AGENT-BASED DYNAMICS OF SOCIAL COMPLEXITY
Modeling Adaptive Behavior & Long-Term Change in Inner Asia

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1. MOTIVATION AND PURPOSE

Inner Asia is the heartland of the Old World, a "bridge" and large-scale social network linking Asia and Europe across an expansive steppe land, and a unique laboratory for understanding long-term social and political adaptations in the face of great challenges. Depending on the epoch, Inner Asia has fluctuated from being an active center or core with significant influence on neighboring regions (China, Russia, South Asia, Eastern Europe, and the Middle East), to being a passive-reactive periphery of these same regions. The mobile populations, long-distance contacts and exchange, rapid transport technologies, and macro-scale complex polities of Inner Asia offer an opportunity to develop and test a new approach to understanding the emergence of complex horizontal and vertical forms of social organization as dynamic adaptive responses to social and environmental changes. In this project we do so using *diachronic data* from written sources and from three archaeological projects located along a north-south transect spanning the Mongolian steppe zone; and developing *object-oriented agent-based simulation models* that build upon and extend extant computational social science models to generate the emergence of multi-scale networks over space and time. This project is aimed at producing new knowledge for better understanding social dynamic responses and collective behavior to change.

Figure 1. Inner Asia and Mongolia showing active survey datasets. (1) Darkhat-Muron, (2) the Khanui and Terhiyn Valleys, and (3) Baga Gazaryn Chuluu. Unnumbered area represents a prior survey dataset at Egiin Gol (1996-2000).

Research Goals The purpose of this project is to build on previous and ongoing efforts in *computational* historical dynamics (or agent-based "cliodynamics"; paraphrasing Turchin 2003: 2004) by pursuing three synergistic goals:
(1) to develop, test and disseminate a new interdisciplinary theory of long-term societal change and adaptation to complex and evolving social and physical environments, a "generative" theory formalized by a spatial multi-agent computational model;
(2) to contribute to the shared understanding of social complexity across the social sciences by integrating concepts and principles within the proposed theoretical framework and research methodology; and
(3) to produce and disseminate new interdisciplinary data resources created by this project, such as a new long-term dataset and diachronic atlas of Inner Asian polities.

Observed Facts The space-time universe of human and social dynamics is vast and heterogeneous in terms of origins and long-term evolution of social complexity and environmental diversity. Regions of social space-time with great dynamism and originality (e.g., Asia in recent millennia) are interspersed with others where complex organization developed along alternate pathways and at different rates of change (e.g., North America 5000 years ago). The long-term fabric of social transformation is not all woven of the same material. Significant differences and regularities occur in human and social dynamics across space and time (Peregrine et al. 2004; Flannery 1999; Blanton et al 1996; Marcus 1998), so comparative research is essential.

We define Inner Asia as composed of Mongolia, Inner Mongolia, Tibet, Manchuria, Eastern Turkestan [Xinjiang], and parts of eastern Central Asia and southern Siberia (Lattimore 1940) with a research emphasis on the Mongolian steppe (fig. 1 above). The importance of this region as a case-study for socio-political development is undeniable, as Inner Asia gave rise to some of the world's most expansive imperial organizations. From the third millennium BC onward, the eastern steppe of Inner Asia was characterized by significant variability in environment, productive resources, subsistence practices, and group organization as

