CS 483 - Data Structures and Algorithm Analysis

Lecture V: Chapter 5, part 1

R. Paul Wiegand

George Mason University, Department of Computer Science

February 22, 2006

Outline

1. Introduction to Decrease-And-Conquer
2. The InsertionSort Algorithm
3. Depth-First and Breadth-First Searching
4. Homework

Decrease-And-Conquer

- Decrease-and-conquer exploits the relationship between a solution to a given problem instance and a solution to a smaller instance of the same problem
- Divide-and-conquer attempts to solve separate pieces of the problem, then combine the pieces into an answer, while Decrease-and-conquer attempts to say something about the total solution in terms of the solution to the smaller piece
- Can be approached top-down (recursively) or bottom-up
- Three variations:

decrease by a constant — Each iteration, the size of a problem instance is reduced by a constant (e.g., $n-1$)

decrease by a constant *factor* — Each iteration, the size of a problem instance is reduced by a constant factor (e.g., $\frac{n}{2}$)

variable size decrease — The reduction pattern varies with each iteration (e.g., EUCLID)

Simple Examples of Decrease-and-Conquer

Consider the problem of computing $f(n) = a^n$:

- Decrease by a constant:
  $$f(n) = \begin{cases} f(n-1) \cdot a & \text{if } n > 1 \\ a & \text{if } n = 1 \end{cases}$$
- Decrease by a constant factor:
  $$a^n = \begin{cases} \left(a^{n/2}\right)^2 & \text{if } n > 0 \text{ is even} \\ \left(a^{(n-1)/2}\right)^2 \cdot a & \text{if } n > 1 \text{ is odd} \\ a & \text{if } n = 1 \end{cases}$$

Simple Examples of Decrease-and-Conquer

Consider the problem of computing $f(n) = a^n$:

- Decrease by a constant:

$$f(n) = \begin{cases} f(n-1) \cdot a & \text{if } n > 1 \\ a & \text{if } n = 1 \end{cases}$$

★ Decrease by a constant factor:

$$a^n = \begin{cases} \left(a^{n/2}\right)^2 & \text{if } n > 0 \text{ is even} \\ \left(a^{(n-1)/2}\right)^2 \cdot a & \text{if } n > 1 \text{ is odd} \\ a & \text{if } n = 1 \end{cases}$$

★ We are *not* solving *each piece*

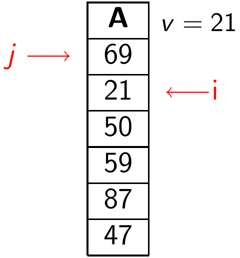
★ We are using knowledge about how the solution to the piece relates to the whole problem

★ $O(\lg n)$

Specifying InsertionSort

InsertionSort($A[0 \ldots n-1]$)

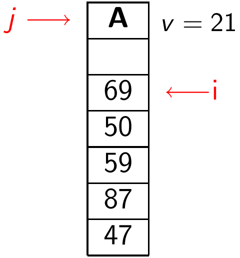
for $i \longleftarrow 1$ to $n-1$ do
  $v \longleftarrow A[i]$
  $j \longleftarrow i-1$
  while $j \geq 0$ and $A[j] > v$ do
    $A[j+1] \longleftarrow A[j]$
    $j \longleftarrow j-1$
  $A[j+1] \longleftarrow v$



Specifying INSERTIONSORT

INSERTIONSORT($A[0 \ldots n-1]$)

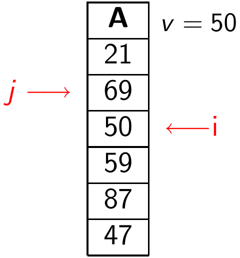
for $i \longleftarrow 1$ **to** $n-1$ **do**
  $v \longleftarrow A[i]$
  $j \longleftarrow i-1$
  while $j \geq 0$ **and** $A[j] > v$ **do**
    $A[j+1] \longleftarrow A[j]$
    $j \longleftarrow j-1$
  $A[j+1] \longleftarrow v$



Specifying INSERTIONSORT

INSERTIONSORT($A[0 \dots n-1]$)

