

# Energy Considerations for Wheeled Mobile Robots Operating on a Single Battery Discharge

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## **Abstract**

This paper presents an energetic model that analyzes energy utilization in mobile robot traverse and estimates maximum range achievable by wheeled mobile robots operating on a single battery discharge. After taking into account different energy utilizations, such as propulsion and steering, the model indicates that the most energy-consuming part of mobile robot is robotics functions, such as computing, sensing, communication, etc. Based on this it points out ways to improve maximum robot traverse range: increasing rover velocity, driving duty cycle (ratio of driving time to total mission time), and decreasing robotics functions' power. Considering the significant energy proportion of robotics functions, the leftover propulsive consumptions are analyzed, which directly determine the maximum range using a classic terramechanics model. The proportions of energy expended in internal robot system and external interaction with terrain are quantified with experiments using a small-sized 4-wheel robot. The maximum traverse range of wheeled-mobile robots could be significant, for example 17km with only 1.12kg battery (166 watts-hour), if the normally immense robotics consumptions are minimized or isolated from the propulsive branch. The resulting propulsion energy, which is only a small fraction of total battery energy expended, is used to estimate achievable range for wheeled mobile robots operating only on a single discharge.



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# 1 Introduction

Maximum robot traverse range is critical in planetary exploration, sewer inspection, mine rescue and many other applications. Short range will largely restrict robot's functionality. For example, planetary exploration would not be able to cover a large planet area, reducing the amount of collected data and mission coverage. The miners trapped in distant places in mine disasters would have less chance to get rescued. So improving maximum traverse range is always of interest in mobile robots.

Without solar panel, combustion engine generator or isotope reactor, energy carried by robots in batteries is always limited. So is the traverse range. Recharging is sometimes impossible for autonomous robots in work, especially in places and time periods lack of sunlight, such as in deep crater or night time in planetary exploration. Replacing new batteries could be difficult in places which are dangerous or inaccessible to human, for example, in exploded mines or confined sewers. Battery-powered robots without replenishment represent the generality of most mobile robots and provide the simplicity to focus on a constant energy amount while examining energy utilization. So only battery-powered wheeled robots operating on single discharge cycle are in the scope of this research.

Generally speaking, rover energy is expended for the following two purposes:

1. Ancillary power for robotics functions like computing, sensing, communication, and payload which scales with duration of operations.
2. Energy for motion, which predominantly scales with distance driven.

Two things that roboticist care about are how far robot goes and how fast it gets there. Curiously, total energy for driving is primarily independent on speed since driving energy is primarily related to rover mass, gravity, distance traveled and terrain resistance. By comparison, the energy for robotics functions, like sensing, computing, communication, utilizes considerable power whether driving or sitting. This ancillary power is less concerning when recharge is possible from solar, generator or radio isotope source, but this energy sink is paramount when operating from only single discharge from battery. In traditional exploration the mission energy for sensing, computing, communication and payload far exceeds driving energy. Most time is spent sitting or creeping. This traditionally requires days, weeks, months or years to drive kilometers. Energies of kilowatt hours are required because ancillary power is drained over such a long duration. The total energy expended during whole mission time could be quantified as:

$$\begin{aligned} \textit{Total Energy} &= \textit{Robotics Energy} + \textit{Mobility Energy} \\ &= \textit{Robotics Power} \times \textit{Mission Time} \\ &\quad + \textit{Constant} \times \textit{Rover mass} \times \textit{Distance} \end{aligned} \quad (1)$$

However, the significance of robotics and mobility energy could not be accurately reflected by this simple addition equation. This paper analyzes actual percentage of robotics and mobility consumption in whole mobile robot energetics and formulate a way to estimate achievable range operating on one single battery discharge.

This paper is divided into the following sections. In Section 2 previous research concerning maximum range is surveyed in an energetic point of view and their relation with this research is discussed. In Section 3, the ideal model of terramechanics propulsion, which is derived from Bekker's Derived Terramechanic model, and ideal mobility energetic model are introduced and their limitations are pointed out. In Section 4, the most overriding part of energy consumption, robotics energy, is illustrated, whose relation with mobility energy and role in the whole mobile robot energetics are further scrutinized. A conclusion is made that given certain consumption of robotics functions, the only way to improve traverse range is by driving fast. In Section 5, a generalized energetic model is generated, which applies to almost any wheeled mobile robots. In Section 7, the generalized model is simplified so that the negligible and unquantifiable energy consumption sources are excluded for further practical analyses. In Section 8, the simplified model is verified by a specific wheeled mobile robot, Killer Krawler 2. The test data are presented and analyzed. Experimental usage and limitation are discussed in the end of this section. Conclusion and discussion of future work are presented in Section 8.

## 2 Prior Work

Mobile robots can find application in a variety of fields, such as mine rescue, sewer inspection, planetary exploration. Tab. 1 compares prior planetary rovers in categories of mass, distance traveled, mission duration, average speed, and non-propulsion power. Then the nominal range is set to 2km. It is assumed that terrain resistance coefficient is 0.5. The time to complete 2km is derived from average rover speed. Then the non-propulsion energy based on this time, propulsion energy for 2km based on reduced weight, and total energy required are calculated. Assuming 100 watt-hour/kg battery energy density gives a rough idea of the required battery mass without recharge. The Mars exploration rovers utilize hazard avoidance software for segments of partial autonomy, but this process is fairly slow because the software causes the rover to periodically stop, observe, and understand the terrain into which it has driven before moving on [1]. The propulsion energy required to travel 2km is several orders of magnitude smaller than non-propulsion consumptions of robotic functions for all planetary rovers listed. As mentioned above, this is due to the fact that the robotics power required for nominal operations is proportional to total operational time and not distance traveled. Assuming total energy all comes from battery, the battery mass required is tremendous. This explained why if without trickle-charging from solar panel or RTG over a long time period most state-of-art planetary rovers can hardly achieve a long distance. In this context, this research investigates the interplays among average velocity, robotic and propulsive power as they pertain to achievable range of robot traverse.

