Extensive Games With Perfect Information

Capture the sequential structure of games. So anything we in CS ordinarily think of as a "game."

Why can't we just model it as a strategic game? Because now we allow a player's choice of action to depend on the history.

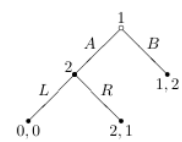
The Ultimatum Game:
P_1 chooses x between 1 and 100.
P_2 is offered $100 - x$ and has to take it or reject it.
If P_2 rejects, then neither player gets anything.

Suppose we just used the strategic game model. Then anything is an equilibrium where P_1 chooses p where p is the highest in the set of offers that P_2 will accept. So, for example, P_2 uses the strategy "I will reject any offer that gives me less than 80" and P_1 keeps 20 and offers P_2 80.

Problem? P_2's threat is not credible! The equilibrium should be that P_1 takes 99 and P_2 accepts the 1 (although behavioral factors come into play in reality – but imagine if the offers were in millions of dollars – would you turn down 1 million because it wasn't fair?)

Subgame perfect equilibrium: behavior in any subgame must be a Nash equilibrium! (Technically, the strategy should specify moves everywhere in the game tree, which is a bit weird)

Another example (from Osborne and Rubinstein):



(A,R) is the only subgame perfect equilibrium. As a strategic game it would admit other equilibria (what?)

Games of Incomplete Information

Different from *imperfect information* games, where you know the other players, their possible actions, and their payoffs, but may not know what actions they have chosen.

In incomplete information games, players may not have some information about the other players: for example, their strategies or payoffs.

For example, suppose Alice doesn't know if Bob really likes her or not. They each have to decide between going out to a bar or to a frat party. If Bob likes Alice, the payoff matrix is (Alice is the row player):

	Bar	Party
Bar	$2, 1$	$0, 0$
Party	$0, 0$	$1, 2$

But if Bob doesn't like Alice, then the payoff matrix is:

	Bar	Party
Bar	$2,0$	$0,2$
Party	$0,1$	$1,0$

Bob knows whether or not he likes Alice, and, therefore, which game is being played. Assume Alice believes that Bob likes her with probability p. In general, uncertainty is on the *types* of players. In this case, Bob's type is unknown (Bob-who-likes-Alice or Bob-who-does-not-like-Alice). Players attach probabilities to the types that other players take.

Big (unrealistic?) assumption often used to solve these types of games: common prior. In this case, Bob know's Alice's estimate p.

Solution concept: **Bayes-Nash equilibrium**. A generalization of Nash equilibrium: a strategy profile plus beliefs for each player over the types of the other players that maximizes the expected payoff for each player given (1) their beliefs about the other players' types and (2) the strategies played by other players.

Let's think more about the example above. Alice could play Bar, or Party, or a mixed strategy. For Bob, we need to specify what each type of Bob would play. So, pure strategies are (B, B), (B, P), (P, B), (P, P). Could also play a mixed strategy with two probabilities specifying the probability of playing Bar for Bob-who-likes-Alice and Bob-who-does-not-like-Alice.

Let's think about possible equilibria in pure strategies. Can do it in two parts. First, what if Alice plays Bar?

Bob-who-likes-Alice would play Bar, and Bob-who-does-not-like-Alice would play Party. When is playing Bar a best-response for Alice?

Expected payoff from Bar? $2p + 0(1-p) = 2p$
Expected payoff from Party? $0p + 1(1-p) = 1-p$
So Bar is a best response if $2p > 1-p$ or $p > 1/3$.

So, if $p > 1/3$ then Alice playing Bar and the two Bobs playing (Bar, Party) are a NE.

Suppose Alice plays Party.

Bob-who-likes-Alice would play Party, and Bob-who-does-not-like-Alice would play Bar. When is playing Party a best-response for Alice?

Expected payoff from Party? $1p + 0(1-p) = p$
Expected payoff from Bar? $0p + 2(1-p)$
So Party is a best response if $p > 2-2p$ or $p > 2/3$.

Therefore, if $p > 2/3$ there are two Nash equilibria: (1) Alice plays Bar and the Bobs play (Bar, Party); (2) Alice plays Party and the two Bobs play (Party, Bar).

If $1/3 < p < 2/3$ then the only NE is (1)

If $p < 1/3$ there is no pure strategy NE.