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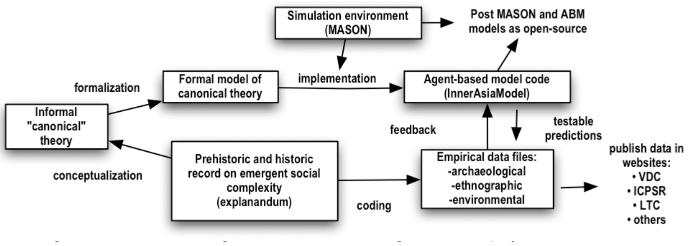
well as diverse networks of inter-cultural contact, including eventually the famed Silk Road trade route (from ca. 200 BC; Malkov 2004, 2005). From the end of the first millennium B.C. onward a series of hierarchical, centrally integrated, and militarily powerful polities emerged across Inner Asia including the Xiongnu, The Turk, Uighur, and Khitan states. The 13th century A.D. witnessed the building and consolidation of a Mongolian empire connecting the coasts of China to the fringes of Europe. The immensity of Genghis Khan's imperial project and it's cultural, political, and economic repercussions have long challenged scholars to explain how a relatively obscure group of steppe nomads managed to conquer and rule such a substantial swathe of the Old World. According to anthropological models, *steppe conditions* (productive instability, low population density, high mobility) lower the potential for complex socio-political organization and instead favor more egalitarian polities (Barfield 2001; Salzman 1999; Johnson and Earle 2000). *However*, there is compelling evidence that by the early first millennium BC, patterns of complex organization in political leadership—and presumably high collective action capacity (*asabyia*; Turchin 2003, based on Ibn Khaldun)—characterized small groups dispersed across the study area (Shelach 1999; Tsybiktarov 1998; Askorov et al. 1992; Hiebert 1992).

Theoretical Deficits Social complexity is a measure of differentiation and integration in a human society and is generally characterized by hereditary social hierarchy, occupational specialization, centralized decision-making and governance capable of ensuring social viability in the face of emerging challenges. Complex societies range from small-scale groups with simple ranked hierarchies to the highly specialized, multi-stratified, integrated polities of the modern world. Some of the most viable theories today are more interdisciplinary (Epstein and Axtell 1996; Feinman and Marcus 1998; Flannery 1999; Turchin 2003), and they aim at integrating social and environmental dynamics using a broad range of scientific concepts and principles appropriate to the complex subject matter we seek to investigate.

Needed is an interdisciplinary theory that builds upon extant progress and encompasses a range of diverse anthropological, economic, sociological, political, psychological and environmental dynamics. Such a theory should be constructed with substantively viable concepts and principles appropriate for the phenomenology of social complexity, not simply chosen on the basis of disciplinary boundaries. Formally, the new methodological paradigm of object-based modeling offers new powerful opportunities for modeling and understanding human and social dynamics, based on well-defined attributes and behavioral relations instantiated in computational models of evolutionary adaptive agents capable of self-generating higher-order social complexity on multiple scales (Epstein 2006).



An overview of how we integrate collaborative activities regarding data, theory, and computational modeling is shown in Figure 2.
The empirical record (observed facts or main *explanandum*) on scale, long-term adaptation and social complexity in the Inner Asian world region is our point of departure (fig. 2, bottom). We use observations on this region of space and time to develop an informal conceptual theory used as the basis for agent-based simulation (conceptualization → formalization → implementation), as well as for producing datasets (coding) for validating, calibrating, and testing the simulation (feedback from testing). Finally, results from the simulation and empirical files coded from the observed record are used to test and refine the computational model. The next sections explain this research plan.

2. THEORETICAL FRAMEWORK

Conceptual Framework: Change, Response, and the Social Dynamics of Complexity The social and contextual processes emphasized in recent anthropological models include prestige-biased cultural transmission, ambitious agents, social leadership dynamics and technological change, the manipulation of ideological and material media, and the strategic management of intra and extra group relationships (Cioffi-Revilla et al. 2005). Collective action capacity (*asabyia*) is also a key causal variable in Turchin's "metaethnic frontier" model (2003)—arguably the most advanced formal *and* empirically tested theory today ("clearly the state of the art in formal modeling and computer simulation of long-term historical changes in territorial states" Collins 2003).

Situational changes produce transfers of information between individuals and their environments, causing the former to sometimes coalesce into social relationships that span new networks. Such a system is "complex" in the sense of von Neumann (1951), because it is iteratively capable of generating increasingly more complex successors. Individual agency is an important factor in these processes, though always within an enabling framework of opportunity and willingness, such as provided by existing relationships, cultural precedent, and resource and environmental constraints (Cioffi-Revilla & Starr 1995). As with other networked phenomena, social networks grow opportunistically, preferentially, and through a non-linear "punctuated" process with connectivity governed by power laws (Barabási & Albert 1999). When successful, collective action promotes new network contacts between individuals at augmented or cross-cutting scales, forging new relationships based on intra-group/inter-individual knowledge and experience, and contributing to the emergence of norms, processes and institutions that constitute social complexity. These, in turn, increase collective action capacity (*asabyia*) as a mission-critical sociopolitical resource.