There has been a body of work in maximizing mobile robots traverse range. The most direct approach to increase range is to improve batteries. For example, automotive propulsion batteries are just beginning the transition from nickel metal hydride to Li-ion batteries, after nearly 35 years of research and development on the latter [2]. The research on Li-air batteries is in progress. If successful, it is predicted that the practical energy density will reach 1700 wh/kg. The maximum range of battery-powered mobile



Table 1: Comparison energy expended by previous Mars and Lunar Rover missions based on their mass and speed. The total time required to drive 2km is determined from averaged rover mission speeds. The driving energy is proportional to the weight times the distance.

| Rover       | Mass<br><i>kg</i> | Reduced Weight<br><i>N</i> | Distance<br><i>km</i> | Duration<br><i>day</i> | Average Speed<br><i>m/hr</i> | Non-propulsion Power<br><i>W-h</i> | Time for 2km<br><i>hour</i> | Non-propulsion Energy<br><i>W-h</i> | Propulsion Energy (Crr=0.5)<br><i>W-h</i> | Total Energy<br><i>W-h</i> | Battery Mass w/o Recharge (100W-h/kg)<br><i>kg</i> |
|-------------|-------------------|----------------------------|-----------------------|------------------------|------------------------------|------------------------------------|-----------------------------|-------------------------------------|---|----------------------------|--|
| Lunokhod 1  | 756               | 1226                       | 10.5                  | 304                    | 1.4                          | 50                                 | 1429                        | 71450                               | 340                                       | 71790                      | 718  |
| Lunokhod 2  | 840               | 1362                       | 39                    | 116                    | 14                           | 50                                 | 143                         | 7150                                | 378                                       | 7528                       | 75   |
| Sojourner   | 12                | 43                         | 0.1                   | 85                     | 0.05                         | 10                                 | 40000                       | 400000                              | 12  | 400012                     | 4000   |
| Spirit      | 185               | 687                        | 7.7                   | 2268                   | 0.14                         | 30                                 | 14286                       | 428580                              | 190                                       | 428770                     | 4288   |
| Opportunity | 185               | 687                        | 40.25                 | 3865                   | 0.43                         | 30                                 | 4651                        | 139530                              | 190                                       | 139720                     | 1397   |
| Curiosity   | 899               | 3336                       | 8.6                   | 740                    | 0.48                         | 50                                 | 4167                        | 208350                              | 927                                       | 209277                     | 2093   |
| Yutu        | 140               | 227                        | 0.1                   | 90                     | 0.05                         | 30                                 | 40000                       | 1200000                             | 63.1                                      | 1200063                    | 12001  |

robot would largely benefit from improvement in battery energy density.

The electrical vehicle industry, where the ancillary consumptions for robotic functions don't exist, is another field eagerly pursuing maximum battery range only from drivetrain. Except the fact that electrical vehicles always have a human driver inside operating it and therefore has no needs for sensing, computing, communication, and other typical robotic functions, battery-powered electric motor propulsion is almost identical to wheeled mobile robots. Tesla Motors has developed electric vehicle whose range reaches 265 miles (426 km) [3] in EPA 5-cycle test [4]. An range estimator is published online [5], which mainly considers driving speed and climate control. The driving speed directly influences the aerodynamics drag the vehicle is facing. However, speed of autonomous mobile robots, especially when driving off-road, is slow. So the main factor that influences maximum range in electrical vehicle doesn't apply to wheeled-mobile robots. Neither does climate control because it is only for the human driver in the vehicle, which mobile robots don't have. Range estimation for electrical vehicle doesn't require much information about road type, because most paved asphalt or concrete roads on which electrical vehicles drive may have similar effects on vehicle wheels. The terrain type for mobile robots, however, varies significantly from paved terrain to gravel, pebble road, terrain with big rocks, or even puddle. These factors must be taken into account in order to estimate range for wheeled mobile robots.

The maximum traverse range is also a topic in walking robot. The swinging of legs and arms which emulates human behavior helps to conserve energy. The world robot maximum distance record is set by Cornell University's battery-operated Ranger robot: It walked a 40.5 mile ultra-marathon on a single charge and without human touch in an indoor athletic field [6]. Ranger has a total mass of 9.9kg, which includes the 2.8kg

lithium-ion battery. The 25.9V battery carries 493 watt-hours energy [7]. The fraction of battery mass over total mass is 0.28. The 1m-tall robot is partially autonomous in that all sensing and computation is on board. However, it needs to be started manually and steering is done with a model plane type radio control [6]. Among the 16 watts total power, only 11.3W goes to motors, while 4.7W is used for on board computing and communication [7]. So robotics energy, in contrast to walking, is 29% of total consumption. The zero ratio of stationary to moving time minimizes the consumption for computing, sensing and communication. However, legged robots has a much lower energy efficiency compared with wheeled robots. Although legs-over-wheels approach may allow better adaptability to all-terrain purposes, especially in uneven environment, the Cost of Transport is much higher due to the complicated drivetrain and the related energy losses. The energy consumption configuration is different from wheeled mobile robots, which is more suitable to achieve more traverse distance.

Besides increasing range directly, in order to improve robot mobility, various research has been done in order to optimize motion planning strategies. Yongguo Mei et. al. [8] presented a new approach to find energy efficient motion plans for mobile robots which find routes and determine velocities. The relation of motors' speed and their power consumption is modeled and analyzed. In another research [9], Yongguo Mei et. al. presented an energy-efficient approach to explore an unknown area, which determines the next target for the robot to visit based upon orientation information. These approaches can select suitable propulsion strategy and steering activity to minimize energy consumption during traverse or shorten necessary travel distance while accomplishing the same task. However, they only focuses on one single source of all energy consumptions without a complete higher level energetic model. The only conclusion that can be drawn is that a certain energy consumption is minimized while the uncertainty remains about other consumption sources and thus the maximum range cannot be estimated.

In robotics, not much work has been done to analyze the influence exerted by average velocity on energetics and achievable range, to scrutinize the complete actual energy consumption in wheeled mobile robots, especially in propulsion. Almost no research aims to obtain analytical energy model based on existing mobile robots to estimate achievable range.

The model in this paper analyzes energy utilization for wheeled mobile robots. It pointed out the most important energy usage is by ancillary consumptions, especially for those rovers with high sitting to driving ratio. After ruling out this part, the amount of propulsion energy is quantified, with which the traverse range can be estimated. In addition to estimating maximum range achievable by battery-powered wheeled mobile robots on a single charge, the model can also estimate necessary battery mass on board according to specific planned traverse.

