

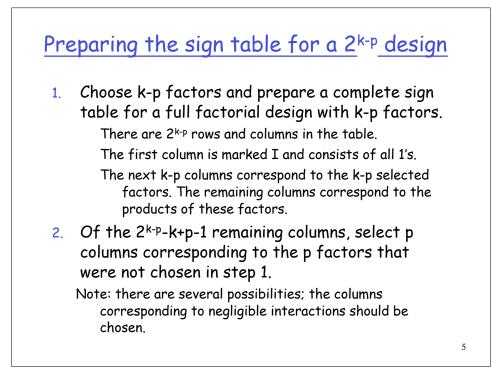
# <u>A 27-4</u> Experimental Design

Consider the 2<sup>3</sup> design below:

| Experiment # | I | A  | В  | С  | AB | AC | BC | ABC |
|--------------|---|----|----|----|----|----|----|-----|
| 1            | 1 | -1 | -1 | -1 | 1  | 1  | 1  | -1  |
| 2            | 1 | 1  | -1 | -1 | -1 | -1 | 1  | 1   |
| 3            | 1 | -1 | 1  | -1 | -1 | 1  | -1 | 1   |
| 4            | 1 | 1  | 1  | -1 | 1  | -1 | -1 | -1  |
| 5            | 1 | -1 | -1 | 1  | 1  | -1 | -1 | 1   |
| 6            | 1 | 1  | -1 | 1  | -1 | 1  | -1 | -1  |
| 7            | 1 | -1 | 1  | 1  | -1 | -1 | 1  | -1  |
| 8            | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1   |

If the factors, AB, AC, BC, ABC are replaced by D, E, F, and G we get a  $2^{7-4}$  design

|                      | AC. AD | AB0   | CD are | negligil | ble we | can u | ise th | ne tabl | e bel |
|----------------------|--------|-------|--------|----------|--------|-------|--------|---------|-------|
| Experiment #         | I      | A     | В      | с<br>С   | D      | E     | F      | G       | у     |
| 1                    | 1      | -1    | -1     | -1       | 1      | 1     | 1      | -1      | 20    |
| 2                    | 1      | 1     | -1     | -1       | -1     | -1    | 1      | 1       | 35    |
| 3                    | 1      | -1    | 1      | -1       | -1     | 1     | -1     | 1       | 7     |
| 4                    | 1      | 1     | 1      | -1       | 1      | -1    | -1     | -1      | 42    |
| 5                    | 1      | -1    | -1     | 1        | 1      | -1    | -1     | 1       | 36    |
| 6                    | 1      | 1     | -1     | 1        | -1     | 1     | -1     | -1      | 50    |
| 7                    | 1      | -1    | 1      | 1        | -1     | -1    | 1      | -1      | 45    |
| 8                    | 1      | 1     | 1      | 1        | 1      | 1     | 1      | 1       | 82    |
| Total                | 317    | 101   | 35     | 109      | 43     | 1     | 47     | 3       |       |
| Total/8              | 39.62  | 12.62 | 4.37   | 13.62    | 5.37   | 0.12  | 5.9    | 0.37    |       |
| Percent<br>variation |        | 37.26 | 4.74   | 43.4     | 6.75   | 0     | 8.1    | 0.03    |       |



| Experiment # | I | A  | В  | С  | AB | AC | BC | ABC |
|--------------|---|----|----|----|----|----|----|-----|
| 1            | 1 | -1 | -1 | -1 | 1  | 1  | 1  | -1  |
| 2            | 1 | 1  | -1 | -1 | -1 | -1 | 1  | 1   |
| 3            | 1 | -1 | 1  | -1 | -1 | 1  | -1 | 1   |
| 4            | 1 | 1  | 1  | -1 | 1  | -1 | -1 | -1  |
| 5            | 1 | -1 | -1 | 1  | 1  | -1 | -1 | 1   |
| 6            | 1 | 1  | -1 | 1  | -1 | 1  | -1 | -1  |
| 7            | 1 | -1 | 1  | 1  | -1 | -1 | 1  | -1  |
| 8            | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1   |

If the ABC interaction is negligible, we should replace ABC with D. If AB is negligible, we can replace AB with D.

