Strategies and Dominant Strategy Solutions

A **strategy** is a specification for how to play the game for a player. A **pure strategy** defines, for every possible choice a player could make, which action the player picks. A **strategy profile** is a set of strategies for all players which fully specifies all actions in a game.

Notation: If $s \in S$ is a strategy profile then we say s_i for player i's strategy and s_{-i} for everyone else's (an $n-1$ dimensional vector).

A game has a **dominant strategy solution** if every player has a unique best strategy, independent of the strategies played by others. So, for example, the Prisoner's Dilemma has a dominant strategy solution.

$s \in S$ is a dominant strategy solution if $\forall i$ and every other $s' \in S$:

$$u_i(s_i, s'_{-i}) \geq u_i(s'_i, s'_{-i})$$

This is a very strong requirement (the field of mechanism design seeks to design games with dominant strategy solutions, as we will see later).

Nash Equilibrium

The key notion is a kind of "stability" – a strategy profile in which no player wants to deviate.

$s \in S$ is a **Nash equilibrium** if

$$u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i}) \forall i$$

Dominant strategy solutions are Nash equilibria.

Iterated Elimination of Strictly Dominated Strategies

Dominated strategies: Strategy s_i (strictly) dominates strategy s'_i if, for all possible strategy combinations of opponents, s_i yields a (strictly) higher payoff than s'_i to player i.

Iterated elimination of strictly dominated strategies: Eliminate all strategies which are dominated, relative to opponents' strategies which have not yet been eliminated.

If iterated elimination of strictly dominated strategies yields a unique strategy n-tuple, then this strategy n-tuple is the unique Nash equilibrium (and it is strict).

Every Nash equilibrium survives iterated elimination of strictly dominated strategies.

Example:

	X	Y	Z
A	1, 1	2, 0	2, 2
B	0, 3	1, 5	4, 4
C	2, 4	3, 6	3, 0

First eliminate A, then X, then Z, then B, leaving (C, Y).

Coordination Games

	Ballgame	Symphony
Ballgame	5, 6	1, 1
Symphony	2, 2	6, 5

Girl is the row player, Boy is the column player.

There are two equilibria: both go to the ballgame or both go to the symphony. In both cases neither one of them would want to unilaterally change his/her action.

More on Multiple Equilibria

Another coordination game:

	L	R
U	9, 9	0, 8
D	8, 0	7, 7

U, L and D, R are both Nash equilibria. What would be reasonable to play? With and without coordination?

While U, L is pareto-dominant, playing D and R are "safer" for the row and column players respectively...

Matching Pennies

	H	T
H	$+1, -1$	$-1, +1$
T	$-1, +1$	$+1, -1$

No pure strategy equilibria

A **mixed strategy** is a probability distribution over strategies.

Nash equilibrium: Both players randomize half and half between actions.

Correlated Equilibrium

A coordinator can choose strategies (equivalently, the players have access to the same randomizer: think traffic lights). Each player must find it rational to play the coordinator's recommendation. Let $p(s)$ be the probability of a strategy vector s being recommended. Then it must be that, for each player i,

$$\sum_{s_{-i}} p(s_i, s_{-i})u_i(s_i, s_{-i}) \geq \sum_{s_{-i}} p(s_i, s_{-i})u_i(s_i^{'}, s_{-i})$$

That is, the expected utility of a player cannot be improved by switching actions.

Note that Nash equilibria are special cases of correlated equilibria in which the distribution over S is the product of the independent distributions for each player.

Example: another cooperate-defect game, but different from the Prisoner's Dilemma (how?)

	D	C
D	$0, 0$	$7, 2$
C	$2, 7$	$6, 6$

What are the Nash equilibria? (1) (C, D) (2) (D, C) (3) Play D with probability 1/3. Expected payoff for both players in the third one?

$(1/9)\times 0+(2/9)\times 7+(2/9)\times 2+(4/9)\times 6 = 42/9$

A correlated equilibrium: the coordinator draws (C, C), (C, D), or (D, C), each with equal probability.

Suppose I am told to take action C. Would I be better off switching to D given what I know about the distribution?
Expected payoff playing C: $(1/2) \times 2 + (1/2) \times 6 = 4$.
Switch to D? $(1/2) \times 0 + (1/2) \times 7 = 3.5$.
Therefore, no!

What if I'm told to take action D? Then I get 7, which is the best possible, so obviously I don't want to switch to C.

What is my expected payoff in this game?

$$(1/3) \times 7 + (1/3) \times 6 + (1/3) \times 2 = 5$$