### 3 Ideal Terramechanics Propulsion and Mobility Energetic Model

The ideal terramechanics propulsion is derived from Bekker's Derived Terramechanic Model (BDTM) [10]. It is an analytical tool for evaluating vehicle off-road mobility. BDTM has been developed using Bekker's equations for vehicle soil interactions. Bekker's model is a simple, linear one degree-of-freedom (1-DOF) model, which assumes that in a perfect cohesionless or frictional soil (i.e. dry sand), soil thrust is a function of vehicular weight [11]. Here, it is assumed that thrust is proportional to robot weight. In most cases, mobile robot moves forward with a constant velocity. So resistance( $R$ ) acted on rover equals to thrust. The linear function is approximated by  $R = C_{rr}mg$ .

The main energy consumption in propulsion is the work to overcome resistance from terrain. In this section where only the mobility of robot is focused on, mobile robots are treated only as simple vehicles, not including the robotic parts. So the energy used for robot propulsion can be approximated by an ideal model that equates propulsion work to total battery energy. In this ideal model, the propulsion work is the product of resistance force( $R$ ) and distance( $d$ ).

In terms of driving, achievable range is dependent on specific energy, which is battery capacity divided by weight. In the most ideal case, battery mass composes total mobile robot weight. So the total energy is energy density( $u$ ) times rover mass (see Fig. 1).

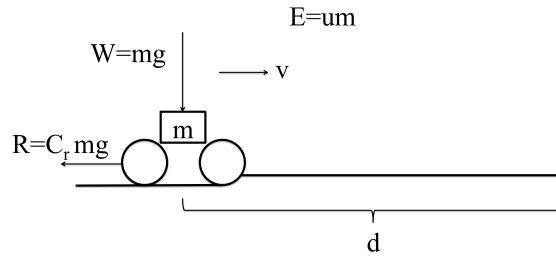


Figure 1: Ideal model of terramechanics propulsion and robot(vehicle) mobility energetic model with full mass battery

So the ideally achievable distance of battery-powered wheeled mobile robots on one single charge is

$$d = \frac{u}{C_{rr}g} \quad (2)$$

In this ideal terramechanics propulsion and energetic model, the maximum wheeled mobile robot range is determined only by energy density( $u$ ), terrain type( $C_{rr}$ ), and gravitational acceleration( $g$ ). However, in the real world of mobile robots, this is not the case. Eqn.2 relies on too many assumptions to obtain its validity. This paper discusses an effective approach to correct these assumptions and extend this ideal model into realistic robotics world.

## 4 Mobile Robot Energetic: Robotics vs. Mobility

One of the most important characteristic for mobile robot is its mobility. Mobile robots have to move when working on the assigned tasks, such as sensing, mapping, exploring, etc. So it is reasonable to divide the whole energetic utilization into two categories: robotics and mobility consumption. Mobility consumption includes all the energy needed to keep robot in motion, such as drive motor, steering motor, and their related energy losses. Robotics consumption is the part of energy used for the assigned tasks during traverse, such as computing, sensing, communication, etc.

The energy required for mobility is dependent on rover drive train, terrain type, traverse distance and rover mass. This only consists of a small fraction of total energetic consumption. Given a rover on a certain terrain, energy required for mobility is approximately constant for a certain range, although this may vary slightly due to different steering activity. The mobility consumption doesn't depend on rover velocity or traveling time, since the mobility power increases with velocity, while traveling time decreases accordingly, which keeps the final mobility energy unchanged.

However, the energy for sensing, computing, and communication is expended at all time whether moving or sitting. So energy consumption of robotic functions is mainly determined by total mission time, therefore rover speed, if given a certain range. Some rovers don't move all the time during traverse. They have to stop intermittently for reasons like navigation, planning, teleoperation, data collection, etc. So they have so called "driving duty cycle", which refers to the percentage of time that the rover is actually driving at payload operation in the total traverse time. In the time period when rover stops and doesn't have mobility consumption, the robotic functions still continuously consume energy. This increases the ratio of robotics to mobility consumption.

### 4.1 Rover Velocity and Driving Duty Cycle

In order to achieve a certain range  $d$ , the amount of mobility energy( $E_m$ ) is approximately constant, which is proportional to rover mass  $m$  and traverse distance  $d$ . The robotics energy is the product of power( $P$ ) and traverse time  $t$ , and the average rover speed is dependent on both actual rover velocity( $v$ ) and driving duty cycle( $D$ ). That is

$$\begin{aligned} E_{total} &= E_m + t \cdot P \\ &= E_m + \frac{d}{v \cdot D} \cdot P \end{aligned} \quad (3)$$

Assuming a 10-kilogram( $m$ ) rover is supposed to achieve a 2 km( $d$ ) traverse on earth ( $g = 9.81m/s^2$ ). The terrain resistance coefficient( $C_{rr}$ )(ratio of terrain resistance to rover weight) is 0.15. The direct propulsion energy to overcome terrain resistance and drive forward consists only 30%( $\eta$ ) of total mobility energy, considering motor loss, internal friction, etc. The robotics power is assumed to be 30 watts. The effect of rover velocity and driving duty cycle on required battery energy can be seen in Fig. 2.

As Fig. 2 shows, with a certain driving duty cycle value, total required energy increases with decreasing rover velocity. Especially in the low rover velocity range, the required energy increases more dramatically. Greater driving duty cycle value reduces required battery energy.

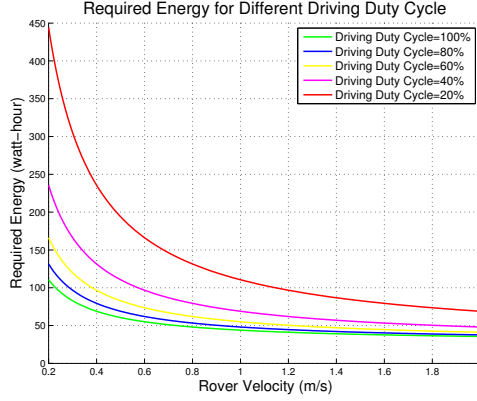


Figure 2: Required total battery energy for a 10kg rover to achieve a 2km traverse for different driving duty cycles with respect to different rover velocities (30-watt robotics power)

Another way to express this is the required rover velocity given certain amount of energy under different duty cycles. This is especially persuasive by battery-operated robots without recharge, like solar panel. The amount of carried energy on board is constant with a certain battery. This can be illustrated by back solving velocity  $v$  in Eqn. 3:

$$v = \frac{P \cdot d}{D(E_{total} - E_m)} \quad (4)$$

Fig. 3 shows that with certain total battery energy, required rover velocity decreases with increasing driving duty cycle. Larger battery capacity allows slower rover velocity.

