

ColorfulCurves: Palette-Aware Lightness Control and Color Editing via Sparse Optimization

Cheng-Kang Ted ChaoGeorge Mason UniversityJason KleinCornell UniversityJianchao TanKuaishou TechnologyJose EchevarriaAdobe ResearchYotam GingoldGeorge Mason University





















Clustering-based: [Chang et al. 2015; Nguyen et al. 2017; Zhang et al. 2017]





Clustering-based: [Chang et al. 2015; Nguyen et al. 2017; Zhang et al. 2017]

Histogram: [Morse et al. 2007]





Clustering-based: [Chang et al. 2015; Nguyen et al. 2017; Zhang et al. 2017]

Histogram: [Morse et al. 2007]

Data-driven: [Lin & Hanrahan 2013; O'Donovan et al. 2011]





Geometric approach: [Tan et al. 2016; Tan et al. 2018; Wang et al. 2019]



































$I = W \cdot P$







$I = W \cdot P$









$I = W \cdot P$









$I = W \cdot P$

[Tan et al. 2018]













$I = W \cdot P$

ColorfulCurves









$I = W \cdot P$

ColorfulCurves






























































































Palette-based editing



Palette-based editing



Palette-based editing





Hard to control lightness



Incompatible with directly changing pixel colors

Palette-based editing





Hard to control lightness



Incompatible with directly changing pixel colors



Tone curve editing



Palette-based editing





Hard to control lightness



Incompatible with directly changing pixel colors

Tone curve editing



Palette-based editing





Tone curve editing



ColorfulCurves

Palette-based editing Tone curve editing































































































$$I_{ab} = W \cdot P$$

$$I_L = \sum_{i=1}^p \hat{W}_i \odot f_i(L_0)$$







$$I_{ab} = W \cdot P$$

$$I_L = \sum_{i=1}^p \hat{W}_i \odot f_i(L_0)$$



$$I_{ab} = W \cdot P$$

$$I_L = \sum_{i=1}^p \hat{W}_i \odot f_i(L_0)$$



$$I_{ab} = W \cdot P$$

$$I_L = \sum_{i=1}^p \hat{W}_i \odot f_i(L_0)$$



$$I_{ab} = W \cdot P$$

$$I_L = \sum_{i=1}^p \hat{W}_i \odot f_i(L_0)$$









$$I_{ab} = W \cdot P$$

$$I_L = \sum_{i=1}^p \hat{W}_i \odot f_i(L_0)$$


$$I_{ab} = W \cdot P$$





$$I_{ab} = W \cdot P$$

$$I_L = \sum_{i=1}^p \hat{W}_i \odot f_i(L_0)$$



$$I_{ab} = W \cdot P^*$$

$$I_L = \sum_{i=1}^p \hat{W}_i \odot f_i^*(L_0)$$
Sparest change!



$$I_{ab}^{*} = W \cdot P^{*}$$

$$I_{L}^{*} = \sum_{i=1}^{p} \hat{W}_{i} \odot f_{i}^{*}(L_{0})$$
Sparest change!







Sparest change!



Find the sparsest change to palette and curves that satisfy constraints

Find the sparsest change to palette and curves that satisfy constraints





$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_i^T B^T B L_i} + w_{sp} \cdot \|q_i \cdot \Delta P_{i,*}\|_2^2$$







L_{2,1} norm

$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_i^T B^T B L_i} + w_{sp} \cdot \|q_i \cdot \Delta P_{i,*}\|_2^2$$







L_{2,1} norm

$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_i^T B^T B L_i} + w_{sp} \cdot \|q_i \cdot \Delta P_{i,*}\|_2^2$$







L_{2,1} norm

$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_i^T B^T B L_i} + w_{sp} \cdot \|q_i \cdot \Delta P_{i,*}\|_2^2$$

L_{2,1} norm

$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_i^T B^T B L_i} + w_{sp} \cdot \|q_i \cdot \Delta P_{i,*}\|_2^2$$

L_{2,1} norm

$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_i^T B^T B L_i} + w_{sp} \cdot \|q_i \cdot \Delta P_{i,*}\|_2^2$$

L_{2,1} norm

$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_i^T B^T B L_i} + w_{sp} \cdot \|q_i \cdot \Delta P_{i,*}\|_2^2$$