Canonical Theory: The Process for Emergent Social Complexity (fig. 2, left) We model the process of emergence and development of social complexity from the perspective of the "canonical theory of social complexity" developed by Cioffi-Revilla (2002; 2005), formally derived from the general theory of political uncertainty (Cioffi-Revilla 1998). As a *canonical* theory, the iterative and uncertain process of institutional emergence and political development (and occasional decay) is explained as resulting from a succession of non-deterministic phase transitions that occur in space-time, based on path-dependent *variations* on a *common theme* called the *"fast" branching process*. As each canonical variation of the same "fast" process occurs for a given society, social complexity may accrue, decrease, or remain the same, producing the "slow" accrual process based on emerging experience, statecraft, bonds of trust, norms, institutions and other collective goods. Negative externalities may also be produced, leading to decay in sociopolitical complexity, as explained below.

The "fast"(micro) process at the agent level. Figure 3 represents the main events in the "fast" canonical process, denoted by **G**, **C**, **N**, **U** and **S**, and their corresponding failure modes ~ **C**, …, ~ **S**. The process as a whole generates a political sample space Ω (outcomes on the right). Social complexity emerges (event **S**, the top outcome in Ω) as a path-dependent phase transition resulting from a process of many different albeit well-specified outcomes. The full model (not shown here) contains five additional detailed sub-trees (triggering or production rules) for generating each of the five main events in the "fast" branching process (Cioffi-Revilla 2005).

By definition, the *first stage* begins when an existing group lacks a system of government (i.e., the community is not yet a polity, event **G** in fig. 3, left) and ends in a different situation (politically complex phase) when such a community has formed a system of government (the community is a polity after iterations of the event **P** in fig. 3, top right). This primary phase for Inner Asian groups occurred up to the early Neolithic period, during which time not even chiefdoms are observable in the archaeological record of Inner Asia, and the process was well underway by 4,500 years ago.

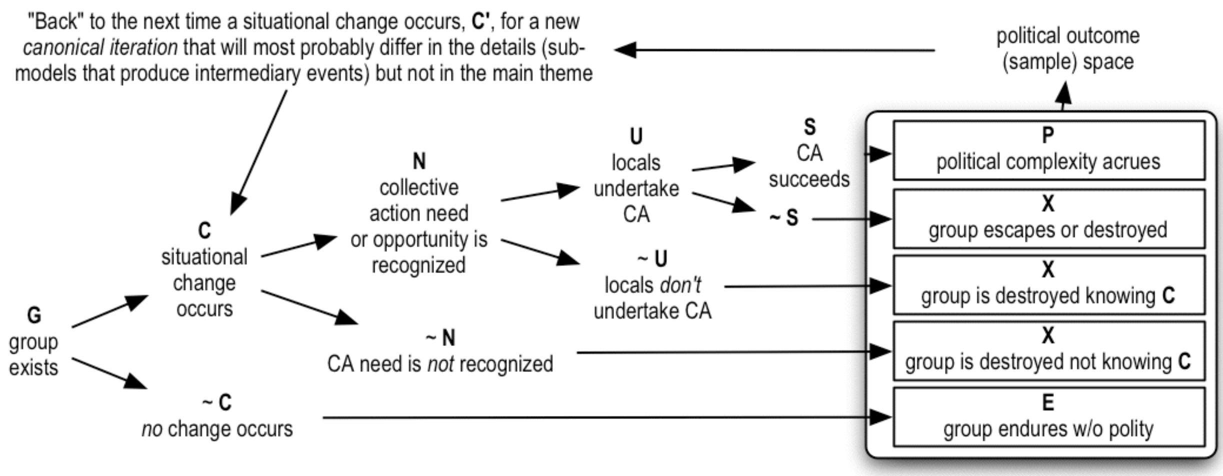


Fig. 3. Canonical theory of social complexity: the "fast" iterative branching process. In turn, each main event (G, C, N, U, S and complimentary failure modes) has an associated causal model in conditional logic, such that the composite causal structure (branching process with all component sub-models) constitutes a sequential-conditional model (Cioffi-Revilla 1998:239-41; 2002; 2005).