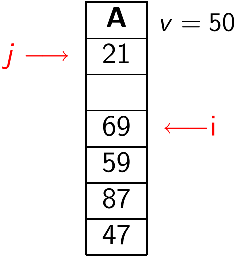
for $i \longleftarrow 1$ to $n-1$ do
  $v \longleftarrow A[i]$
  $j \longleftarrow i-1$
  while $j \geq 0$ and $A[j] > v$ do
    $A[j+1] \longleftarrow A[j]$
    $j \longleftarrow j-1$
  $A[j+1] \longleftarrow v$



Specifying InsertionSort

InsertionSort($A[0 \dots n-1]$)

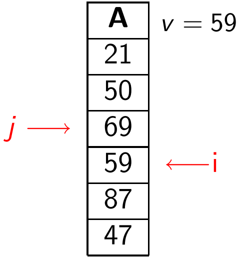
for $i \longleftarrow 1$ to $n-1$ do
  $v \longleftarrow A[i]$
  $j \longleftarrow i-1$
  while $j \geq 0$ and $A[j] > v$ do
    $A[j+1] \longleftarrow A[j]$
    $j \longleftarrow j-1$
  $A[j+1] \longleftarrow v$



Specifying INSERTIONSORT

INSERTIONSORT($A[0 \ldots n-1]$)

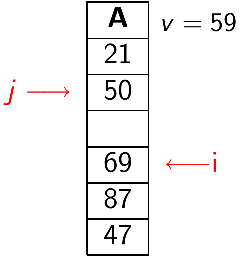
for $i \longleftarrow 1$ to $n-1$ do
  $v \longleftarrow A[i]$
  $j \longleftarrow i-1$
  while $j \geq 0$ and $A[j] > v$ do
    $A[j+1] \longleftarrow A[j]$
    $j \longleftarrow j-1$
  $A[j+1] \longleftarrow v$



Specifying InsertionSort

InsertionSort($A[0 \ldots n-1]$)

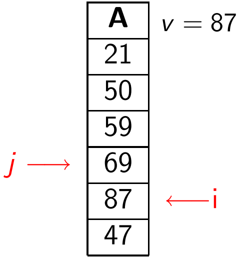
for $i \longleftarrow 1$ to $n-1$ do
  $v \longleftarrow A[i]$
  $j \longleftarrow i - 1$
  while $j \geq 0$ and $A[j] > v$ do
    $A[j+1] \longleftarrow A[j]$
    $j \longleftarrow j - 1$
  $A[j+1] \longleftarrow v$



Specifying InsertionSort

InsertionSort($A[0 \dots n-1]$)

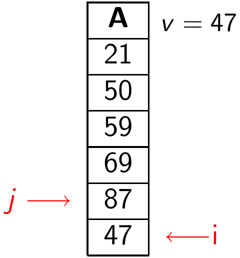
for $i \longleftarrow 1$ to $n-1$ do
  $v \longleftarrow A[i]$
  $j \longleftarrow i-1$
  while $j \geq 0$ and $A[j] > v$ do
    $A[j+1] \longleftarrow A[j]$
    $j \longleftarrow j-1$
  $A[j+1] \longleftarrow v$



Specifying InsertionSort

InsertionSort($A[0 \ldots n-1]$)

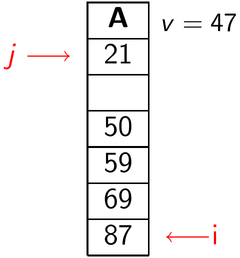
```
for i ⟵ 1 to n − 1 do
  v ⟵ A[i]
  j ⟵ i − 1
  while j ≥ 0 and A[j] > v do
    A[j + 1] ⟵ A[j]
    j ⟵ j − 1
  A[j + 1] ⟵ v
```



Specifying InsertionSort

InsertionSort($A[0 \dots n-1]$)

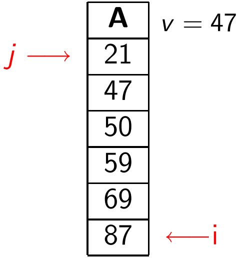
```
for i ⟵ 1 to n − 1 do
  v ⟵ A[i]
  j ⟵ i − 1
  while j ≥ 0 and A[j] > v do
    A[j + 1] ⟵ A[j]
    j ⟵ j − 1
  A[j + 1] ⟵ v
```



Specifying InsertionSort

InsertionSort($A[0 \ldots n-1]$)

for $i \longleftarrow 1$ to $n-1$ do
  $v \longleftarrow A[i]$
  $j \longleftarrow i-1$
  while $j \geq 0$ and $A[j] > v$ do
    $A[j+1] \longleftarrow A[j]$
    $j \longleftarrow j-1$
  $A[j+1] \longleftarrow v$



It's like arranging cards in your hand!

