

# Computer Science 2300: Homework 1

Due: February 7, 2011

**Note:** Please use rigorous, formal arguments. You will not receive full credit otherwise. Homework is due **at the beginning of lecture**.

1. (10 points) [Based on a question from Cormen *et al*'s *Introduction to Algorithms*] Consider the following procedure for generating a permutation of the numbers  $1 \dots n$ . First, choose an integer  $r$  uniformly at random between 1 and  $n$ . Now consider the array  $A$  with indices from 0 to  $n - 1$ . Fill  $A[i]$  with the number  $i + r + 1$  if  $i + r + 1 \leq n$  and with the number  $i + r + 1 - n$  otherwise. First show that each number between 1 and  $n$  has a  $1/n$  probability of winding up in any particular position in  $A$ . Then show that the resulting permutation in  $A$  is not uniformly random.
2. (10 points) Let  $F_k$  be the Fibonacci numbers, with  $F_0 = 0, F_1 = 1$ , and  $F_i = F_{i-1} + F_{i-2}$  if  $i > 1$ . Prove by induction that  $F_{n-1}F_{n+1} = F_n^2 + (-1)^n$ .
3. (10 points) Problem 1.4 in DPV.
4. (20 points) Problem 2.5 in DPV, parts a through j (2 points each) only (pages 71-72). For 5 points of extra credit if you get it right, you can also do part k.
5. (10 points) Problem 2.16 in DPV.