A situational change makes the group *metastable*, in the sense that a *potential* for increased (decreased) sociopolitical complexity is created, albeit not immediately realized. The realization of such a potential for complexity will depend on how the rest of the fast process evolves and on how people and environments interact. Xiongnu and Mongol societies succeeded; others failed.

Given a situational change (**C**), the group may or may not *understand the need for collective action* in the face of such a change (event **N** in fig. 3, after **C**). Causally, **N** is an information-processing event, involving signal detection, cognition, and other causal events, and is modeled accordingly. If the group does *not* reach a proper understanding of the situational change, it may be destroyed or dispersed without further political development (outcome $\mathbf{X^*} \in \Omega$).

If the group does perceive the situational change, it may or may not be willing and able to *undertake collective action* (**U**) in response to such change, depending on its collective action capacity (*asabyia*). Collective action occurs through a variety of mechanisms (Lichbach 1996) that are used in detail to model the occurrence of **U** (CC-R 2002, 2005). Sometimes group members fail to undertake collective action (~**U**), even when they understood the situational change **C**, since not every group that understands has the operational capacity to undertake collective action. If the group undertakes collective action under a persistent situational change, then *they may succeed* (**S**) *or fail* (~**S**), depending on the situation and the actions undertaken. If they fail they may be destroyed (outcome $\mathbf{Z} \in \Omega$). For example, the Mongol empire succeeded with a federational organization, while others failed and were absorbed or destroyed as the Mongol imperial polity expanded. Several outcomes in the total outcome space (events denoted by **X**) can all eventually lead to state failure, for example by long-term degradation in collective action capacity.

Finally, if they succeed, then the consequences or societal effects of such success will augment the political complexity of the group (outcome $\mathbf{A} \in \Omega$), because—even if only on a small scale and the most temporary basis—mobilization of resources, lessons about who to trust, hierarchies of leaders and followers, specialized assignments, division of labor, sharing of information, coordination experience, and other elements of government will have been realized. Significantly, collective action capacity for dealing with the next situational change (threat or opportunity) will increase. The phase transition in the quantum increase in collective action capacity is observable by the formation of multiplex networks on several scales: cognitive, individual, group, and institutional. Such a phase transition has enduring organizational effects on the group, and the next time their situation changes demanding collective action they might be able to draw on more collective action capacity and cope with it better; they will have more governance experience than before. The phase transition also means the *realization* of the *potential* that had been created by the earlier situational change, when the group had become *metastable* after the initial phase.

The "slow"(macro) process at the societal level. The above sequence of main events describes a single passage through the "fast" canonical process. Over time, a group will experience many such processes, each as a variation on the common theme of challenge-response just described. Failure paths lead to political decay and even destruction (through all events beneath **P**), so the emergence of political complexity is never a preordained outcome (*asabyia* is not produced automatically).

In Inner Asia the slow macro process at the societal level eventually generated a sequence of state-level polities (Xiongnu, Türk, Uighur, Khitan, Mongol empire, and others), as well as some failures. For example, the Xiongnu polity formed around 200 BC *because* the Xiongnu society at that time was able to overcome, through collective action, a situational change composed of several events (to be recorded in our diachronic database) that took place while the society had a pre-state system of government. Had Xiongnu society failed in achieving collective action it would have transitioned into one of the other forms of social complexity in the outcome-space Ω. For example, it may have been destroyed, conquered or dispersed (event **X**). The latter were instances of failure in dealing with invasions, economic, demographic or environmental changes. For example, power struggles after the 15th century produced political decline and Mongolia was eventually conquered by the Manchus in the 17th and 18th centuries.

