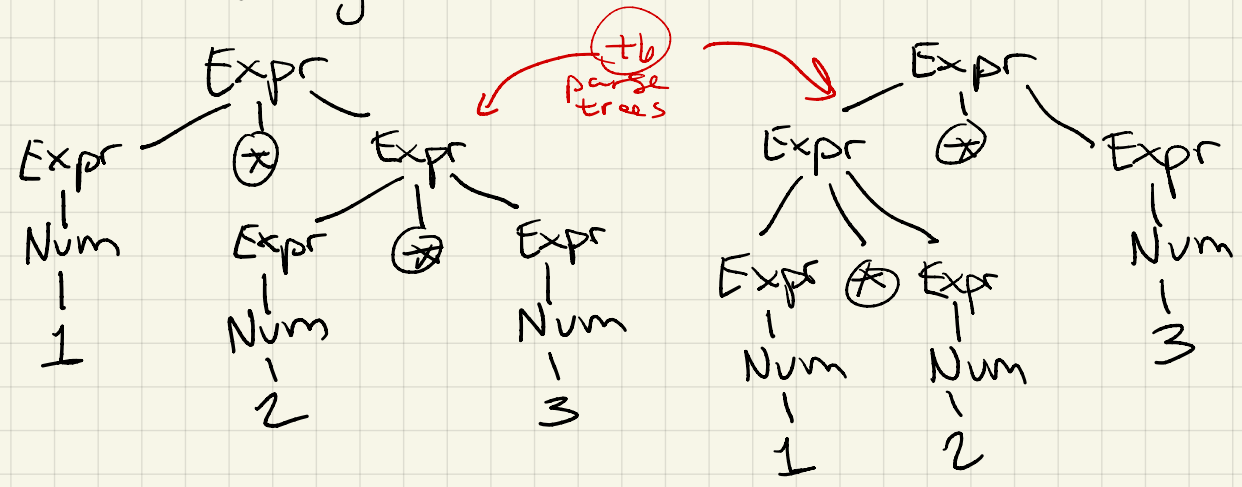


- $\Rightarrow T \text{ and } S$
- $\Rightarrow U \text{ or } T \text{ and } S$
- $\Rightarrow \text{false or } T \text{ and } S$
- $\Rightarrow \text{false or } U \text{ and } S$
- $\Rightarrow \text{false or true and } S$
- $\Rightarrow \text{false or true and } T$
- $\Rightarrow \text{false or true and } U \text{ or } T$
- $\Rightarrow \text{false or true and true or } T$
- $\Rightarrow \text{false or true and true or } U$
- $\Rightarrow \text{false or true and true or false}$



-2,5 : ambiguous. (doesn't have to have left assoc. operators)
 -3 : introduced ()'s

7 a) 1*2*3 ⁺² has two valid parse trees, so the language is ambiguous.



b $Expr \rightarrow Num * Expr \mid Num / Expr \mid Num$ ^{+4: right-associative}
 $Num \rightarrow (Expr) \mid 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$ ^{+2: unambiguous, same language.}
^{+2: parens handled correctly.}

8

- a) yes, +3
- b) yes, +3
- c) no, +3
- d) yes, +3