Increasing rover velocity and driving duty cycle both mean driving fast. So it is safe to say that given a rover with certain parameters and terrain type, in order to achieve certain range with limited battery energy, the only way is driving fast, namely increase rover velocity or driving duty cycle.

## 4.2 Achievable Range

Given battery energy capacity, achievable range can be analytically calculated. It is assumed that 100 watt-hour (about 1kg battery) is applied for traverse. The mobility energy  $E_m$  is no longer constant, because it depends on traverse range, which is the value to estimate. The mobility energy now equals:

$$E_m = \frac{C_{rr} \cdot m \cdot g \cdot d}{\eta} \quad (5)$$

Plugging Eqn. 5 into Eqn. 3 yields the relation between rover velocity and achievable distance:

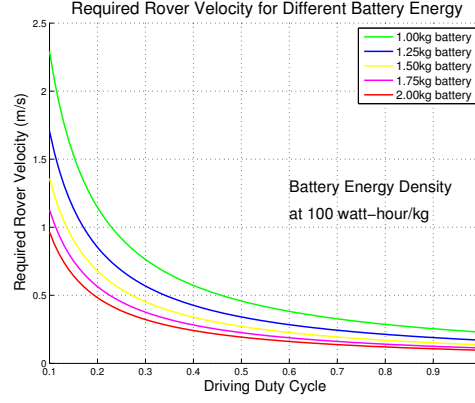


Figure 3: Required rover velocity to achieve a 2km traverse for different battery energy with respect to different driving duty cycles (30-watt robotics power)

$$d = \frac{v \cdot E \cdot D \cdot \eta}{v \cdot C_{rr} \cdot m \cdot g \cdot D + P \cdot \eta} \quad (6)$$

With 100 watt-hour battery energy on board, the relation is illustrated in Fig. 4

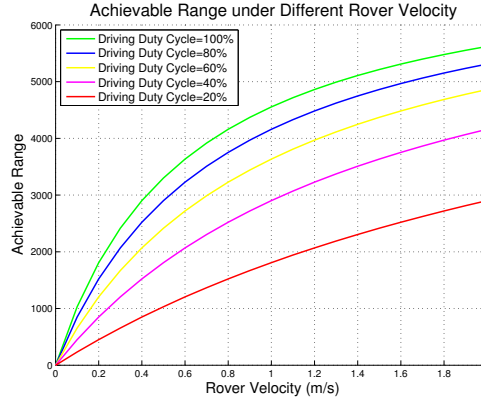


Figure 4: Achievable range of a 10kg rover with 100 watt-hour battery energy under different driving duty cycles with respect to different rover velocities (30-watt robotics power)

Fig. 4 proves that greater range requires greater rover velocity. By taking the limit of rover velocity  $v$ , the maximum asymptotic range can be derived:

$$d_{asymptotic} = \lim_{v \rightarrow \infty} \frac{v \cdot E \cdot D \cdot \eta}{v \cdot C_{rr} \cdot m \cdot g \cdot D + P \cdot \eta} = \frac{E \eta}{C_{rr} m g} \quad (7)$$

After taking into account other factors such as robotics power, rover mass, terrain resistance coefficient, Fig. 5 shows that rover velocity and driving duty cycle, which together determine average rover velocity, have the most significant influence on energy required to finish 2km traverse. Robotics power is also important. The mass and terrain resistance coefficient, which play a role in mobility energy, influence the required energy least.

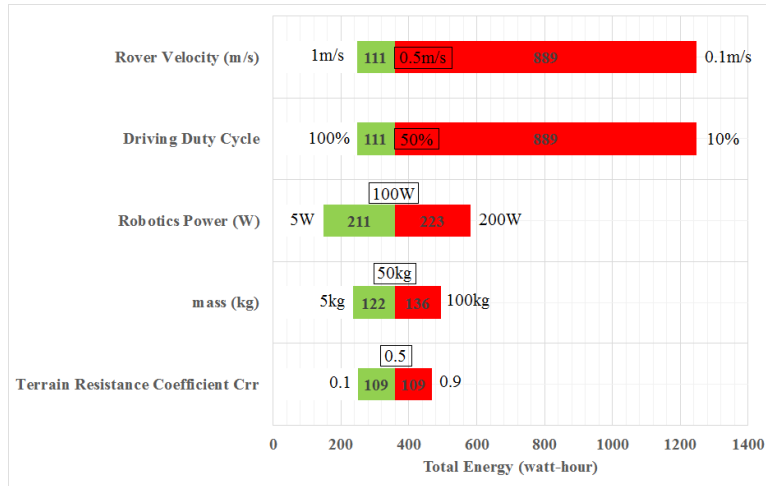


Figure 5: Influence of Different Rover Parameters on Total Required Battery Energy to Accomplish a 2km Traverse

## 5 Wheeled Mobile Robots Energetic Model

The big picture of energy utilization from battery for wheeled mobile robots is illustrated in Fig. 6. In Fig. 6, rectangle boxes represent system components inside mobile robot, while ellipses show the related energy consumption.

At the beginning stage of the chain, energy is drawn from battery pack. Although the capacity of a certain battery pack should be a constant value in the specification, it is unreasonable to assume that all energy stored can be extracted. For reasons of packaging, cell enclosures, and not completely draining the battery, only a fraction of the energy is drawn. The left energy in the battery is termed as "Residual Battery Energy".