3. TOWARDS COMPUTATIONAL IMPLEMENTATION

The project uses two classes of theory-drive research methods that are integrated (fig. 2 earlier) to accomplish the project goals: empirical methods and computational methods, as in table 1. In this paper we summarize only the computational methods. As for other parts of this project, a more detailed description is provided in the full project proposal Cioffi-Revilla et al. (2005).

As shown earlier in Figure 2, this project will use computational methods to formalize, instantiate, test, and analyze the canonical theory presented earlier, based on tools developed by the GMU team and discussed with the SI in preparation for this project. Specifically, we will use the MASON simulation toolkit (Luke et al. 2005) to instantiate and analyze the canonical-theoretic model, and will (complexity of the model permitting) apply the ECJ evolutionary computation system as part of the testing and analysis process. Both tools may be found at http://cs.gmu.edu/~eclab/ The model, *InnerAsianWorld,* will consist of individuals and groups interacting with one another and with their environment generating a virtual history of emergent community-level and more aggregate social structures (norms, mutual trust, institutions, land-use patterns and other collective outcomes).

Model Instantiation and Analysis with MASON The canonical theory of emergent political complexity will be instantiated with an agent-based model in the new *MASON simulation environment* (*http://cs.gmu.edu/~eclab/projects/mason/*). MASON is a fast, easily extendable multiagent simulation library plus a suite of visualization tools and other modules, and is the result of a joint effort of Sean Luke and Cioffi-Revilla, and has been co-funded by George Mason University's Evolutionary Computation Laboratory and GMU's Center for Social Complexity. MASON was explicitly designed to foster cross-fertilization between computer science and the social sciences, and so supports the supports the interdisciplinary goals of the Human and Social Dynamics Priority Area.

Other discrete-event multi-agent simulators are available: Swarm, RePast, Ascape, StarLogo, and NetLogo. MASON meets our design criteria better than these systems because it is faster (in some cases, much faster), while retaining portability, complete separation of visualization from the model, checkpointing, and guaranteed replicability (Luke et al. 2005). The MASON features are essential for our goals, including the special needs of evolutionary computation.

Inner Asia Agent-Based Model "ModelofInnerAsia" (MIA; figure 4) will be the simulation model that instantiates the canonical theory described earlier (fig. 3), a model for understanding the rise and fall of polities as a result of how societies respond or adapt to challenging threats and opportunities originating from within or without. Thus, MIA goes beyond the tradition of earlier territorial competition simulation models developed mostly in political science (international relations) and "computational history" (Bremer & Mihalka 1977; Cederman 2002; Cusack & Stoll 1990; Liverani & Parisi 1998; Min 2002; Min et al. 2004; Turchin 2003) in several significant ways. The main simulation objective for the MIA simulation is to *grow pristine social complexity over a territory*, rather that evolve a landscape populated by pre-existing polities. Moreover, the design principle is to build the agent-based model (ABM) using the "fast" and "slow" dynamics of the canonical theory, thereby endogenizing the process of collective responses (or failures), rather than relying solely or primarily on economic or demographic dynamics (e.g., Reynolds 2003). MIA will be an ABM for simulating emergence of sociopolitical complexity through adaptation and long-term change—the empirical referent being Inner Asia from approximately 4,500 BC—starting from politically simple (socially unstructured) kin-based nomadic groups of hunter-gatherers discussed earlier.

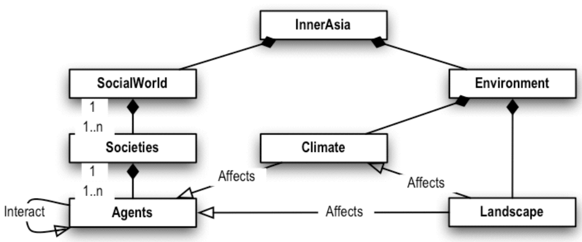


Fig. 4 Computational objects (social agents, groups, environments) that compose the InnerAsia model

Of course, we are not claiming to model an entire 4,500 year period down to individual relationships and land-use in every hectare. However, we believe it crucial to provide individual-agent granularity during the initial process when collective action capacity (*asabyia*) is relatively low (arguably mostly limited to group hunting skills: elements of leadership, intelligence, coordination, some logistics). Thus, MIA will be developed as a "research programme" (Lakatos 1970) through a succession of models with progressive problemshifts. The models will move from the highly local to the abstract and global, each model aggregating and abstracting results of the previous model.