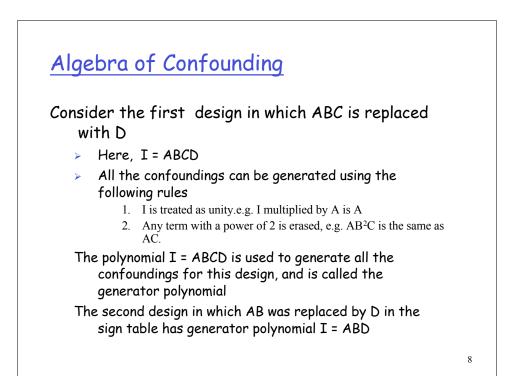


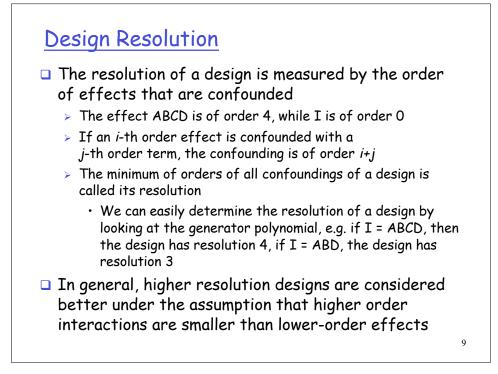
- The drawback of 2<sup>k-p</sup> designs is that the experiments only yield the combined effects of two or more factors. This is called confounding
  - On the previous slide, the effects of ABC and D are confounded (denoted as ABC = D)
- □ In a 2<sup>k-1</sup> design, every column represents a sum of two effects.
  - > For our example,
    - A = BCD, B = ACD, C = ABD, AB = CD, AC = BD,
    - BC = AD, ABC = D, I = ABCD
    - This means that columns for A, B, C, D actually correspond to A + BCD, B + ACD, C + ABD, D+ ABC, etc.
  - > If we replace AB with D,
    - I = ABD, A = BD, B = AD, C = A BCD, D = AB, AC = BCD,

7

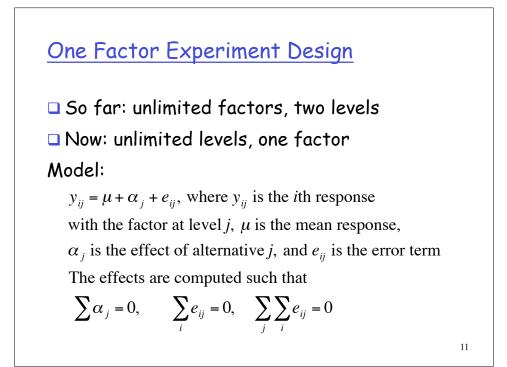
BC = ACD, ABC = CD

□ In a 2<sup>k-p</sup> design, 2<sup>p</sup> effects are confounded



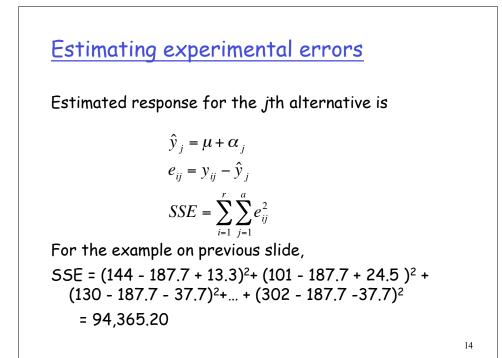






| Notation: $j$ (f | actor leve                | , , <u>,</u> | eation), $a$ (1 | number of l |
|------------------|---------------------------|--------------|-----------------|-------------|
| Example: a =     | = <i>3</i> , <i>r</i> = 5 |              |                 |             |
|                  | R                         | V            | Z               |             |
|                  | 144                       | 101          | 130             |             |
|                  | 120                       | 144          | 180             |             |
|                  | 176                       | 211          | 141             |             |
|                  | 288                       | 288          | 374             |             |
|                  | 144                       | 72           | 302             |             |

| Nodel cont                                      | <u> </u>             |             |               |                    |
|---|----------------------|-------------|---------------|--------------------|
| Notation:<br>$\overline{y}_{\mu} = \mu$ , grand | mean (avg            | . of all re | esponses i,j) | 1                  |
| $\overline{y}_{j} = \mu + \alpha_{j}, c$        | olumn me             | an (avg     | of all respon | ises               |
|   | fc                   | or a parti  | cular factor  | level)             |
|   | R                    | V           | Z             |                    |
|   | 144                  | 101         | 130           |                    |
|   | 120                  | 144         | 180           |                    |
|   | 176                  | 211         | 141           |                    |
|   | 288                  | 288         | 374           |                    |
|   | 144                  | 72          | 302           |                    |
| Column Sum                                      | 872                  | 816         | 1127          | 2815               |
| Column mean                                     | 174.4 $\bar{y}_{.1}$ | 163.2       | 225.4         | 187.7 $\bar{y}_{}$ |
|   |                      |             |               |                    |