Comments About InsertionSort

- When dealing with $A[n-1]$, we assume that the $A[0 \ldots n-2]$ problem has already been solved
- We find an appropriate position for $A[n-1]$ and insert it
- The *idea* of this algorithm is recursive, but a bottom-up, iterative implementation is typically best
- One way to speed up insertion is to use BinarySearch to find the position (aka *binary insertion sort*)

Analyzing (Straight) InsertionSort

We count $A[j] > v$ comparisons, analysis depends on data ...

- Worst case:
- Best case:
- Average case:

Analyzing (Straight) InsertionSort

We count $A[j] > v$ comparisons, analysis depends on data ...

- Worst case:
  - All elements in the sublist are shifted every insertion
  - This occurs when A is initially strictly decreasing
  - $C_{worst}(n) = \sum_{i=1}^{n-1} \sum_{j=0}^{i-1} i = \frac{(n-1)n}{2} \in \Theta(n^2)$
- Best case:
- Average case:

Analyzing (Straight) InsertionSort

We count $A[j] > v$ comparisons, analysis depends on data ...

- Worst case:
  - All elements in the sublist are shifted every insertion
  - This occurs when A is initially strictly decreasing
  - $C_{worst}(n) = \sum_{i=1}^{n-1} \sum_{j=0}^{i-1} i = \frac{(n-1)n}{2} \in \Theta(n^2)$
- Best case:
  - We check each insertion, but no shift is necessary
  - $C_{best}(n) = \sum_{i=1}^{n-1} 1 = n - 1 \in \Theta(n)$
- Average case:

Analyzing (Straight) InsertionSort

We count $A[j] > v$ comparisons, analysis depends on data ...

- Worst case:
  - All elements in the sublist are shifted every insertion
  - This occurs when A is initially strictly decreasing
  - $C_{worst}(n) = \sum_{i=1}^{n-1} \sum_{j=0}^{i-1} i = \frac{(n-1)n}{2} \in \Theta(n^2)$
- Best case:
  - We check each insertion, but no shift is necessary
  - $C_{best}(n) = \sum_{i=1}^{n-1} 1 = n - 1 \in \Theta(n)$
- Average case:
  - Investigate number of pairs of elements that are out of order
  - On randomly ordered arrays, InsertionSort makes on average half as many comparisons as on decreasing arrays
  - $C_{avg}(n) \approx \frac{n^2}{4} \in \Theta(n^2)$

Searching Graphs

- Solutions to many problems involve searching through a graph
- There are a variety of ways of to search a graph ...
- But there's a simple generalization for many methods:

GraphSearch(G, a)

$WaitingList \longleftarrow \langle a \rangle$
$VisitedList \longleftarrow \langle\rangle$
while not Empty$(WaitingList)$
  $v \longleftarrow$ GetAndRemoveItem$(WaitingList)$
  $ChildrenList \longleftarrow$ GetChildVertices(G, v)
  AddListToList$(WaitingList, ChildrenList)$
  AddItemToList$(VistedList, v)$

Searching Graphs

- Solutions to many problems involve searching through a graph
- There are a variety of ways of to search a graph ...
- But there's a simple generalization for many methods:

GraphSearch(G, a)

$WaitingList \longleftarrow \langle a \rangle$
$VisitedList \longleftarrow \langle\rangle$
`while not` Empty($WaitingList$)
  $v \longleftarrow$ GetAndRemoveItem($WaitingList$)
  $ChildrenList \longleftarrow$ GetChildVertices(G, v)
  AddListToList($WaitingList, ChildrenList$)
  AddItemToList($VistedList, v$)