Model I will consist of a single group of human agents in a stylized environment in order to understand the fundamental human and social dynamics of the "fast" branching process (fig. 3 earlier) as situational changes occur (with threats and opportunities) and the group of agents is affected. This model will also explore the complete outcome space, particularly the relative frequency for the "top *explanandum* event" (accrual of social complexity). As initial "proof of concept", the Wetlands model in MASON (Cioffi-Revilla et al. 2004; see Figure 5 below) is instructive for developing Model I, given its focus on hunter-gatherer groups that exchange information in a changing environment where minimal sociality emerges (*sociophily*). We note that land-use patterns are already present in the Wetlands version of Model I, since groups make differential use of the landscape (feeding, transit, shelter, etc.).

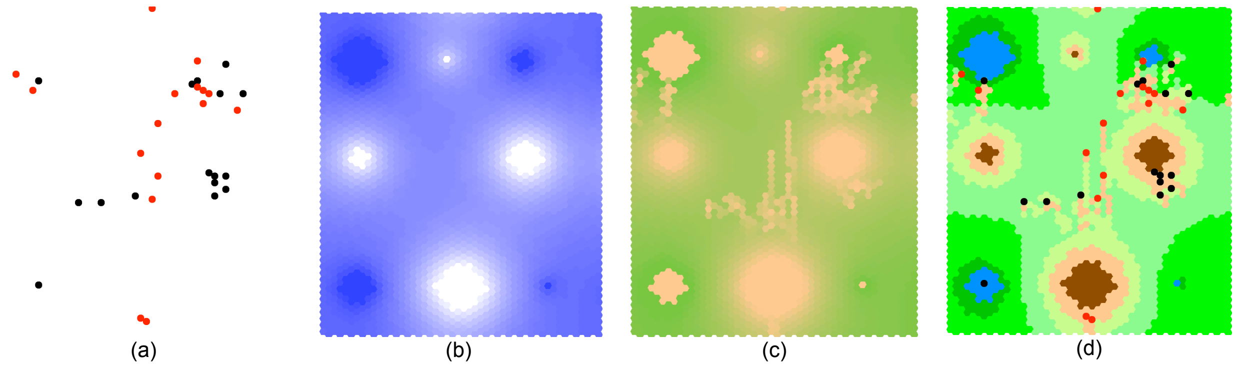


Figure 5. The MASON Wetlands simulation separates computation from visualization to increase spped and computational power. The visualization layers portray the spatial distribution of (a) ethnic groups, (b) climate (dark blue is very rainy, white is dry and offers shelter), (c) food sources (green), and (d) the composite Wetlands world with groups moving around searching food, seeking shelter occasionally and adapting to climate. Source: Cioffi-Revilla et al. (2004).

Model II will expand the number and variety of environments (examining a larger region) and will add more groups, while assuming small groups as the agent-level entity. The main objective of this second model is to grow the first set of chiefdoms or interaction network of simple autonomous polities. For example, Model II will investigate the puzzle of why some groups evolved politically complex societies in some ecotopes whereas others did not. An explanation based on the canonical theory would use *asabyia* as a key causal variable (group cultural attribute). Model III, will be calibrated so as to run for a "long historical period," similar to the duration it took for the first state-level Xiongnu system to emerge (ca. 200 BC). Each model will increase the space-time scale of the simulation from local (Model I or "fast" process) to global (Model III or "slow" process) with a regional "mesoscale" in between (Model II). Land-use patterns are expected to increase in complexity as the models progress, requiring corresponding solutions in terms of computation and visualization.