### 5.1 Propulsion

A fraction of available battery energy is used by power train, where energy is output in mechanical form. The energy is firstly regulated by power electronics, who consumes energy itself. This is defined as "Power Electronics Loss". This part of loss is mainly made up of heat loss and cooling fan power. The effective energy is further fed into

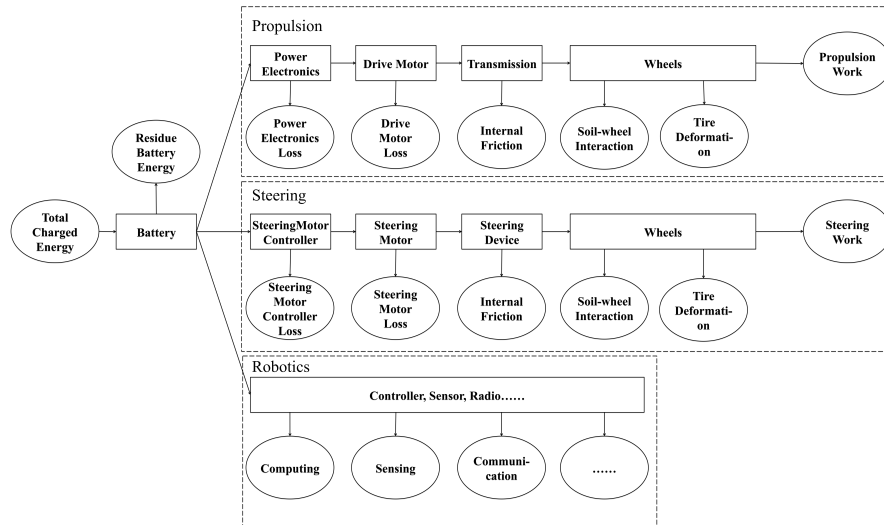


Figure 6: Rover Energy Utilization

drive motor. The motor also loses energy due to resistive losses in windings, core losses and mechanical losses in bearings. The effective transformed energy coming out of the motor is mechanical and is supplied to the transmission. All resistance in gears, bearings, etc, are summarized as "internal friction". This loss is dependent on the type of drive train, manufacturing precision, lubrication and so on. It is inevitable when the rover is in motion. The energy is further transmitted into rover wheels. When rover is interacting with terrain, soil-wheel interaction continuously consumes energy. The tires are always deformed. Due to the damping effect in tires a certain amount of energy is not conserved. The aerodynamic loss is proportional to the squared velocity [12], so it makes up a significant fraction in energy consumption when vehicle speed is high. But the speed of off-road rovers is slow, so this part of energy loss is usually negligible. At the end of the propulsion branch, the final effective energy is used for propulsion. Thrust at least equal to resistance from terrain has to be exerted to keep robot moving forward.

## 5.2 Steering

The energy consumption of steering is another important factor in mechanical work. The configuration of the energy chain is almost the same as that of propulsion. This part of consumption is very difficult to quantify since it depends on a variety of factors, such as terrain type, traverse path, steering activity, and so on. Some skid-steer rovers even don't have steering system. For them, this consumption is zero.



### 5.3 Robotics

In all the energy actually extracted from battery, a large percentage is consumed by robotics functions. Unlike propulsion and steering energy which are mainly dependent on distance, robotics consumption is determined by mission time.

During traverse, rover has to consume energy to accomplish the assigned tasks, such as sensing environment, collecting data, taking photos, or rescuing survivors in extremely dangerous environment. Some rovers have to communicate with the base. The computer on board consumes energy continuously as well. The rover may have other robotic consumption, like lighting for operating in caves, mines and sewers. It is impossible to generalize all consumption types in one model since they vary significantly from rover to rover.

## 6 Simplified Model

In order to get rid of the negligible and unquantifiable energy consumptions, a simplified model is formulated. In this model, only the residual battery energy, steering and robotics consumption, power electronics loss, motor loss, and internal friction are considered, as shown in Fig. 7.

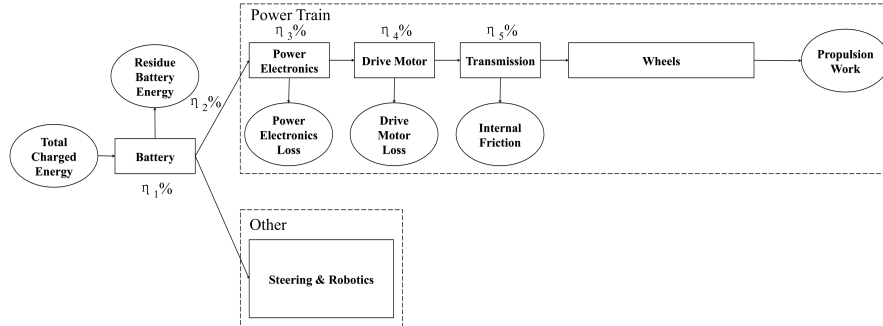


Figure 7: Simplified Model

In Fig. 7, the energy from battery has to go through a series of bypasses and losses in the whole energy chain to reach the bottom, the propulsion work, which determines the maximum traverse distance. Here, several energy efficiencies for the intermediate steps in the energy chain are defined:

$$\text{Battery Efficiency: } \eta_1\% = 1 - \frac{\text{Residual Battery Energy}}{\text{Total Charged Energy}}$$

$$\text{Propulsion Branch Efficiency: } \eta_2\% = 1 - \frac{\text{Steering Consumption} + \text{robotics Consumption}}{\text{Available Battery Energy}}$$

$$\text{Power Electronics Efficiency: } \eta_3\% = 1 - \frac{\text{Power Electronics Loss}}{\text{Propulsion Branch Energy}}$$

$$\text{Motor Efficiency: } \eta_4\% = 1 - \frac{\text{Motor Loss}}{\text{Power Electronics Energy}}$$

Mechanical Efficiency:  $\eta_5\% = 1 - \frac{\text{Internal Friction Loss}}{\text{Mechanical Energy}}$

## 6.1 Comparison with the Ideal Model

Through these five intermediate efficiencies, or losses, the energy left is the effective fraction used directly to overcome terrain resistance and to propel. This partially resemble the ideal model in Fig. 1, where we derived Eqn. 2. Here, the term  $f$  is introduced to describe the fraction of battery mass over total rover mass, in sight of the fact that no mobile robot can carry zero non-battery mass on board. The realistic case is interpreted: the reduced energy used for propulsion through  $\eta_1$  to  $\eta_5$  should be

$$E_p = \prod_{i=1}^5 \eta_i E = \prod_{i=1}^5 \eta_i f m u \quad (8)$$

$E_p$  is used purely for direct propulsion, which is realistic, in contrast to the ideal model. So when the terrain is simplified to a homogeneous and flat one with a constant resistance coefficient  $C_{rr}$ , we have

$$\prod_{i=1}^5 \eta_i f m u = C_{rr} m g d \quad (9)$$

Canceling  $m$  in both sides yield:

$$d = \frac{\prod_{i=1}^5 \eta_i f u}{C_{rr} g} \quad (10)$$

Compared with Eqn. 2

$$d = \frac{u}{C_{rr} g}$$

we have an extra term

$$\prod_{i=1}^5 \eta_i f$$

This term takes into account realistic energy distribution in robot system and battery mass fraction and therefore extends our ideal model into real world.

