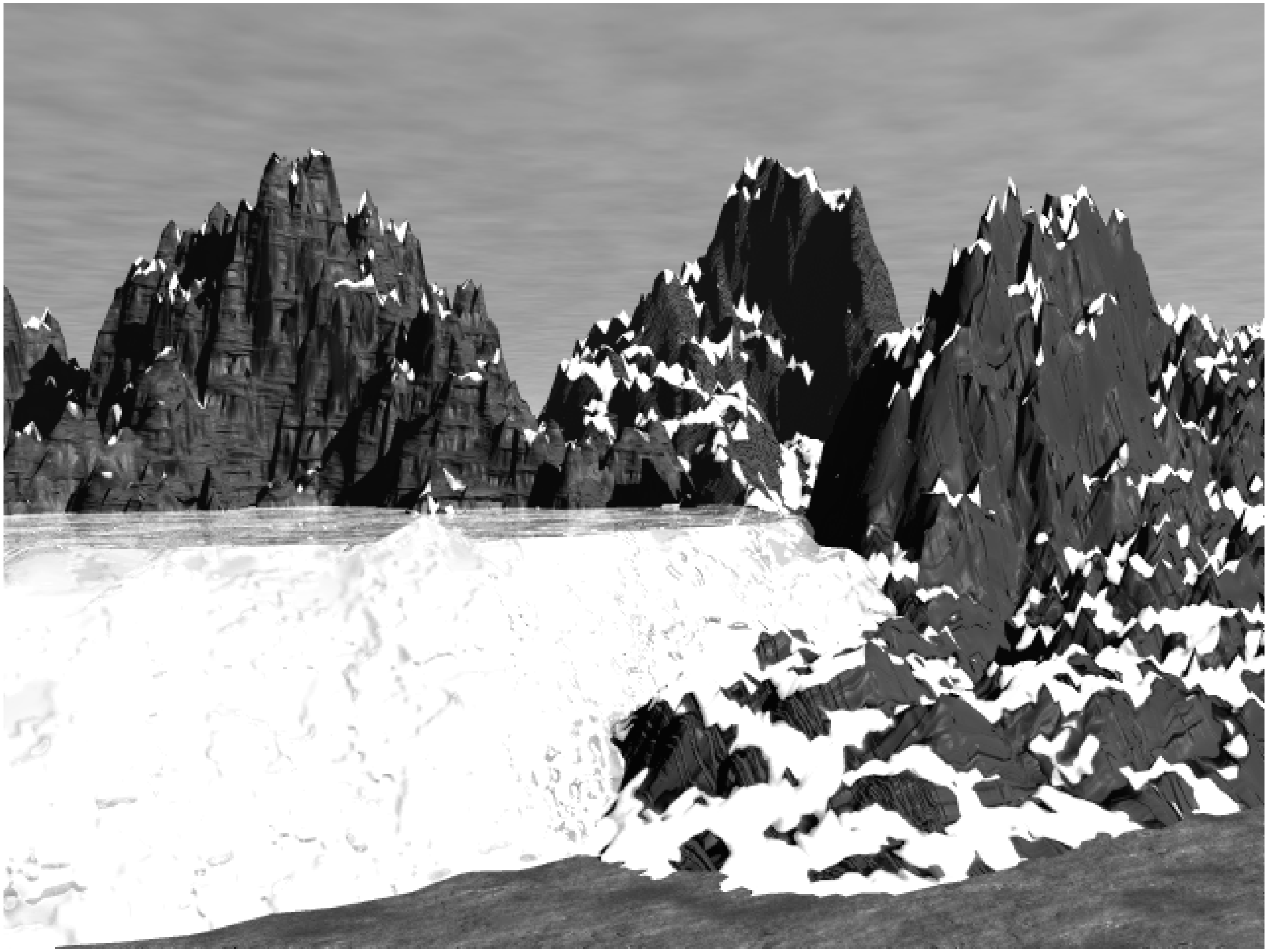
Finding Corner Points



Normalized Cross-correlation

Let $w_1 = I_1(x_1 + i, y_1 + j)$ and $w_2 = I_2(x_2 + i, y_2 + j)$, $i = -W, \ldots, W,\ j = -W, \ldots, W$ be two square image windows centered at locations (x_1, y_1) and (x_2, y_2) of images I_1 and I_2, respectively.