DFS, BFS, Best-First, and A* are all instances of this method, depending on the list structure. DFS uses a Stack; BFS uses a queue

Searching Graphs

- Solutions to many problems involve searching through a graph
- There are a variety of ways of to search a graph ...
- But there's a simple generalization for many methods:

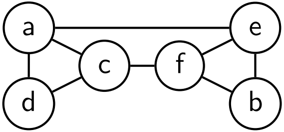
GraphSearch(G, a)

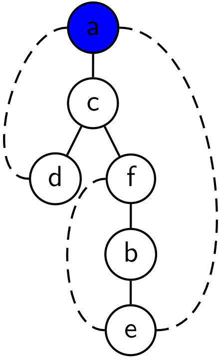
$WaitingList \longleftarrow \langle a \rangle$
$VisitedList \longleftarrow \langle \rangle$
`while not` Empty($WaitingList$)
  $v \longleftarrow$ GetAndRemoveItem($WaitingList$)
  $ChildrenList \longleftarrow$ GetChildVertices(G, v)
  AddListToList($WaitingList, ChildrenList$)
  AddItemToList($VistedList, v$)

DFS, BFS, Best-First, and A* are all instances of this method, depending on the list structure. DFS uses a Stack; BFS uses a queue

WARNING: The algorithms in the book are presented differently, but they are the same in spirit.

Depth-First Searching: An Example

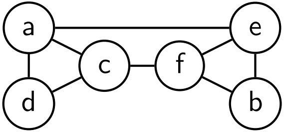


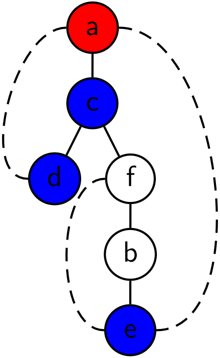


Stack $= \langle a \rangle$
Visited $= \langle \rangle$
Dead $= \langle \rangle$

- For adjacency matrix representation, traversal time is $\Theta(|V|^2)$
- For adjacency list representation, traversal time is $\Theta(|V| + |E|)$
- We can use the algorithm to check for connectivity & cycles, and to find articulation points

Depth-First Searching: An Example





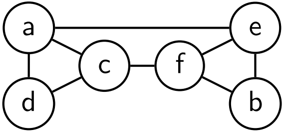
Stack $= \langle cde \rangle$

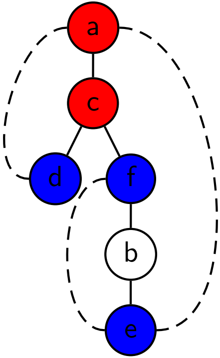
Visited $= \langle a \rangle$

Dead $= \langle \rangle$

- For adjacency matrix representation, traversal time is $\Theta(|V|^2)$
- For adjacency list representation, traversal time is $\Theta(|V| + |E|)$
- We can use the algorithm to check for connectivity & cycles, and to find articulation points

Depth-First Searching: An Example

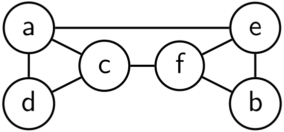


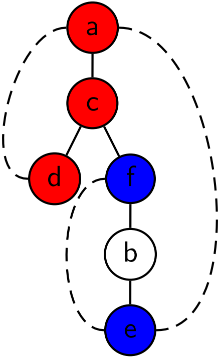


Stack $= \langle dfe \rangle$
Visited $= \langle ac \rangle$
Dead $= \langle \rangle$

- For adjacency matrix representation, traversal time is $\Theta(|V|^2)$
- For adjacency list representation, traversal time is $\Theta(|V| + |E|)$
- We can use the algorithm to check for connectivity & cycles, and to find articulation points

Depth-First Searching: An Example





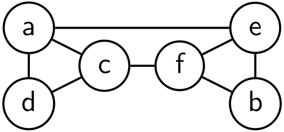
Stack $= \langle fe \rangle$

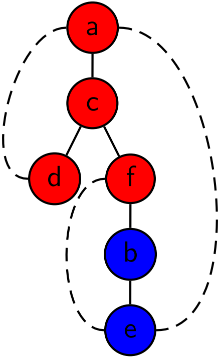
Visited $= \langle acd \rangle$

Dead $= \langle d \rangle$

- For adjacency matrix representation, traversal time is $\Theta(|V|^2)$
- For adjacency list representation, traversal time is $\Theta(|V| + |E|)$
- We can use the algorithm to check for connectivity & cycles, and to find articulation points