L_{2,1} norm

$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_i^T B^T B L_i} + w_{sp} \cdot \|q_i \cdot \Delta P_{i,*}\|_2^2$$

L_{2,1} norm

$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_i^T B^T B L_i} + w_{sp} \cdot \|q_i \cdot \Delta P_{i,*}\|_2^2$$

L_{2,1} norm

$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_i^T B^T B L_i} + w_{sp} \cdot \|q_i \cdot \Delta P_{i,*}\|_2^2$$

L_{2,1} norm

$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_i^T B^T B L_i} + w_{sp} \cdot \|q_i \cdot \Delta P_{i,*}\|_2^2$$

L_{2,1} norm

$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_i^T B^T B L_i} + w_{sp} \cdot \|q_i \cdot \Delta P_{i,*}\|_2^2$$

$$\sum_{i=1}^{c} \left\| \left(\sum_{i=1}^{p} \widetilde{W}_{ij}(P_{i,*} + \Delta P_{i,*}) \right) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*}$$

L_{2,1} norm

$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_i^T B^T B L_i} + w_{sp} \cdot \|q_i \cdot \Delta P_{i,*}\|_2^2$$

tion of the constraints

$$\sum_{i=1}^{c} \left\| \left(\sum_{i=1}^{p} \widetilde{W}_{ij}(P_{i,*} + \Delta P_{i,*}) \right) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i$$

L_{2,1} norm

$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_i^T B^T B L_i} + w_{sp} \cdot \|q_i \cdot \Delta P_{i,*}\|_2^2$$

tion of the constraints

$$\sum_{i=1}^{c} \left\| \left(\sum_{i=1}^{p} \widetilde{W}_{ij}(P_{i,*} + \Delta P_{i,*}) \right) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i$$

L_{2,1} norm

$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_i^T B^T B L_i} + w_{sp} \cdot \|q_i \cdot \Delta P_{i,*}\|_2^2$$

tion of the constraints

$$\sum_{i=1}^{c} \left\| \left(\sum_{i=1}^{p} \widetilde{W}_{ij}(P_{i,*} + \Delta P_{i,*}) \right) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in$$

L_{2,1} norm

$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_i^T B^T B L_i} + w_{sp} \cdot \|q_i \cdot \Delta P_{i,*}\|_2^2$$

tion of the constraints

$$\sum_{i=1}^{c} \left\| \left(\sum_{i=1}^{p} \widetilde{W}_{ij}(P_{i,*} + \Delta P_{i,*}) \right) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in$$

L_{2,1} norm

$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_i^T B^T B L_i} + w_{sp} \cdot \|q_i \cdot \Delta P_{i,*}\|_2^2$$

ab portion of the constraints $E_{p} = \sum_{j=1}^{c} \left\| \left(\sum_{i=1}^{p} \widetilde{W}_{ij}(P_{i,*} + \Delta P_{i,*}) \right) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{P}_{i,*} \right\|_{2}^{2}$

$$E_{l} = \sum_{j=1}^{c} \|\widetilde{S}_{j} \odot \left(\sum_{i=1}^{p} \widetilde{W}_{ij}L_{i}\right) - \widetilde{C}_{j}\|_{2}^{2} + \sum_{i=1}^{p} \|\overline{S}_{i} \odot L_{i} - \overline{C}_{i}\|_{2}^{2}$$

L_{2,1} norm

$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_i^T B^T B L_i} + w_{sp} \cdot \|q_i \cdot \Delta P_{i,*}\|_2^2$$

ab portion of the constraints $E_{p} = \sum_{j=1}^{c} \left\| \left(\sum_{i=1}^{p} \widetilde{W}_{ij}(P_{i,*} + \Delta P_{i,*}) \right) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{P}_{i,*} \right\|_{2}^{2}$

$$E_i = \sum_{j=1}^{c} \|\widetilde{S}_j \odot \left(\sum_{i=1}^{p} \widetilde{W}_{ij} L_i\right) - \widetilde{C}_j\|_2^2 + \sum_{i=1}^{p} \|\overline{S}_i \odot L_i - \overline{C}_i\|_2^2$$