Normalized cross-correlation of w_1 and w_2 is given by

$$NCC(w_1, w_2) = \frac{(w_1 - \overline{w}_1) \cdot (w_2 - \overline{w}_2)}{\|w_1 - \overline{w}_1\| \, \|w_2 - \overline{w}_2\|}$$

where w_1 and w_2 are treated as vectors. ($a \cdot b$ stands for inner product of vectors a and b, $\overline{a}$ for the mean value of vector elements and $\|a\|$ for the 2-norm of vector a.)

For two windows whose pixel values differ by a scale factor only NCC will be equal to 1; if the windows are different NCC has value lower than 1.

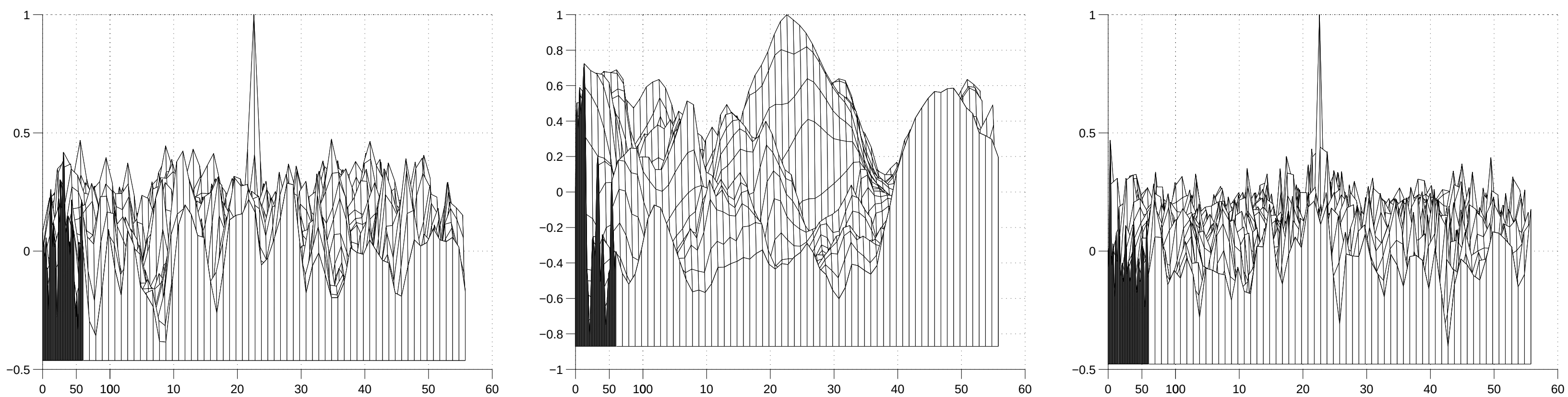
For two non-zero binary patterns which differ in all pixels NCC is -1.

Normalized cross-correlation corresponds to the cosine of the angle between w_1 and w_2; this angle varies between 0° and 180° — the corresponding cosines vary between 1 and -1.

Corner points differ from other points — have low NCC with all other points.