Depth-First Searching: An Example





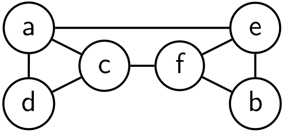
Stack $= \langle be \rangle$

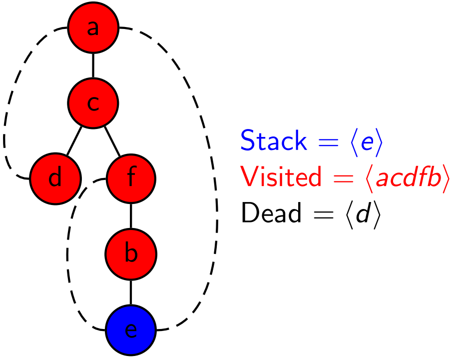
Visited $= \langle acdf \rangle$

Dead $= \langle d \rangle$

- For adjacency matrix representation, traversal time is $\Theta(|V|^2)$
- For adjacency list representation, traversal time is $\Theta(|V| + |E|)$
- We can use the algorithm to check for connectivity & cycles, and to find articulation points

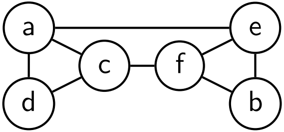
Depth-First Searching: An Example

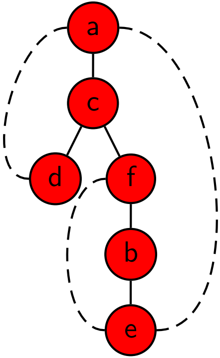




- For adjacency matrix representation, traversal time is $\Theta(|V|^2)$
- For adjacency list representation, traversal time is $\Theta(|V| + |E|)$
- We can use the algorithm to check for connectivity & cycles, and to find articulation points

Depth-First Searching: An Example





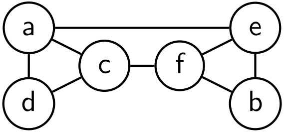
Stack $= \langle \rangle$

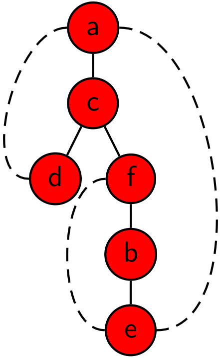
Visited $= \langle acdfbe \rangle$

Dead $= \langle de \rangle$

- For adjacency matrix representation, traversal time is $\Theta(|V|^2)$
- For adjacency list representation, traversal time is $\Theta(|V| + |E|)$
- We can use the algorithm to check for connectivity & cycles, and to find articulation points

Depth-First Searching: An Example

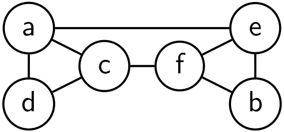


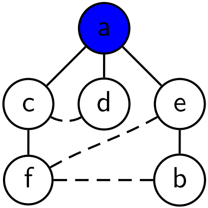


Stack = $\langle\rangle$
Visited = $\langle acdfbe \rangle$
Dead = $\langle debfca \rangle$

- For adjacency matrix representation, traversal time is $\Theta(|V|^2)$
- For adjacency list representation, traversal time is $\Theta(|V| + |E|)$
- We can use the algorithm to check for connectivity & cycles, and to find articulation points

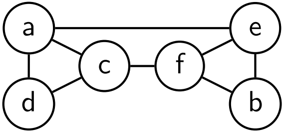
Breadth-First Searching: An Example

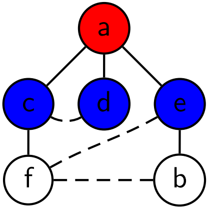




- For adjacency matrix representation, traversal time is $\Theta(|V|^2)$
- For adjacency list representation, traversal time is $\Theta(|V| + |E|)$
- We can use the algorithm to check for connectivity & cycles, and to find minimum paths

