# A Whirlwind Tour of Game Theory

[Much of this material from "Algorithmic Game Theory," edited by Nisan *et al*]

Game theory = "multiagent decision theory"

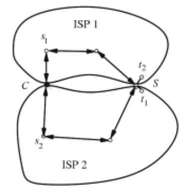
A *strategic game* is one where players choose actions and receive rewards based on their own actions and those of the other players.

Example, the Prisoner's Dilemma:

| | Cooperate | Defect |
|---|---|---|
| Cooperate | $3, 3$ | $0, 5$ |
| Defect | $5, 0$ | $1, 1$ |



The ISP Routing game:




If each ISP only pays costs for the traffic within itself, it will try to get the outbound traffic out as soon as possible. So if one packet needs to go from $s_1$ to $t_1$ and another one from $s_2$ to $t_2$, then they'll both get sent to $c$, even though this is globally suboptimal.

Tragedy of the commons: suppose there are $n$ countries. Each can control pollution at a cost of 3, and every polluting country adds a cost of 1 to every other country. What will happen?

A bandwidth sharing game: $n$ players sharing bandwidth. Each picks an amount to use $a \in [0, 1]$. If $\sum_j x_j \geq 1$ then no one gets anything (network failure). Otherwise player $i$ gets $x_i(1 - \sum_j x_j)$.

Solution? Intuitive notion of equilibrium (we'll formalize it later): everyone should choose an action so that given the actions that others choose no one wants to change their action. When would this be the case?

Let $t$ be the sum over the bandwidth used by everyone else. Then I get to pick $x$, the amount I'm going to use.

$$\max_x x(1 - t - x)$$

Taking the derivative and setting to 0,

$$x = (1 - t)/2$$

Turns out that the unique solution to this is when everyone uses $1/(n+1)$ bandwidth. Therefore, everyone is getting $1/(n + 1)^2$ utility. So the social utility is $\approx 1/n$.

Alternative strategy? Everyone uses $1/2n$, social utility is 1/4. But not an equilibrium!