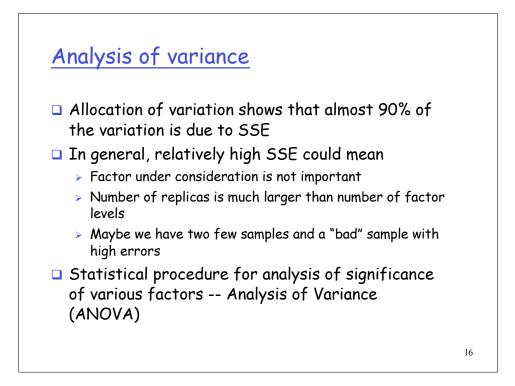


### Allocation of variation

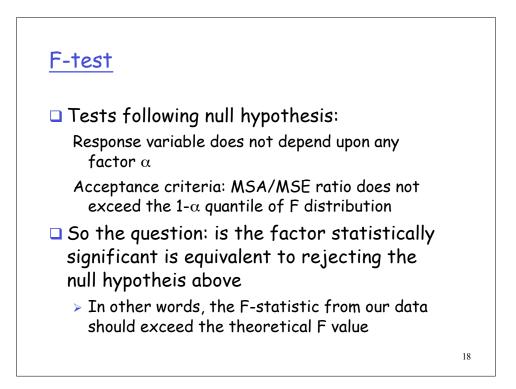
- We can show that SSY = SSO + SSA + SSE, where SSY is sum of squares of y, SSO is the sum of squares of the grand mean, SSA is the sum of squares of effects, and SSE is the sum of squares of errors
- SSE can be easily calculated from SSY, SSO, and SSA

$$SS0 = ar\mu^2, SSA = r\sum_{i=1}^{a} \alpha_j^2$$

- □ Further, SST = SSY SSO = SSA + SSE
- In our example, SST = 105,357; SSA = 10,992 (10.4%);
  SSE = 94,365 (89.6%)
  - > Is SSA statistically significant?



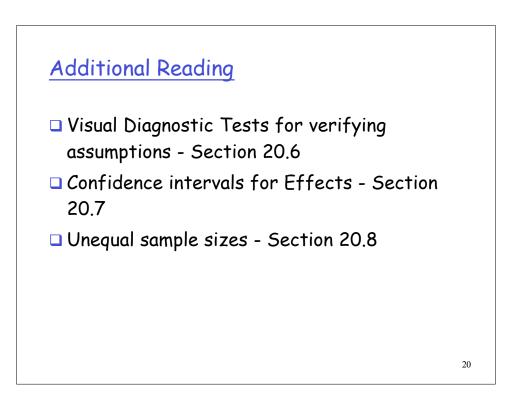
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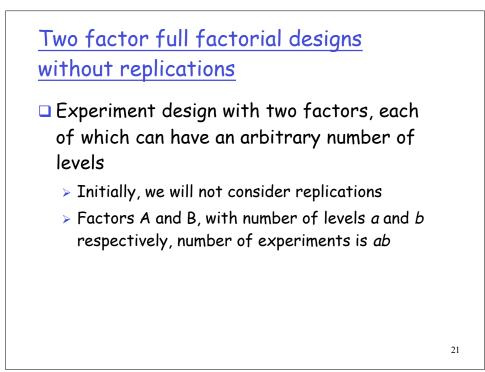


## F-test example

For our example,

MSA = SSA/(a-1) = 10,992/2 = 5496.1 MSE = SSE/a(r-1) = 94,265/12 = 7863.8 Computed F statistic = 5496.1/7963.8 = 0.7 Theoretical F(0.9;2,12) = 2.8 Thus, we can conclude that the factor under consideration is not statistically significant





## Model

 $y_{ij} = \mu + \alpha_j + \beta_i + e_{ij}$ , where  $y_{ij}$  is the observed response with the factor A at level *j* and the factor B at level *i*,  $\mu$  is the mean response,  $\alpha_j$  is the effect of factor A at level *j*, and  $\beta_i$  is the effect of factor B at level *i*, and  $e_{ij}$ is the error term The effects are computed such that  $\sum \alpha_j = 0$ ,  $\sum \beta_i = 0$ ,  $\sum \sum_j \sum_i e_{ij} = 0$ 

We obtain  $\overline{y}_{i} = \mu$ ,  $\overline{y}_{i} = \mu + \beta_{i}$ ,  $\overline{y}_{j} = \mu + \alpha_{j}$ 

| Vorkload     | Two    | One<br>Casha | No<br>Caaba | Row    | Row  | Row<br>Effect     |
|--------------|--------|--------------|-------------|--------|------|-------------------|
|              | Caches | Cache        | Cache       | Sum    | Mean | Effect            |
| SM           | 54.0   | 55.0         | 106.0       | 215.0  | 71.7 | -0.5 $\beta_1$    |
| ECO          | 60.0   | 60.0         | 123.0       | 243.0  | 81.0 | 8.8               |
| IEVE         | 43.0   | 43.0         | 120.0       | 206.0  | 68.7 | -3.5              |
| HRYSTONE     | 49.0   | 52.0         | 111.0       | 212.0  | 70.7 | -1.5              |
| ORT          | 49.0   | 50.0         | 108.0       | 207.0  | 69.0 | -3.2              |
| olumn Sum    | 255.0  | 260.0        | 568.0       | 1083.0 |      |                   |
| olumn Mean   | 51.0   | 52.0         | 113.6       |        | 72.2 | $\overline{y}_{}$ |
| olumn effect | -21.2  | -20.2        | 41.4        |        |      | <i>·</i>          |

$$\begin{aligned}$$