## 6.2 Propulsion Branch Efficiency

The most variable efficiency is the propulsion branch efficiency since the fraction of robotics consumptions varies significantly from rover to rover, or even from mission to mission with the same rover. Even given a certain rover, this efficiency is closely dependent on rover velocity( $v$ ) and driving duty cycle( $D$ ) as well, as discussed in Section 3. If the normal propulsion power, steering power, and robotics power is known, the propulsion branch efficiency could be calculated by:

$$\eta_2 = \frac{P_{propulsion} \cdot D}{P_{propulsion} \cdot D + P_{steering} \cdot D + P_{robotics}} \quad (11)$$

### 6.3 Asymptotic Maximum Traverse Distance

With a certain rover, it is commonsense that maximum traverse distance increases with increasing number of batteries. However, maximum distance is asymptotic, since increasing total energy with more batteries also increases rovers mass, which has a counter effect on range.

Assuming battery number to be  $n$ , energy of a single battery is  $e$ , rover mass without battery is  $m_r$  and single battery mass is  $m_b$ , we have

$$d = \prod_{i=1}^5 \eta_i \frac{ne}{C_{rr}(m_r + nm_b)g} = \prod_{i=1}^5 \eta_i \frac{ne}{nC_{rr}m_b g + C_{rr}m_r g} \quad (12)$$

The asymptotic maximum distance can be derived by taking the limit:

$$d_{asymptotic} = \prod_{i=1}^5 \eta_i \lim_{n \rightarrow \infty} \frac{ne}{nC_{rr}m_b g + C_{rr}m_r g} = \prod_{i=1}^5 \eta_i \frac{e}{C_{rr}m_b g} = \prod_{i=1}^5 \eta_i \frac{u}{C_{rr}g} \quad (13)$$

Compared with Eqn. 10, Eqn. 13 sets  $f$  to 1, which is the asymptotic value of battery mass fraction when total battery mass is much greater than rover mass. This asymptotic maxima will be further shortened by the payload capacity of the rover. That is, if too many batteries are placed on rover, it will finally lead to stall of drive motors. This also aligns with Eqn. 7 since assuming infinite number of batteries means ignoring rover mass and thus equaling  $\frac{E}{m}$  to energy density  $u$ . Assuming infinite rover velocity is to set  $\eta_2$  to 100%.

### 6.4 Cost Of Transport

Estimation of rover range can help to back solve and estimate the required amount of batteries when a maximum traverse distance is already known before the transit. Given a specific rover, all model efficiencies remain approximately constant within normal payload range. While motor efficiency may vary with different loads, the change is miniscule. With a certain type of terrain, the average resistance coefficient is a constant value that can be calibrated by experimentation.

With the model described above, "Cost of Transport" can be calculated analytically, which helps to estimate approximate maximum traverse range for a mobile robot when negotiating with a certain terrain.

Cost of Transport is defined as total energy used per unit weight per unit distance travelled:

$$COT \triangleq \frac{E}{mgd} \quad (14)$$

If the energy used comes from battery,

$$COT = \frac{fmu}{mgd} = \frac{fu}{gd} \quad (15)$$

So

$$d = \frac{fu}{COTg} \quad (16)$$

Compared with Eqn. 10,  $COT$  in this model can be expressed as:

$$COT = \frac{C_{rr}}{\prod_{i=1}^5 \eta_i} \quad (17)$$

The Cost of Transport depends only on rover ( $\prod_{i=1}^5 \eta_i$ ) and terrain type ( $C_{rr}$ ). Eqn. 10 now becomes

$$d = \frac{fu}{COTg} \quad (18)$$

## 7 Model Verification

### 7.1 Killer Krawler 2

In order to verify the model, a series of tests were done on a crawler robot: Killer Krawler 2 (KK2), seen in 8. A crawler robot is a type of robot that uses an extremely light and flexible chassis to surmount extreme obstacles relative to its size using only four wheels. The flexible chassis and elastic suspension keep all four wheels in constant contact with the ground even over extreme terrain.

KK2 employs a 2-motor 4-wheel drive configuration. Two independent motors drive the front and rear axles respectively. The separation of propulsion power enables more driving profiles, like 2-wheel-drive, 4 wheel-drive, or even anchored-crawling. However, in most cases, 4-wheel-drive is selected to ensure better mobility. KK2 has steering systems on both front and rear axles, which further expands its capacity to negotiating with rough terrains.

The energy distribution schematic is shown in Fig. 9.

As shown in Fig. 9, the steering and robotics consumptions are completely isolated with the primary battery pack. The steering largely depends on different traverse path. The robotics consumption varies from robot to robot as well. Some sensing robot requires only a little energy to power the sensors when compared with propulsion work. Others may have large actuators on board like robot arms which demand even more energy to actuate than propulsion. So it is ideal that the most variable source of energy consumption can be excluded from the model, which set the propulsion branch efficiency  $\eta_2$  to be 100%.

