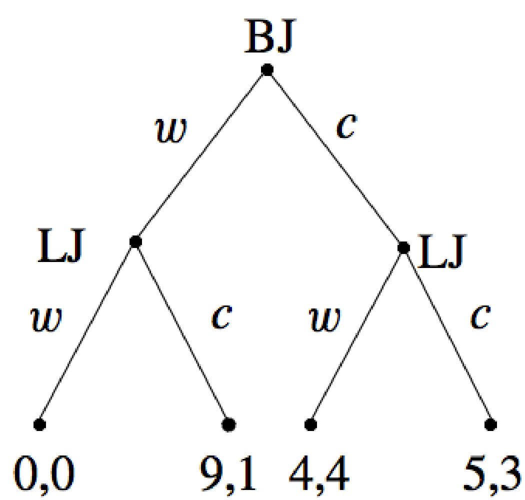
# CSE 516A: Homework 2

Due: Nov 1, 2019

- Please check the submission instructions for Gradescope provided on the course website. You must follow those instructions exactly.
- Homework is due **by 11:59 PM on the due date.**
- Please keep in mind the collaboration policy as specified in the course syllabus. If you discuss questions with others you **must** write their names on your submission, and if you use any outside resources you **must** reference them. Note that several of the questions are quite open-ended. You will be graded in part on the quality of your analysis and in part on the quality of your writeup, so please write your answers up carefully. There are eight questions on three pages.

1. (10 points) **Subgame perfection warmup:** Consider the following game between two players BJ and LJ, in extensive form:




   What are the pure-strategy Nash equilibria of this game (when you consider it in normal form)? Are any of them not subgame perfect? Why?

2. (20 points) **A very unfair final:** 15 people show up for the final exam in CSE 777, "Gamification". The instructor orders them by their current score in the class, with the person with the highest score first and the person with the lowest score last (there are no ties). So now let's say Student 1 is the highest scoring, Student 2 the second highest, etc. In this order, they

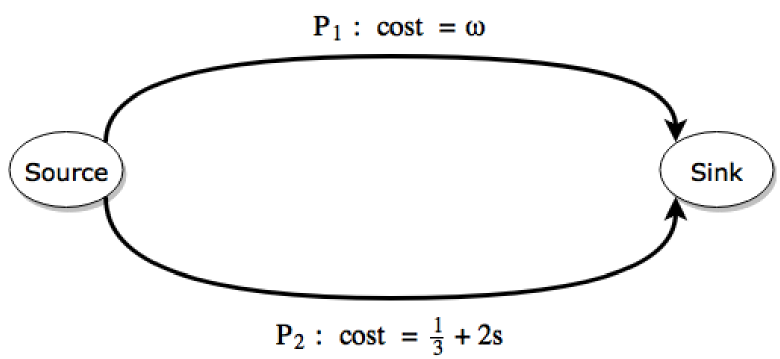
play a splitting game of the following form. 100 points are to be split between the students, with only integer splits allowed. Student $i$ gets to propose a split among all the remaining students (students $i$ through 15, including the proposer). Now all the remaining students ($i$ through 15) vote on the split. If it gets at least 50% of the votes (therefore, ties are broken in favor of the proposal) the proposal is accepted and the proposed split constitutes the scores of the students on the final. If the proposal is rejected, the student who proposed it receives an F for the course (which is a strictly worse outcome for all students than receiving 0 on the final) and leaves the game, and student $i+1$ gets to make the next proposal. (Note: You can assume anything you want about how agents choose among options they are indifferent among, as long as you state what you're assuming clearly.)

(a) What is the (subgame perfect) outcome of this game?

(b) How does this change if ties are now broken in favor of rejecting the proposal?

3. (10 points) **Lighten my load** Consider a game with $n$ players, each of whom has a task to complete. These tasks take time $t_1, \ldots, t_n$ respectively. There are $m$ machines available, and each player has to choose one machine to send her task to, so $a_i \in \{1, \ldots, m\}$. The utility of each player is given by the negative of the total *load* of the machine she chose, that is $u_i(\mathbf{a}) = -\sum_{a_j = a_i} t_j$. Prove that this is a what's called a *weighted* potential game, which is defined the same way as a potential game, but with the difference that the change in potential could be the product of a player-dependent constant and the change in that player's utility. (Hint: try using the sum of squares of machine loads as the potential function).