$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_{i}^{T} B^{T} B L_{i} + w_{sp} \cdot \|q_{i} \cdot \Delta P_{i,*}\|_{2}^{2}}$$

$$E_{p} = \sum_{j=1}^{c} \left\| \left(\sum_{i=1}^{p} \widetilde{W}_{ij}(P_{i,*} + \Delta P_{i,*}) \right) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\|$$

$$E_l = \sum_{j=1}^c \|\widetilde{S}_j \odot \left(\sum_{i=1}^p \widetilde{W}_{ij}L_i\right) - \widetilde{C}_j\|_2^2 + \sum_{i=1}^p \|\overline{S}_i \odot L_i\|_2^2$$

$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_{i}^{T} B^{T} B L_{i} + w_{sp} \cdot \|q_{i} \cdot \Delta P_{i,*}\|_{2}^{2}}$$

$$E_{p} = \sum_{j=1}^{c} \left\| \left(\sum_{i=1}^{p} \widetilde{W}_{ij}(P_{i,*} + \Delta P_{i,*}) \right) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\|$$

$$E_l = \sum_{j=1}^c \|\widetilde{S}_j \odot \left(\sum_{i=1}^p \widetilde{W}_{ij}L_i\right) - \widetilde{C}_j\|_2^2 + \sum_{i=1}^p \|\overline{S}_i \odot L_i\|_2^2$$

$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_{i}^{T} B^{T} B L_{i} + w_{sp} \cdot \|q_{i} \cdot \Delta P_{i,*}\|_{2}^{2}}$$

$$E_{p} = \sum_{j=1}^{c} \left\| \left(\sum_{i=1}^{p} \widetilde{W}_{ij}(P_{i,*} + \Delta P_{i,*}) \right) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\|$$

$$E_l = \sum_{j=1}^c \|\widetilde{S}_j \odot \left(\sum_{i=1}^p \widetilde{W}_{ij}L_i\right) - \widetilde{C}_j\|_2^2 + \sum_{i=1}^p \|\overline{S}_i \odot L_i\|_2^2$$

$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_i^T B^T B L_i} + w_{sp} \cdot ||q_i \cdot \Delta P_{i,*}||_2^2}$$

$$\sum_{i=1}^{p} \left\| \left(\sum_{i=1}^{p} \widetilde{w}_{ij}(P_{i,*} + \Delta P_{i,*}) \right) - \widehat{C}_{*,j} \right\|_2^2 + \sum_{i \in P^c} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{P}_{i,*} \right\|_2^2$$

$$\sum_{j=1}^{c} ||\widetilde{S}_j \odot \left(\sum_{i=1}^{p} \widetilde{W}_{ij} L_i \right) - \widetilde{C}_j ||_2^2 + \sum_{i=1}^{p} ||\overline{S}_i \odot L_i - \overline{C}_i||_2^2$$

$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_{i}^{T} B^{T} B L_{i} + w_{sp} \cdot \|q_{i} \cdot \Delta P_{i,*}\|_{2}^{2}}$$

$$E_{p} = \sum_{j=1}^{c} \left\| \left(\sum_{i=1}^{p} \widetilde{w}_{ij}(P_{i,*} + \Delta P_{i,*}) \right) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{P}_{i,*} \right\|_{2}^{2}$$

$$Subject to$$

$$-128 \le P_{i} + \Delta P_{i} \le 127$$

$$E_{l} = \sum_{j=1}^{c} \|\widetilde{S}_{j} \odot \left(\sum_{i=1}^{p} \widetilde{w}_{ij} L_{i} \right) - \widetilde{C}_{j}\|_{2}^{2} + \sum_{i=1}^{p} \|\overline{S}_{i} \odot L_{i} - \overline{C}_{i}\|_{2}^{2}$$

$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_{i}^{T} B^{T} B L_{i} + w_{sp} \cdot \|q_{i} \cdot \Delta P_{i,*}\|_{2}^{2}}$$

$$E_{p} = \sum_{j=1}^{c} \left\| \left(\sum_{i=1}^{p} \widetilde{w}_{ij}(P_{i,*} + \Delta P_{i,*}) \right) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \|(P_{i,*} + \Delta P_{i,*}) - \widehat{P}_{i,*}\|_{2}^{2}$$