The verification mainly focuses on propulsion branch. One may argue that the propulsive consumption is only a small fraction of the whole energetic model. However, it is the propulsion branch that directly influence the maximum traverse range. The robotics consumption varies significantly from rover to rover, but can be included into the propulsive branch by Eqn. 11. In order to achieve generality for all wheeled

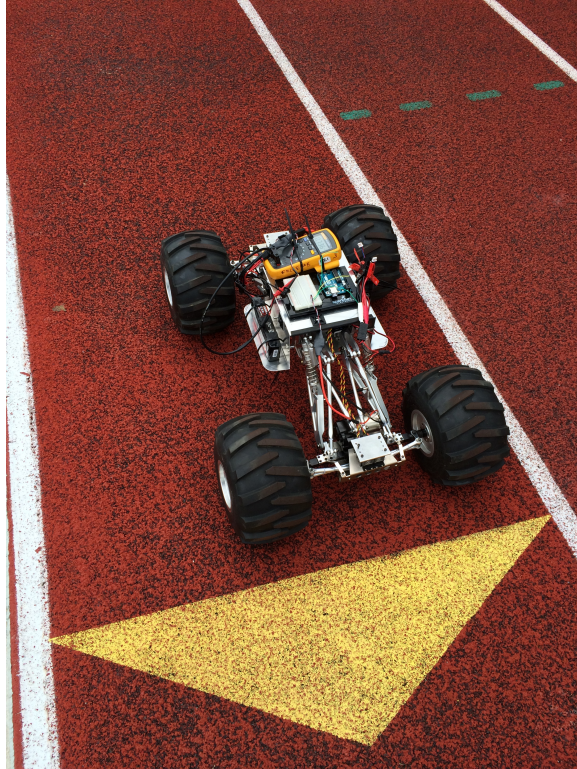


Figure 8: Killer Krawler 2 in Field Test

mobile robots with different payload, the verification is done only on the common parts: propulsion branch.

In order to determine other efficiencies from  $\eta_1$  to  $\eta_5$ , three current voltage monitors are plugged into the rover. They are shown by W in Fig. 9 and help to measure the power in different positions in the energy chain.

The ancillary battery pack comprises four 7.6V 2200mAh 2-cell LiPo batteries and the single element of the primary power source is one 11.1V 5000mAh 3-cell LiPo battery.

## 7.2 Experiment

In the energy model described in Fig. 7, the residue battery energy can be determined by the difference between power calculated by the first current voltage monitor W1 and the claimed battery capacity of the primary pack. There is no steering and robotics consumption due to the isolation of energy chains. The power electronics loss is the difference between the power from W1 and the sum of W2 and W3. One term that can not be measured directly by the existing tools is the internal friction loss. So the "wheels-up test" is designed to quantify this amount of energy.

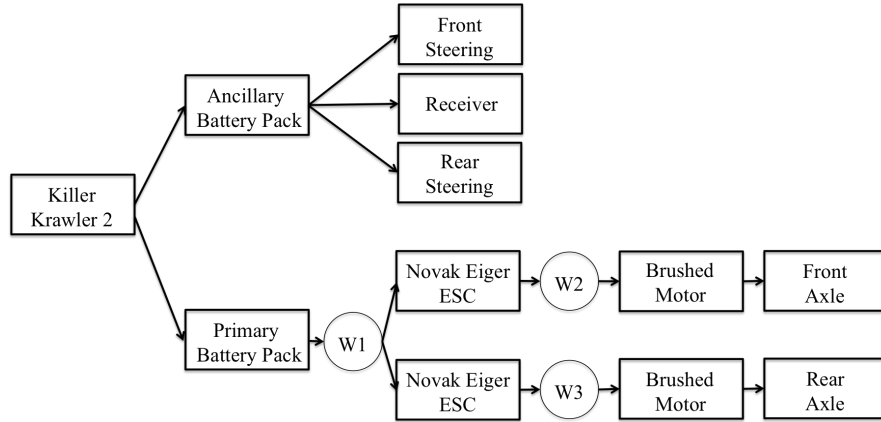


Figure 9: KK2 Energy Distribution Schematic

Table 2: Wheels-up Current and Voltage

|       | Averaged Current (A) | Averaged Voltage (V) |
|-------|----------------------|----------------------|
| Front | 3.0                  | 4.0                  |
| Rear  | 3.1                  | 4.1                  |

In the wheels-up test, the rover was suspended in the air. So all the situations in real robot traverse are simulated except the impact from external terrain. In other words, the energy to overcome the resistance from terrain and to propel is excluded in this test. In this context, all the mechanical energy comes out of the motors are purely consumed by the internal friction, which can give us an idea of the energy consumption inside the rover itself.

In the field test, KK2 was driven on a chosen terrain, the plastic runway of an athletic field. The resistance coefficient between the runway and KK2's beadlock wheels is measured. The current and voltage value of the three monitors are recorded and averaged. The traverse distance on one full discharge cycle of the primary pack with one, two and three single battery elements is measured for the purpose of model verification.

## 7.3 Data

### 7.3.1 Wheels-up Test

In the 2-hour wheels-up test, the front current  $I_2$ , front voltage  $V_2$ , rear current  $I_3$ , rear voltage  $V_3$  are measured and averaged (shown in Tab. 2) until the primary battery pack is empty.

Table 3: Field Test Current and Voltage

|       | Averaged Current (A) | Averaged Voltage (V) |
|-------|----------------------|----------------------|
| Front | 3.6                  | 5.7                  |
| Rear  | 3.5                  | 5.7                  |
| Total | 4.6                  | 9.6                  |

### 7.3.2 Field Test

In the 1.25-hour field test, not only the current and voltage of the front and rear motor, but the total I and V of the primary battery pack are measured. The average values are shown in Tab. 3.

## 7.4 Analysis

### 7.4.1 Internal Friction Power

The aim of the wheels-up test is to determine how much energy is consumed inside the rover, after the motor where electrical energy is transformed into mechanical energy. In the 2-hour wheels-up test, the energy supplied to the motor is (the values are taken from Tab. 2):

$$E_{motor} = E_{front} + E_{rear} = (V_{front} \times I_{front} + V_{rear} \times I_{rear}) \times 2 \text{ hour} = 1.78 \times 10^5 J \quad (19)$$

A fraction of this energy is lost because of motor inefficiency. The most significant part of motor loss is heat loss due to resistance. So the motor energy consumption is simplified as:

$$E_{motor} = E_{heat \text{ loss}} + E_{mechanical} \quad (20)$$

According to Joule's First Law, the amount of heat released due to passage of an electric current through a conductor is proportional to the square of the current such that

$$Q \propto I^2 R \quad (21)$$

Joule heating has a coefficient of performance of 1.0, so

$$P_{heat \text{ loss}} = I^2 R \quad (22)$$

The resistance of the front and rear motors are both  $0.5\Omega$ . So we can calculate the total motor loss, which is  $6.70 \times 10^4 J$ .

The efficiency of the two motors in the wheels-up test is about 62%, which is normal for small size brushed motors.