4. (10 points) **Nonatomic Routing** Consider the following routing network, where edges are annotated with costs.




The difference from what we've done in class is that this is a *nonatomic* routing game, where each agent controls only an $\epsilon$ fraction of the flow in the network (the concepts are otherwise the same – in equilibrium, no $\epsilon$ fraction of agents would want to deviate). Suppose one unit of flow needs to be routed from the source to the sink, and two paths are available, annotated with their costs. $0 \leq s \leq 1$ is the share of agents taking $P_2$. Suppose $\omega = 1$, and all of this information is common knowledge. What is the equilibrium outcome? Is there a better outcome in terms of the total cost paid by all agents? If so, explain why it is not an equilibrium.

5. (10 points) **Iterated elimination** Consider the following game in normal form:

| | L | R |
|---|---|---|
| H | $1,1$ | $0,0$ |
| T | $1,1$ | $2,1$ |
| E | $0,0$ | $2,1$ |

Are there any pure strategy Nash equilibria in this game? If so, what are they? Are there any weakly dominated strategies? What would happen if you followed the iterated elimination of weakly dominated strategies in this game?

6. (15 points) **Uncertain PD** Consider the following variant of the Prisoner's Dilemma game.

| | C | D |
|---|---|---|
| C | $-1+\theta, -1+\theta$ | $-5+\theta, 0$ |
| D | $0, -5+\theta$ | $-4, -4$ |

$\theta$ is an unknown state of nature (common to both players), and its value is drawn uniformly at random from $[-3, 3]$. What is the expected total social welfare of the Nash equilibrium in 3 cases: (1) When both players are only aware of the common prior on $\theta$? (2) When both players are aware of the actual realization of $\theta$ prior to choosing their actions? (3) When both players are aware of whether $\theta > -1$ or $\theta \le -1$ (but not the specific realization of $\theta$)?

7. (10 points) **Lawyers, money, and subgame perfection** Suppose the famous tech company Pear is contemplating suing Sam and Sons for patent infringement. It costs Pear $c$ to initiate the lawsuit. The probability that Pear wins the lawsuit is $p$, and the expected payoff if it wins is $x$ (and that cost is borne by Sam and Sons). In between initiating the lawsuit and taking it to trial, Pear can offer a settlement in which Sam and Sons pay $s$ to have Pear drop the case. If they do not settle, Pear has to decide whether to go to trial, in which case they have to pay a lawyer $l_p$. Meanwhile, Sam and Sons would have to pay $l_s$ to defend themselves in court. Draw out the entire game tree. Suppose $px < l_p$, what is the subgame perfect equilibrium of this game? What happens if Pear puts a lawyer on staff and has to pay them $l_p$ regardless of whether they go to trial or not? In what circumstances would having a lawyer on staff be beneficial? (You can answer this question with different cases for different possible values / comparative values of the parameters)

8. (15 points) **Alice and Silent Bob** Consider the following game. There are 2 players, Alice and Bob. Alice may be one of two types, either feisty (with probability $\lambda$), or sedate (with probability $1-\lambda$). She goes to a pub, where she sees Bob. Alice gets to order either beer or wine from the bartender, and Bob gets to observe her order. If Alice is feisty she prefers beer, while if she is sedate she prefers wine. Bob is a little bit belligerent and wants to get into an argument with Alice, but only if he can win, and he can only win if Alice is sedate (and he doesn't know her type, he only observes her order). The payoffs are as follows. Alice's payoff starts at 0, increases by one if she orders the type of beverage she prefers, and by two more if Bob chooses to be silent instead of argue. Bob gets a payoff of 0 if either (i) he chooses to be silent and Alice is sedate, or (ii) he chooses to argue and Alice is feisty. He gets a payoff of 1 if either (i) he chooses to be silent and Alice is feisty, or (ii) he chooses to argue and Alice is sedate.

   First, convert this game to normal-form and write down the payoff matrix (remember that you need actions conditional on types for Alice and conditional on Alice's order for Bob). Second, assume $\lambda \ge 0.5$. Are there any pure strategy Bayes-Nash equilibria? Which are they? Are these also Perfect Bayesian equilibria? Why or why not?