## Homework Help

Section 1.3, Problem 9

February 2, 2006

## Exercise 1.1-9

Design an algorithm for the following problem: Given a set of n points in the x-y coordinate plane, determine whether all of them lie on the same circumference.

## Help

I am not going to provide a well-specified algorithm here, but simply describe the simplest (and ugliest) solution of which I could conceive for this problem. There are other, more elegant methods. I'll leave it to you to turn my description into a well-specified algorithm, or to investigate a different method.

First, understand that you only need to consider situations where there are 3 or more points — one can always draw a circle that connects any two points in a plane (actually, one can draw more than one circle). So pick an arbitrary three unique points from your list and *imagine* they lie on a circle together with some center  $(x_c, y_c)$ :



With this and help from Pythagorus, we have three equations and three unknowns  $(x_c, y_c, \text{ and } r)$ :

$$(x_1 - x_c)^2 + (y_1 - y_c)^2 = r^2$$
  

$$(x_2 - x_c)^2 + (y_2 - y_c)^2 = r^2$$
  

$$(x_3 - x_c)^2 + (y_3 - y_c)^2 = r^2$$

Apply a standard algorithm for solving for the three unknown values (investigate this for yourselves). There are two main potential results. First, it is possible that the system cannot be solved ... then you are done, the answer is that no such circle can be drawn. If the system can be solved, it is certainly the case that only one such circle can be drawn if the points all lie in the same plane. Having solved it, you now know the circle's center and its radius. So just march through the list and check to make sure all the points are of distance r from the center point. If all points are r from the center then the answer is "yes," otherwise the answer is "no."

What is the time complexity of my algorithm?