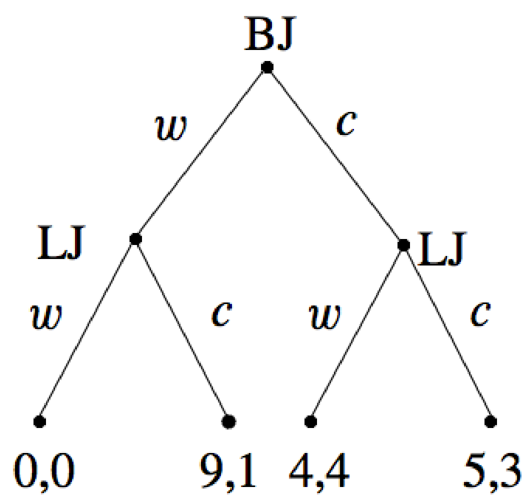
CSE 516A: Homework 2

Due: March 22, 2018 by 10 PM (on Gradescope)

Note: Please keep in mind the collaboration policy as specified in the course syllabus. If you discuss questions with others you **must** write their names on your submission, and if you use any outside resources you **must** reference them. Grading will take into account how clearly you communicate your solutions, so please write your answers up carefully. There are eight questions on three pages.

1. (10 points) **Subgame perfection warmup:** Consider the following game between two players BJ and LJ, in extensive form:

   What are the pure-strategy Nash equilibria of this game (when you consider it in normal form)? Are any of them not subgame perfect? Why?

2. (20 points) **A very unfair final:** 15 people show up for the final exam in CSE 777, "Gamification". The instructor orders them by their current score in the class, with the person with the highest score first and the person with the lowest score last (there are no ties). So now let's say Student 1 is the highest scoring, Student 2 the second highest, etc. In this order, they play a splitting game of the following form. 100 points are to be split between the students, with only integer splits allowed. Student i gets to propose a split among all the remaining students (students i through 15, including the proposer). Now all the remaining students (i through 15) vote on the split. If it gets at least 50% of the votes (therefore, ties are broken in favor of the proposal) the proposal is accepted and the proposed split constitutes the scores

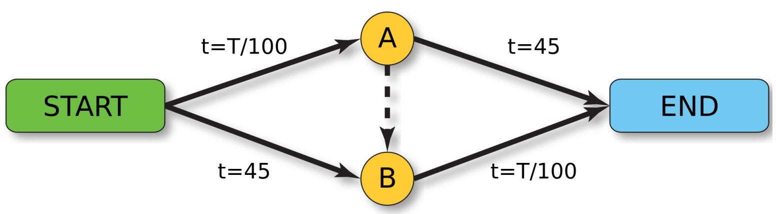
of the students on the final. If the proposal is rejected, the student who proposed it receives an F for the course (which is a strictly worse outcome for all students than receiving 0 on the final) and leaves the game, and student $i+1$ gets to make the next proposal. (Note: You can assume anything you want about how agents choose among options they are indifferent among, as long as you state what you're assuming clearly.)

(a) What is the (subgame perfect) outcome of this game?

(b) How does this change if ties are now broken in favor of rejecting the proposal?

3. (10 points) **Lighten my load** Consider a game with n players, each of whom has a task to complete. These tasks take time $t_1, \ldots, t_n$ respectively. There are m machines available, and each player has to choose one machine to send her task to, so $a_i \in \{1, \ldots, m\}$. The utility of each player is given by the negative of the total *load* of the machine she chose, that is $u_i(\mathbf{a}) = -\sum_{a_j = a_i} t_j$. Prove that this is a what's called a *weighted* potential game, which is defined the same way as a potential game, but with the difference that the change in potential could be the product of a player-dependent constant and the change in that player's utility. (Hint: try using the sum of squares of machine loads as the potential function).

4. (10 points) **Routing** Consider the following routing network, where edges are annotated with costs.



In this case t represents the cost of taking an edge, and T is the number of agents taking that edge. Suppose there are 5000 agents at the start node (and that they are ex ante identical, and this is all common knowledge), and they all need to reach the end node. Find a Nash equilibrium of this game (and show that it is a Nash equilibrium).

