Computer Science 2300: Homework 6

Due: April 29, 2010

Note: Please use rigorous, formal arguments. If you are asked to provide an algorithm then you may either write pseudocode similar to the pseudocode in the DPV text, or provide a clear description in English. You **must** also provide an argument for why the algorithm is correct, and an analysis of the running time. We encourage you to collaborate with other students, while respecting the collaboration policy. Please write the names of all the other students you collaborated with on the homework. **Hardcopies are required by submission time. E-mailed versions will not be accepted.**

1. (5 points) DPV Problem 7.18 parts (a) and (b)
2. (5 points) DPV Problem 7.19
3. (5 points, based on CLRS Problem 26.1-6 in the 3rd Edition) Prof. Adam's two sons really don't like each other. They both refuse to walk on any city block that the other has stepped on that day. However, they have no problem with their paths crossing at a corner. Prof. Adam's house and the school are both on corners, but the professor wants to figure out if he can send both of his children to that school or not (they have to walk), using a map of the city streets. Show how to formulate this problem as a maximum flow problem.
4. (10 points, based on CLRS Problem 26.2-12 in the 3rd Edition) Consider a flow network G that has edges entering the source s. Let f be a flow in G in which one of the edges (v, s) entering the source has $f(v, s) = 1$. Prove that there must exist another flow f' with $f'(v, s) = 0$ such that $|f| = |f'|$. Give an $O(|E|)$ time algorithm to compute f', given f.
5. (5 points) Given a uniform hash function, derive the probability that there will be no collisions when inserting n elements into a hash table of size m.
6. (10 points) Suppose social security numbers are generated uniformly at random (with replacement: this is obviously a massive oversimplification because two people could then have the same social security number). How many people need to be in a room before it is more likely than not that two people in the room have the same last four digits of their SSNs?
7. (10 points) DPV Problem 5.19