CSE 516: Homework 3

Due: December 6, 2019

- **You are allowed to work in teams of 2 for this homework – only one of you needs to submit on Gradescope, noting the members of the team**, and as usual to discuss things verbally with others (remembering to acknowledge them). Remember that if you use any outside resources you **must** reference them. Grading will take into account how clearly you communicate your solutions, so please write your answers up carefully. There are 3 questions on 2 pages.
- Homework is due **by 11:59 PM on the due date.**
- Some of the questions on this homework are more open-ended than those on previous homeworks. Please explain your answers and your reasoning in detail. There are three questions on three pages.

1. (30 points) **Voting in multiattribute domains** A condo complex with three residents is having a debate about whether to build a new swimming pool, and a new squash court, and a new community jacuzzi. Of course, condo fees will go up according to whichever options are selected. Each of these is a binary (0 or 1) choice, so we can express the preferences of the voters as follows

   - Voter 1: $110 \succ 101 \succ 011 \succ 001 \succ 100 \succ 010 \succ 000 \succ 111$
   - Voter 2: $101 \succ 011 \succ 110 \succ 010 \succ 100 \succ 001 \succ 000 \succ 111$
   - Voter 3: $011 \succ 110 \succ 101 \succ 100 \succ 010 \succ 001 \succ 000 \succ 111$

   where these are complete preferences over the combinatorial set of what gets built and does not get built, with 1 indicating it gets built (and the order is the one in which they are named above).

   - Suppose residents are asked to vote individually on each of the three issues simultaneously, and everyone is *hopeful* about their preferences with respect to the other issues (when they are voting for each individual issue, they are acting under the assumption that their most preferred outcome for the other two issues has been achieved). What will be the outcome of the three votes? Is this good or bad in the context of the overall preferences?
   - Suppose instead you hold a Borda count election over the entire set of alternatives, and everyone is truthful. What is the outcome? Is this good or bad?
   - Now suppose you scale this problem to more general yes/no ballot initiatives, and there are more than three such initiatives on the ballot. Leaving aside incentive compatibility, what are the other problems with the Borda count approach?

- Going back to the original problem, suppose you instead have sequential votes on the issues, with the outcome of each vote being revealed before going on to the next one. What would the outcomes be in the example, assuming hopeful voting in terms of as-yet-undetermined outcomes? Are there strategic issues here, and what is the effect of the chosen sequence of votes?

2. (30 points) **Voting rules:**. For this problem, you will be working with the votes that were recorded in the 2008 APA Council elections, available at `http://www.preflib.org/data/election/apa/`. Familiarize yourself with the data format, by reading: `http://www.preflib.org/data/format.php`. You may want to work with the data with strict orders (and potentially incomplete lists). Implement code that takes these votes and runs the following types of elections to choose one candidate for the council: plurality vote, instant runoff voting (repeatedly eliminating the candidate with the fewest first place votes until one has a majority of first place votes), Borda count (think about how to treat incomplete preference lists), and approval voting (where every candidate that shows up on a voter's preference list gets 1 point and the candidate with the maximum number of points wins). Give a brief summary (in text) of your implementations and report the results. What does this experience tell you?

3. (40 points) **Giving back to those who pay:** A company with n employees, who each work at their own desks, with company-allocated computers, just received a gift of m awesome new computers from a wealthy donor. The company is very sophisticated in how it does things, so it gives each employee an annual budget of $20,000 for equipment to improve their working lives, and it achieves excellent results in terms of productivity by using this approach. The company wishes to figure out how best to allocate these new computers; in effect, it must determine which of its n employees will get the m new computers, with each of those employees getting exactly one of those computers. The old computers of these workers will be donated to charity. Each worker knows the value to her, in terms of added productivity (which correlates well with annual bonus), of getting one of these new computers. Without loss of generality, order the workers in terms of decreasing value of getting a new computer – thus, the true valuations, in decreasing order, are $v_1, v_2, \ldots, v_n$, with $v_1 < \$20,000$. The company wishes to elicit truthful valuations from its employees. One option is just to run a VCG mechanism (with Clarke pivot payments), with employees paying from their equipment budgets (assume this takes place on Jan 1, before anyone has had the chance to spend anything from their budget). The issue is that this doesn't really align with the company's goals – the computers were free, the budgets can be used for other purposes, so why make the employees spend from their budgets? But how can they get incentive compatibility otherwise?

   The company comes up with an idea. They'll run a two-stage mechanism. The first stage is the ordinary VCG, with (Clarke pivot) payments being collected from participants. In the second stage, the company will take all the payments, and then give that money *back* to the employees' equipment funds in some manner. They propose two possible alternatives. For notational convenience, let a_i be the $(m+1)$th highest bid from an agent other than i, and b_i be the $(m+2)$th highest bid from an agent other than i. In alternative (1), the second stage payment to employee i should be $\frac{m}{n}a_i$. In alternative (2) the second stage payment to employee i should be $\frac{m}{n-m-1}a_i - \frac{m(m+1)}{(n-m-1)(n-m-2)}b_i$. Assume that $n \geq 2m+3$.

   Answer the following questions in the context of these two two-stage mechanisms:

- Are the mechanisms incentive compatible? Do they make no positive transfers? Are they individually rational? (Prove your claims)
- How much of the VCG payments does the second stage give back to the employees for each of the two mechanisms?
- Is one of these mechanisms always better (in terms of how much it gives back) than the other for any set of valuations? If not, how do they differ, and in which cases would you want to use one rather than the other, if your goal is to give back as much as possible?