Corner Point Candidates

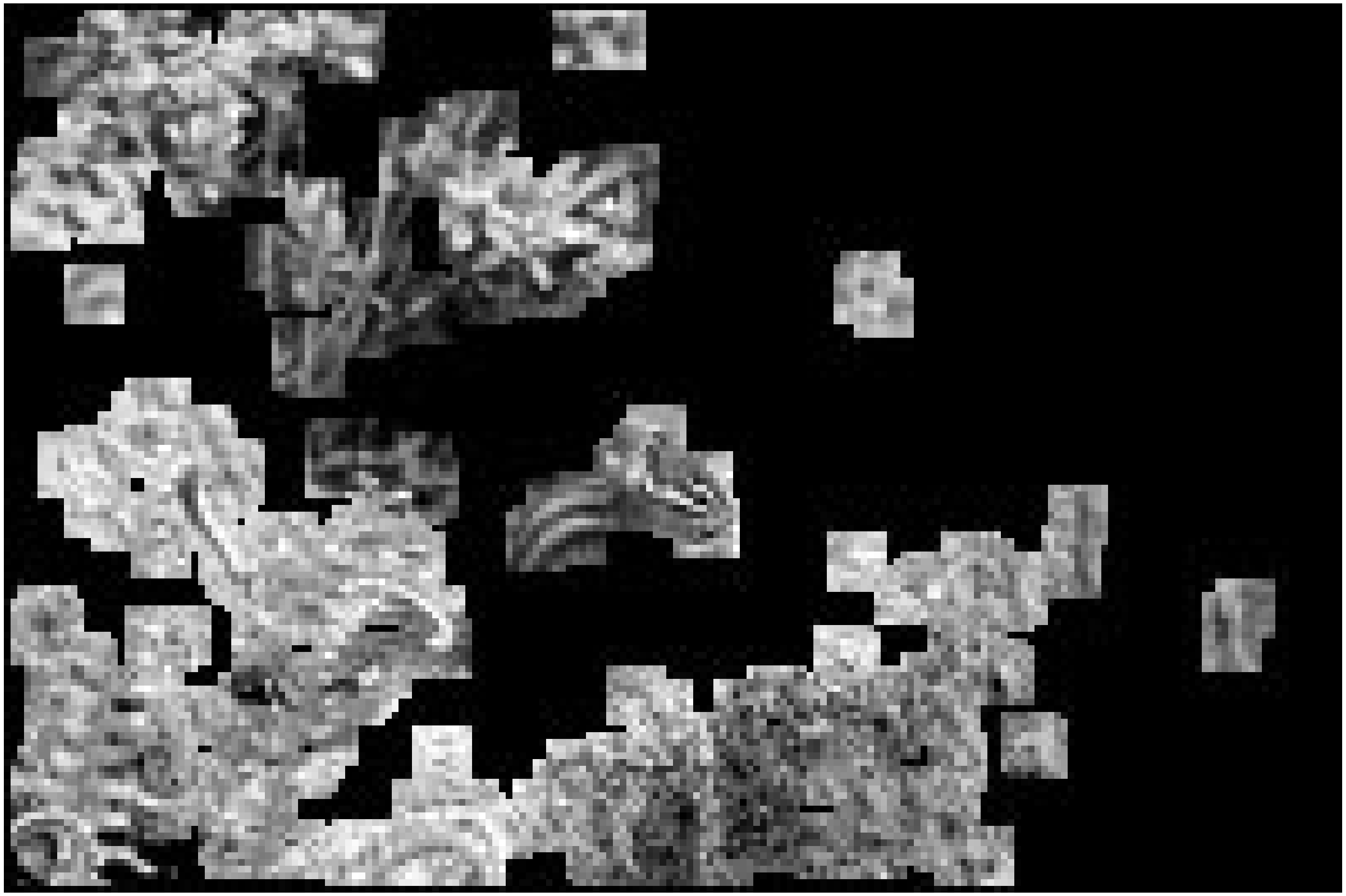
Corner Point Selection



Cross-correlation results for 9×9 neighborhoods of the three image points on the previous slide.

