Computer Science 4963/6963: Homework 1

Due: March 1, 2010

Note: Due in class.

1. Prove that Hanson's logarithmic market scoring rule is incentive compatible in the following sense: at any time, an arriving trader with a personal belief that the probability of the event occurring is p should, in order to maximize profit, trade a quantity q that changes the spot price to p.

2. Suppose two companies are competing in the Internet search business, and they both need to simultaneously decide how much they are going to invest in research. Each company can invest some $x \in [0.001, 1]$ units into research at cost $x/4$. If a firm invests x and its competitor invests y, the probability that the firm wins the search battle is $x/(x+y)$. The payoff for winning the search battle is 1, and the payoff for losing is 0. What are the pure strategy Nash equilibria of this game?

3. **For those taking the class for graduate credit (6963) only. Those taking the class for undergraduate credit (4963) may do this question for extra credit.**
   Prove that there does exist a mixed equilibrium for the pricing game that is defined in Example 1.9 (page 14) of the text (also discussed in class). Do so by showing that the strategy profile where both players draw a price from the probability distribution with support $[1/2, 1]$ and cumulative distribution function $F(x) = 2 - (1/x)$.