5. (10 points) **Iterated elimination** Consider the following game in normal form:

	L	R
H	$1,1$	$0,0$
T	$1,1$	$2,1$
E	$0,0$	$2,1$

Are there any pure strategy Nash equilibria in this game? If so, what are they? Are there any weakly dominated strategies? What would happen if you followed the iterated elimination of weakly dominated strategies in this game?

6. (15 points) **Alternating ultimatums** Consider the following variant of the ultimatum game. Alice can make an offer to Bob to split a reward of 1 (the units are really large, say on the order of \$1M, but it's convenient to think of it as one unit). She makes some offer $a \in [0,1]$ and Bob decides whether to accept or not (here a is the amount Alice *keeps*, with Bob getting

the rest). If Bob accepts, they receive the utility from that split. However, if Bob rejects, then *he* gets to make the next offer – he offers Alice some $b \in [0, 1]$ and then she has to decide whether to accept or not (b being the amount that Bob keeps). If she rejects, then she gets to make the next offer again, and so on forever. There is one more twist. Utilities are discounted, so that one unit some t rounds in the future is only worth γ^t ($\gamma \in (0, 1)$) of one unit now (so, for example, if Alice accepts an offer of $1/2$ three rounds from now, after her offer is rejected and Bob's next offer and her next offer are also both rejected, then she receives utility (from her current perspective) of $\gamma^3(1/2)$).

Show that the following strategies are a subgame perfect equilibrium: At any time, Alice offers $a^* = 1/(1+\gamma)$ and accepts any $b \le b^*$, while Bob offers $b^* = 1/(1+\gamma)$ and accepts any $a \le a^*$.

For 2 points and 10000 MASEcoins of extra credit, show that for *any* $x \in [0, 1]$, there is some *Nash* equilibrium of the game where the reward of 1 is divided as x going to Alice and $1 - x$ going to Bob (γ is set arbitrarily).

7. (10 points) **Lawyers, money, and subgame perfection** Suppose the famous tech company Pear is contemplating suing Sam and Sons for patent infringement. It costs Pear c to initiate the lawsuit, and they have to pay a lawyer l_p. Meanwhile, Sam and Sons would have to pay l_s to defend themselves in court. The probability that Pear wins the lawsuit is p, and the expected payoff if it wins is x. In between initiating the lawsuit and taking it to trial, Pear can offer a settlement in which Sam and Sons pay s to have Pear drop the case. Draw out the game tree. Suppose $px < l_p$, what is the subgame perfect equilibrium of this game? What happens if Pear puts a lawyer on staff and has to pay them l_p regardless of whether they go to trial or not? In what circumstances would having a lawyer on staff be beneficial?

8. (15 points) **Alice and Silent Bob** Consider the following game. There are 2 players, Alice and Bob. Alice may be one of two types, either feisty (with probability λ), or sedate (with probability $1 - \lambda$). She goes to a pub, where she sees Bob. Alice gets to order either beer or wine from the bartender, and Bob gets to observe her order. If Alice is feisty she prefers beer, while if she is sedate she prefers wine. Bob is a little bit belligerent and wants to get into an argument with Alice, but only if he can win, and he can only win if Alice is sedate (and he doesn't know her type, he only observes her order). The payoffs are as follows. Alice's payoff starts at 0, increases by one if she orders the type of beverage she prefers, and by two more if Bob chooses to be silent instead of argue. Bob gets a payoff of 0 if either (i) he chooses to be silent and Alice is sedate, or (ii) he chooses to argue and Alice is feisty. He gets a payoff of 1 if either (i) he chooses to be silent and Alice is feisty, or (ii) he chooses to argue and Alice is sedate.

   First, convert this game to normal-form and write down the payoff matrix (remember that you need actions conditional on types for Alice and conditional on Alice's order for Bob). Second, assume $\lambda \ge 0.5$. Are there any pure strategy Bayes-Nash equilibria? Which are they? Are these also Perfect Bayesian equilibria? Why or why not?