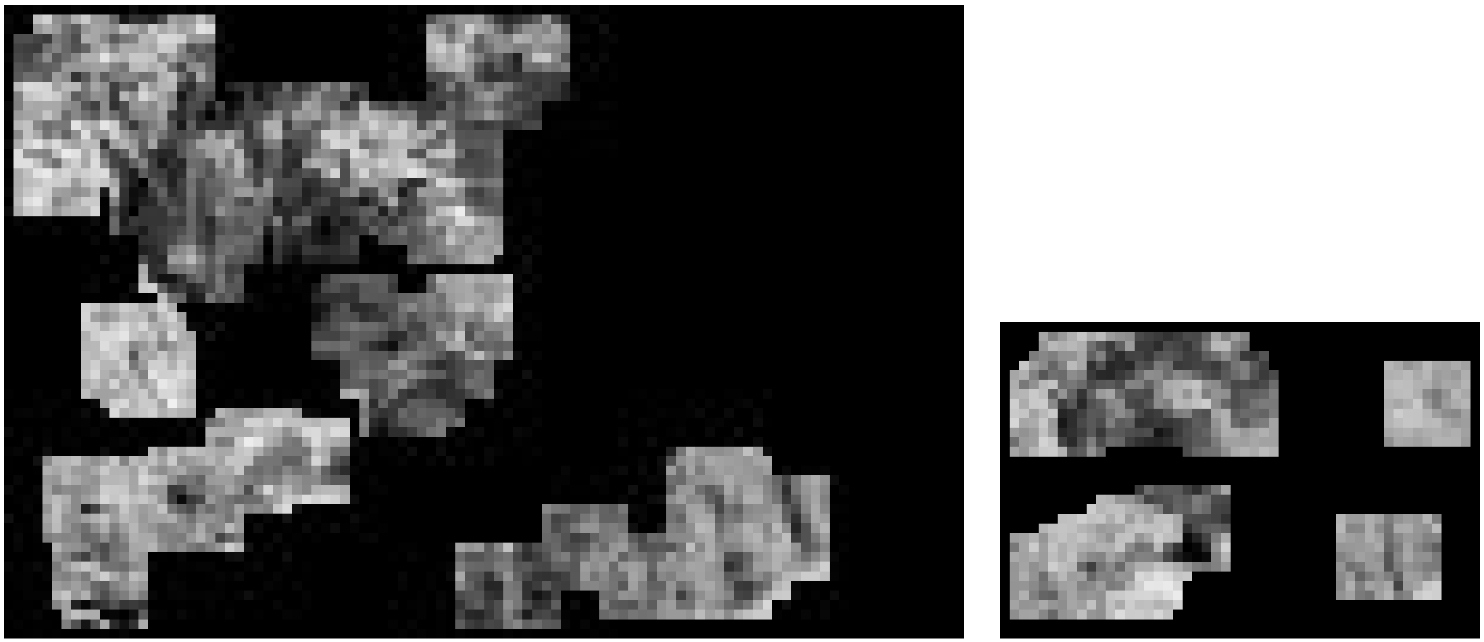
Corresponding similarity functions shown from an $80°$ viewing angle. Two outside points are good feature point candidates, while the center point is not

Selected Corner Points



Selected feature points for 1/2 resolution: each point is shown with its 9×9 neighborhood.

Selected Corner Points



Selected feature points for 1/4, and 1/8 resolutions: each point is shown with its 9×9 neighborhood.

Problems with NCC

We need something more efficient for a practical corner detection algorithm.

An NCC-based algorithm detects posints that are different from their neighborhood. We need a measure of “cornerness”.

Alternative: use a measure based on local structure.

Local Structure Matrix

Consider the spatial image gradient $[E_x, E_y]^T$, computed for all points (x, y) of an image area (neighborhood). The matrix M, defined as

$$M = \begin{pmatrix} \Sigma E_x^2 & \Sigma E_x E_y \\ \Sigma E_x E_y & \Sigma E_y^2 \end{pmatrix}$$

where the sums are taken over the image neighborhood, captures the geometric structure of the gray level pattern. Note that the sums can be replaced by Gaussian smoothing.

M is a symmetric matrix and can therefore be diagonalized by rotation of the coordinate axes, so with no loss of generality, we can think of M as a diagonal matrix:

$$M = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix},$$

where λ_1 and λ_2 are the eigenvalues of M.

We can choose λ_1 as the larger eigenvalue so that $\lambda_1 \geq \lambda_2 \geq 0$.

If the neighborhood contains a corner, then we expect $\lambda_1 > \lambda_2 \geq 0$, and the larger the eigenvalues, the stronger (higher contrast) their corresponding edges.

A corner is identified as two strong edges; therefore as $\lambda_1 > \lambda_2$, a corner is a location where λ_2 is sufficiently large.

Solve: $det(M - \lambda I) = 0$ to obtain λ_1 and λ_2.

There are three cases:

No structure: (smooth variation) $\lambda_1 \approx \lambda_2 \approx 0$

1D structure: (edge) $\lambda_2 \approx 0$ (direction of edge), λ_1 large (normal to edge)

2D structure: (corner) λ_1 and λ_2 both large and distinct

The eigenvectors $\vec{n}$ of M ($M\vec{n} = \lambda\vec{n}$) correspond to the direction of smallest and largest change at (x, y).