$$Subject to$$

$$-128 \le P_{i} + \Delta P_{i} \le 127$$

$$E_{l} = \sum_{j=1}^{c} \|\widetilde{S}_{j} \odot \left(\sum_{i=1}^{p} \widetilde{w}_{ij} L_{i} \right) - \widetilde{C}_{j}\|_{2}^{2} + \sum_{i=1}^{p} \|\overline{S}_{i} \odot L_{i} - \overline{C}_{i}\|_{2}^{2}$$

$$E_{sp} = \sum_{i=1}^{p} \sqrt{L_{i}^{T} B^{T} B L_{i} + w_{sp} \cdot \|q_{i} \cdot \Delta P_{i,*}\|_{2}^{2}}$$

$$E_{p} = \sum_{j=1}^{c} \left\| \left(\sum_{i=1}^{p} \widetilde{W}_{ij}(P_{i,*} + \Delta P_{i,*}) \right) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,j} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \widehat{C}_{*,i} \right\|_{2}^{2} + \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\| (P_{i,*} + \Delta P_{i,*}) - \sum_{i \in P^{c}} \left\|$$

$$E_l = \sum_{j=1}^c \|\widetilde{S}_j \odot \left(\sum_{i=1}^p \widetilde{W}_{ij}L_i\right) - \widetilde{C}_j\|_2^2 + \sum_{i=1}^p \|\overline{S}_i \odot L_i\|_2^2$$

Block coordinate descent

$$\{L_i, \Delta P_i\} = \arg\min_{L_i, \Delta P_i} E_{sp} + w_{eq}(E_l + E_p)$$

subject to

$$-128 \le P_i + \Delta P_i \le 127$$
$$L_{i,1} = 0, \quad L_{i,s} = 1$$

Block coordinate descent

$$\{L_i, \Delta P_i\} = \arg\min_{L_i, \Delta P_i} E_{sp} + w_{eq}(E_l + E_p)$$

subject to

$$-128 \le P_i + \Delta P_i \le 127$$
$$L_{i,1} = 0, \quad L_{i,s} = 1$$

Block coordinate descent

$$\{L_i, \Delta P_i\} = \arg\min_{L_i, \Delta P_i} E_{sp} + w_{eq}(E_l + E_p)$$

subject to

$$-128 \le P_i + \Delta P_i \le 127$$
$$L_{i,1} = 0, \quad L_{i,s} = 1$$

fast linear solve

Block coordinate descent

$$\{L_i, \Delta P_i\} = \arg\min_{L_i, \Delta P_i} E_{sp} + w_{eq}(E_l + E_p)$$

subject to

$$-128 \le P_i + \Delta P_i \le 127$$
$$L_{i,1} = 0, \quad L_{i,s} = 1$$

$$Ax_l = b$$

fast linear solve

Block coordinate descent

$$\{L_i, \Delta P_i\} = \arg\min_{L_i, \Delta P_i} E_{sp} + w_{eq}(E_l + E_p)$$

subject to

$$-128 \le P_i + \Delta P_i \le 127$$
$$L_{i,1} = 0, \quad L_{i,s} = 1$$

Block coordinate descent

$$\{L_i, \Delta P_i\} = \arg\min_{L_i, \Delta P_i} E_{sp} + w_{eq}(E_l + E_p)$$

subject to

$$-128 \le P_i + \Delta P_i \le 127$$
$$L_{i,1} = 0, \quad L_{i,s} = 1$$

Block coordinate descent

$$\{L_i, \Delta P_i\} = \arg\min_{L_i, \Delta P_i} E_{sp} + w_{eq}(E_l + E_p)$$

$$-128 \le P_i + \Delta P_i \le 127$$
$$L_{i,1} = 0, \quad L_{i,s} = 1$$



$$\min_{\Delta P} \sum_{i=1}^{p} \sqrt{L_i^T B^T B L_i + w_{sp} \cdot \|q_i \cdot \Delta P_{i,*}\|_2^2}$$

small QP problem

Block coordinate descent

$$\{L_i, \Delta P_i\} = \arg\min_{L_i, \Delta P_i} E_{sp} + w_{eq}(E_l + E_p)$$

$$-128 \le P_i + \Delta P_i \le 127$$
$$L_{i,1} = 0, \quad L_{i,s} = 1$$



$$\begin{array}{c} \min_{\Delta P} \quad \sum_{i=1}^{p} \sqrt{L_{i}^{T} B^{T} B L_{i}} + w_{sp} \cdot \|q_{i} \cdot \Delta P_{i,*}\|_{2}^{2}} \\
\text{Small QP problem} \\
\end{array}$$