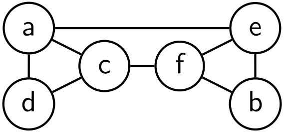
Breadth-First Searching: An Example

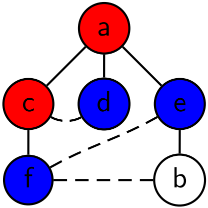




- For adjacency matrix representation, traversal time is $\Theta(|V|^2)$
- For adjacency list representation, traversal time is $\Theta(|V| + |E|)$
- We can use the algorithm to check for connectivity & cycles, and to find minimum paths

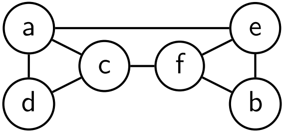
Breadth-First Searching: An Example

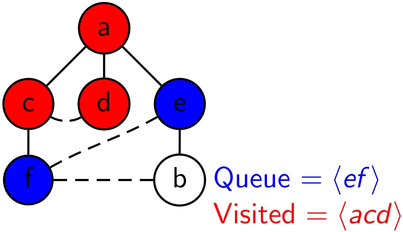




- For adjacency matrix representation, traversal time is $\Theta(|V|^2)$
- For adjacency list representation, traversal time is $\Theta(|V| + |E|)$
- We can use the algorithm to check for connectivity & cycles, and to find minimum paths

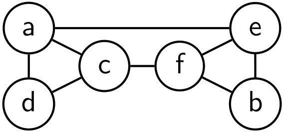
Breadth-First Searching: An Example

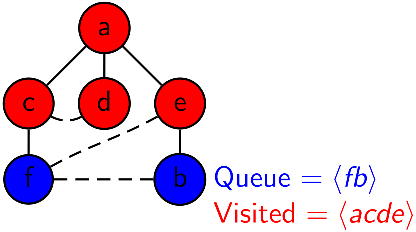




- For adjacency matrix representation, traversal time is $\Theta(|V|^2)$
- For adjacency list representation, traversal time is $\Theta(|V| + |E|)$
- We can use the algorithm to check for connectivity & cycles, and to find minimum paths

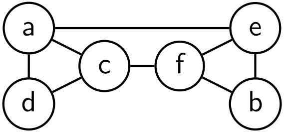
Breadth-First Searching: An Example

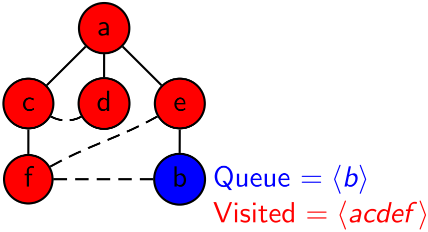




- For adjacency matrix representation, traversal time is $\Theta(|V|^2)$
- For adjacency list representation, traversal time is $\Theta(|V| + |E|)$
- We can use the algorithm to check for connectivity & cycles, and to find minimum paths

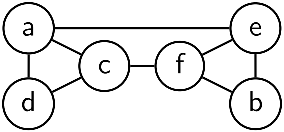
Breadth-First Searching: An Example

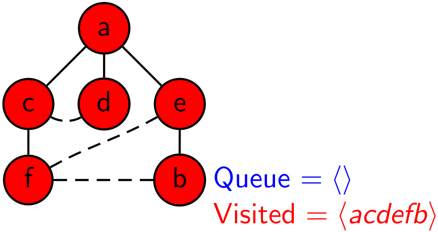




- For adjacency matrix representation, traversal time is $\Theta(|V|^2)$
- For adjacency list representation, traversal time is $\Theta(|V| + |E|)$
- We can use the algorithm to check for connectivity & cycles, and to find minimum paths

Breadth-First Searching: An Example





- For adjacency matrix representation, traversal time is $\Theta(|V|^2)$
- For adjacency list representation, traversal time is $\Theta(|V| + |E|)$
- We can use the algorithm to check for connectivity & cycles, and to find minimum paths

Assignments

- This week's assignments:
  - Section 5.1: Problems 4, 6, and 9
  - Section 5.2: Problems 1, 4, and 7