By the assumption that all motor loss is heat loss, all other energy taken by the motor should be transformed into mechanical energy. This amount of energy is only

consumed to overcome the internal friction in order to keep the wheels rotating in the air. So the power of the internal friction can be derived:

$$P_{internal\ friction} = \frac{E_{motor} - E_{heat\ loss}}{2\ hour} = 15.4W \quad (23)$$

#### 7.4.2 Field Power

Through the total current and voltage coming out of the primary battery pack, we can know how much energy is actually extracted from the battery:

$$E_{total} = V_{total} \times I_{total} \times 1.25\ hour = 1.99 \times 10^5 J \quad (24)$$

The energy goes into the motors in the 1.25-hour field test can be calculated according to Eqn. 19 with the values in Tab. 3. The energy is  $1.82 \times 10^5 J$ . The motor current increases in the field test due to the increasing load caused by terrain interaction. The increasing load in normal range would increase the efficiency as well. The motor efficiency in the field test is about 68.9%.

By subtracting the heat loss from the motor energy, the out-coming mechanical energy from the two motors is  $1.25 \times 10^5 J$ .

In the field test, however, the mechanical energy is divided into two parts, internal friction and propulsion.

#### 7.4.3 Propulsion Energy

The energy actually used for propulsion in the field test is

$$E_{propulsion} = E_{mechanical\ energy} - P_{internal\ friction} \times 1.25\ hour = 5.61 \times 10^4 J \quad (25)$$

The  $5.61 \times 10^4 J$  energy is at the bottom of the energy chain and is directly used for propulsion.

### 7.5 Efficiencies Calibration

With the data from the wheels-up and field test(not including the measured distance), all the efficiencies  $\eta_1$  to  $\eta_5$  can be derived.

$$\eta_1 = \frac{198720J}{5000mAh \times 11.1V \times 3600s} = 99.5\%$$

$$\eta_2 = 100\%$$

$$\eta_3 = \frac{182115J}{198720J} = 91.6\%$$

$$\eta_4 = \frac{125393J}{182115J} = 68.9\%$$

$$\eta_5 = \frac{56070}{125392.5} = 44.7\%$$



Table 4: Comparison of Actual and Estimated Range

| Number of Batteries | Battery Mass Fraction $f$ | Estimated Range (C=2.4) | Actual Range | Error Rate |
|---------------------|---------------------------|-------------------------|--------------|------------|
| 1                   | 2.8%                      | 6300m                   | 6031m        | 4.5%       |
| 2                   | 5.4%                      | 12150m                  | 12623m       | 3.9%       |
| 3                   | 7.9%                      | 17775m                  | 17029m       | 4.4%       |
| 18                  | 34.7%                     | 76518m                  | 79974        | 4.3%       |

## 7.6 Distance Estimation

Since most realistic factors are considered, quantified, modeled, or properly neglected, now it is reasonable to apply the ideal propulsion model. The mass of the rover plus the primary pack with 1 battery on board is 13.27kg. The resistance coefficient  $C_{rr}$  is 0.07. So

$$d = \frac{E_{propulsion}}{C_{rr}mg} = 6153m. \quad (26)$$

Compared with the actual value of 6031m in the field test, the difference of 122m caused by neglecting other mechanical losses and the possible amplification effect of internal friction in actual terrain is still acceptable.

## 7.7 Cost of Transport Verification

According to Eqn. 17, Cost of Transport can be calculated by plugging in the value:

$$C = 0.25 \quad (27)$$

With a certain terrain and certain rover, the Cost of Transport is also a constant value. For KK2 on the plastic runway,  $COT$  is 0.25.

In order to verify this constant, several distance tests are done to see if the constant can give a sufficiently accurate estimation of maximum traverse distance.

In Eqn. 18, the energy density  $u$  is kept constant with LiPo battery.

$$u = \frac{5000mAh \times 11.1V}{0.37kg} = 150Wh/kg \quad (28)$$

So we changed the battery mass fraction  $f$  with multiple number of batteries. 1, 2, 3, and 18 batteries were used in three tests, the results are shown in Tab. 4

The 4th test with 18 batteries was done by placing the weight of 17 batteries and only 1 actual battery on the rover. This is to simulate the situation when the rover is driving with 18 batteries on board while avoid to make the actual traverse. The achieved range is multiplied by 18.

So the error rate is less than 5%.

## 7.8 Experimental Usage and Limitation

The derived model extends the ideal model and makes it useful in real world application. It can estimate the maximum traverse ranges with different number of batteries on

a single charge. It would be more useful to determine the number of necessary batteries before the transit given the required traverse distance. The quantified amount of actual propulsion energy is sufficiently precise.

However, the model requires an accurate estimation of average resistance coefficient of terrain. Furthermore, the average resistance coefficient sometimes may not capture the variation in real environment. For rovers which have only one single battery pack, the steering and robotics consumption cannot be isolated. They also depends on the energy in the only battery. Without information of their consumption or the ratio of their consumption to the propulsion consumption, the estimation can not be accurate enough.

## 8 Conclusion and Future Work

In conclusion, an effective energetic model is generated and verified in order to extend the usage of an ideal model into real world application. The model analyzes energy utilization in mobile robots. It shows the importance of rover velocity and driving duty cycle in the maximum traverse range problem. One conclusion that can be drawn from the model is that in order to achieve greater range the rover should drive faster and increase driving duty cycle. The analytically calculated Cost of Transport by this model also helps to estimate maximum range of rovers based on battery mass fraction and energy density. The energetic model indicates that the internal friction is the most significant part in propulsion energy consumption. Only 28% of the total energy in propulsion chain can reach the bottom and be used directly for propulsion in the test of this paper. If the robotic consumption is not isolated like in KK2, the percentage of actual energy directly used for propulsion work will be further reduced, especially with a low rover velocity and driving duty cycle value. This explains why the range of electric-powered mobile robots is not satisfactory.

In future work, however, methods to better quantify the steering and robotics consumption remains to be revealed. Sometimes the energy in the rover cannot be separated and must rely on one single battery. The better way to find propulsion branch efficiency  $\eta_2$  still remains to be researched, especially when the driving duty cycle is not so clear. The possible amplification effect of internal friction when dealing with actual terrain should be considered as well. Devices like magnetic bearings can be applied to suspend the rover in air so that the normal support force on the wheels can be retained while eliminating terrain resistance. Other mechanical losses should be modeled and quantified to get a more precise model.

## 9 Acknowledgment

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