Block coordinate descent

$$\{L_i, \Delta P_i\} = \arg\min_{L_i, \Delta P_i} E_{sp} + w_{eq}(E_l + E_p)$$

$$-128 \le P_i + \Delta P_i \le 127$$
$$L_{i,1} = 0, \quad L_{i,s} = 1$$



Block coordinate descent

$$\{L_i, \Delta P_i\} = \arg\min_{L_i, \Delta P_i} E_{sp} + w_{eq}(E_l + E_p)$$

$$-128 \le P_i + \Delta P_i \le 127$$
$$L_{i,1} = 0, \quad L_{i,s} = 1$$





Palette-aware lightness control







Photo credit: @<u>Europeana</u>

Palette-aware lightness control







Photo credit: @<u>Europeana</u>

Constraint-driven color editing





Photo credit: @ Pietro De Grandi



Constraint-driven color editing





Photo credit: @ Pietro De Grandi



10 experts, average of 10 years (3-30 years) of photo editing experience



strongly disagree

Q1: The color and luminance editing tools gave me sufficient control over the output image. Q2: Recoloring the image by editing palette colors was convenient. Q3: Luminance editing by adjusting a curve per palette color was convenient. Q4: The automatically chosen palette colors matched my expectations. Q5: Recoloring the image by placing pixel constraints was useful. Q6: Placing and removing constraints in any order was useful. Q7: I found the overall interface easy and fun to use. Q8: I found the tool to be more effective at color and luminance editingcompared to my most comfortable tool

10 experts, average of 10 years (3-30 years) of photo editing experience





strongly disagree

Q1: The color and luminance editing tools gave me sufficient control over the output image. Q2: Recoloring the image by editing palette colors was convenient. Q3: Luminance editing by adjusting a curve per palette color was convenient. Q4: The automatically chosen palette colors matched my expectations. Q5: Recoloring the image by placing pixel constraints was useful. Q6: Placing and removing constraints in any order was useful. Q7: I found the overall interface easy and fun to use. Q8: I found the tool to be more effective at color and luminance editingcompared to my most comfortable tool

10 experts, average of 10 years (3-30 years) of photo editing experience



colors matched my expectations.

Q5: Recoloring the image by placing pixel constraints was useful.

Q6: Placing and removing constraints in any order was useful.

Q7: I found the overall interface easy and fun to use.

Q8: I found the tool to be more effective at color and luminance editingcompared to my most comfortable tool

s (3-30 years) of photo editing experience



"[T]he color changing tool is amazing. I was blown away with

the accuracy of the selection tool and how clean it is." minance editing by adjusting a curve per palette color was convenient.

Q4: The automatically chosen palette colors matched my expectations.

Q5: Recoloring the image by placing pixel constraints was useful.

Q6: Placing and removing constraints in any order was useful.

Q7: I found the overall interface easy and fun to use.

Q8: I found the tool to be more effective at color and luminance editingcompared to my most comfortable tool



"[T]he color changing tool is 10 experts, average of

amazing. I was blown away with the accuracy of the selection Land how clean it is." minance editing by adjusting a curve per palette color was convenient.

"[C]olorfulCurves is intuitive and easy to adjust the color of a

achieved with lightroom." compared to my most

specific area, which cannot be













 $I_{ab} = W \cdot P$



































0.9











0.9



















































Real-time editing under constraint-driven sparse optimization







Semantic-aware color editing







Semantic-aware color editing



Text-guided professional photo editing





Semantic-aware color editing





Text-guided professional photo editing

Dynamic gamut deformation

Acknowledgements

- **Project page**: <u>https://cragl.cs.gmu.edu/colorfulcurves/</u>
- Code and data: <u>https://github.com/tedchao/ColorfulCurves</u>
- Statistics support
 - Yu-Lin Hsu
- **Financial support**
 - Adobe





• Photographers: Mariano Garcia, Cheng-Ju Ko, Eric Wang, Mina Nabil, Jaan AlBalushi, Fabio Amore, Ammar Hashhash, Anastasia Vasilchenko, Dilhara Prasangika, Iwan, Areen Shah