The objects in the MIA ABM at the start of a simulation run will include a social world and environments inhabited by groups of individuals. Each landscape is composed of hexagons (following Cussack & Stoll 1990 and Ming et al. 2004, to avoid neighboring interaction problems with corners), each with an associated bitstring describing location, basic physical attributes, land-use and resources. As the simulation evolves, groups distributed across the landscape might first settle and then evolve, in some cases producing emergent polities distributed over one or more landscape region.

The main simulation loop for each time step in a simulation run will be formalized from the canonical theory, starting from what triggers situational changes (event **C**, fig. 3) to the production of one of the political outcomes in the outcome space of the "fast" branching process (**P** or one of the **X** failure events in fig.3). The loop will include explicit situation-dependent information-processing (Devlin 1991; Simon 1996), multi-mode decision-making, collective action problem-solving processes (Lichbach 1996) including CA capacity dynamics (Turchin 2003), and opportunity-willingness conditioning (Cioffi-Revilla and Starr 1995; Starr 1978). The main loop always begins from situational change **C**. Sigmoid functions for modeling various "tipping point" triggers (called driven threshold systems in complexity theory; Rundle et al. 1996) will be developed, including modeling the probability of new object formations as complexity evolves.

Model Calibration and Verification Two critical tasks in our simulation development process are testing and validation using the empirical data described in the previous section (archaeological, ethnographic, environmental).

As is often the case in simulations, our initial model parameters (the precise model rules guiding the agents' actions and interactions) may not produce results that closely match the empirical data (Cioffi-Revilla 2002a; Gilbert & Troitzsch 2005: ch. 2). In addition to hand-calibration of the model based on the empirical data, the model will also autocalibrate using global stochastic optimization techniques such as *Evolutionary Computation* (EC) (Fogel and Michelewicz 2000; Mitchell 1996; see Gilbert & Troitzsch 2005: ch. 10 for an overview of EC in social simulations). EC iteratively makes small modifications to the model parameters and rules (i.e., it evolves the code), tests those modifications against a set of known data (the *training set*), and updates model parameters and rules based on feedback from testing, eventually producing a model which fits the data as closely as possible. EC and related methods have been successful at discovering or calibrating multi-agent models in ant colony optimization, competitive game-playing, and robot team simulation. The techniques may also be used to harden the model for robustness. In this scenario, we select a subset of model parameters over which we wish the model to produce invariant results. The EC system then tries to optimize the model in the face of changing settings from these model parameters; such changes may also be co-adaptive to the optimization system.

The primary challenge for EC is model complexity (hence speed). If the model takes a long time to run, it will be challenging to use in an EC framework requiring large numbers of runs. But, while EC is not central to our project, we *will* be able to demonstrate the efficacy of these techniques on a model of this size. Co-PI Sean Luke has developed a popular stochastic optimization system called ECJ (EC in Java; www.cs.gmu.edu/~eclab; Luke 2000), specially designed to deal with large-scale models and dovetails well with MASON.

After calibrating the model through hand tuning and computer optimization, we will then validate the final model against a separate *validation set* of data to argue for generality. This will be done by asking question such as: Do MIA ABM distributions and statistical moments derived from simulations match empirical data from the archaeological, ethnographic and environmental datasets? Do simulated processes match the empirical processes (e.g., life tables of actors, settlement and land-use patterns, Markov processes, counting processes and others in the databases)? Do MIA virtual histories match real histories as constructed from the archaeological, ethnographic and environmental data? Do models from MIA match empirical models (e.g., logit models for political transitions)?

5. CONCLUSIONS

We anticipate three contributions to our understanding of human and social dynamics in response to change and long-term adaptation: (1) a new theoretically-grounded simulation model (“ModelofInnerAsia”) validated and calibrated by the best available data; (2) a new long-term cross-cultural database; and (3) new conceptual, theoretical, and methodological contributions for understanding social complexity and long-term change and adaptation in real and artificial societies. We summarize each of these below. Besides their intrinsic value, each contribution can provide the basis for further scientific advances in theory, data, and methodology, as well as contributing foundations for new synergies.

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