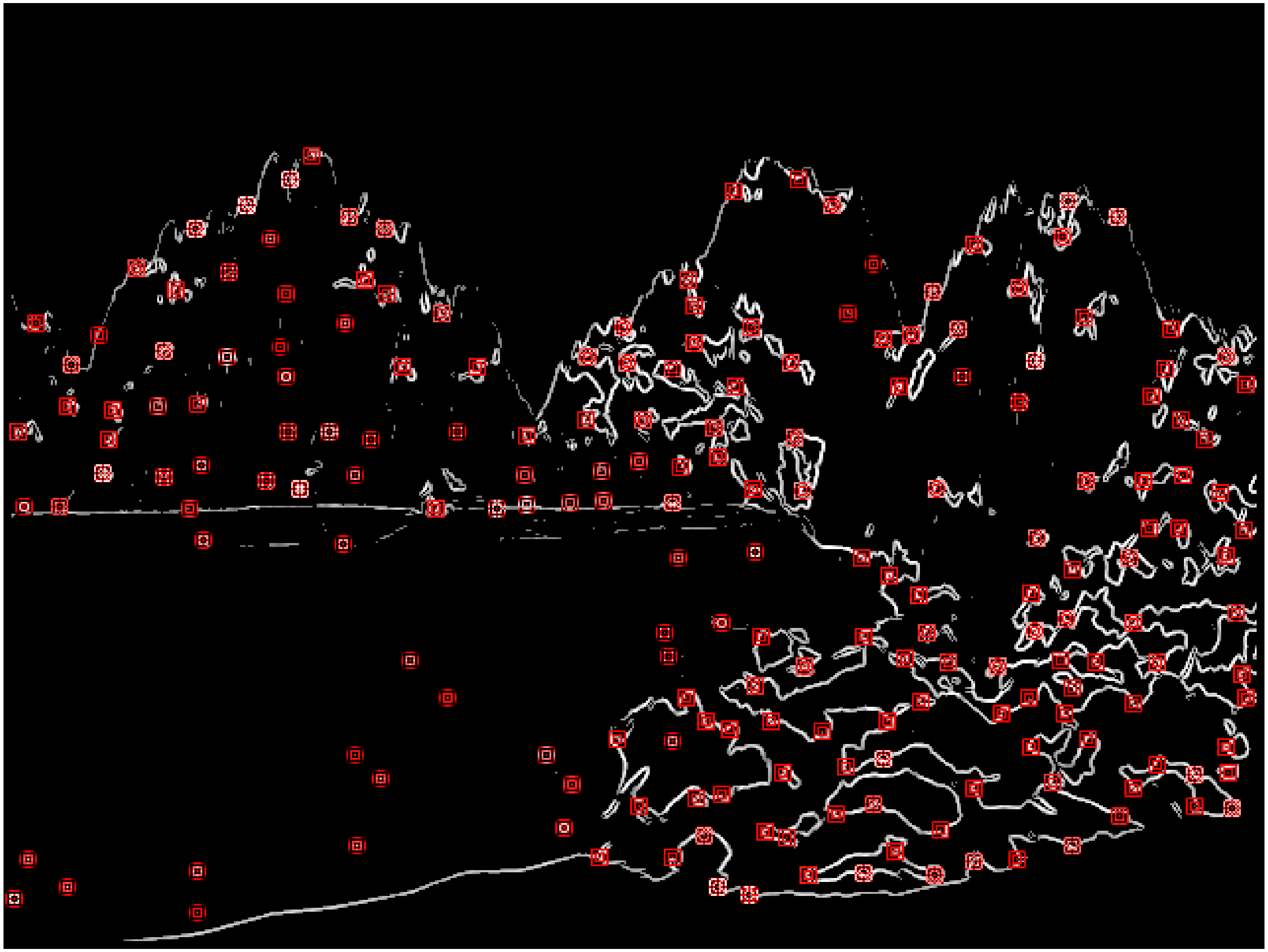
Computing Cornerness

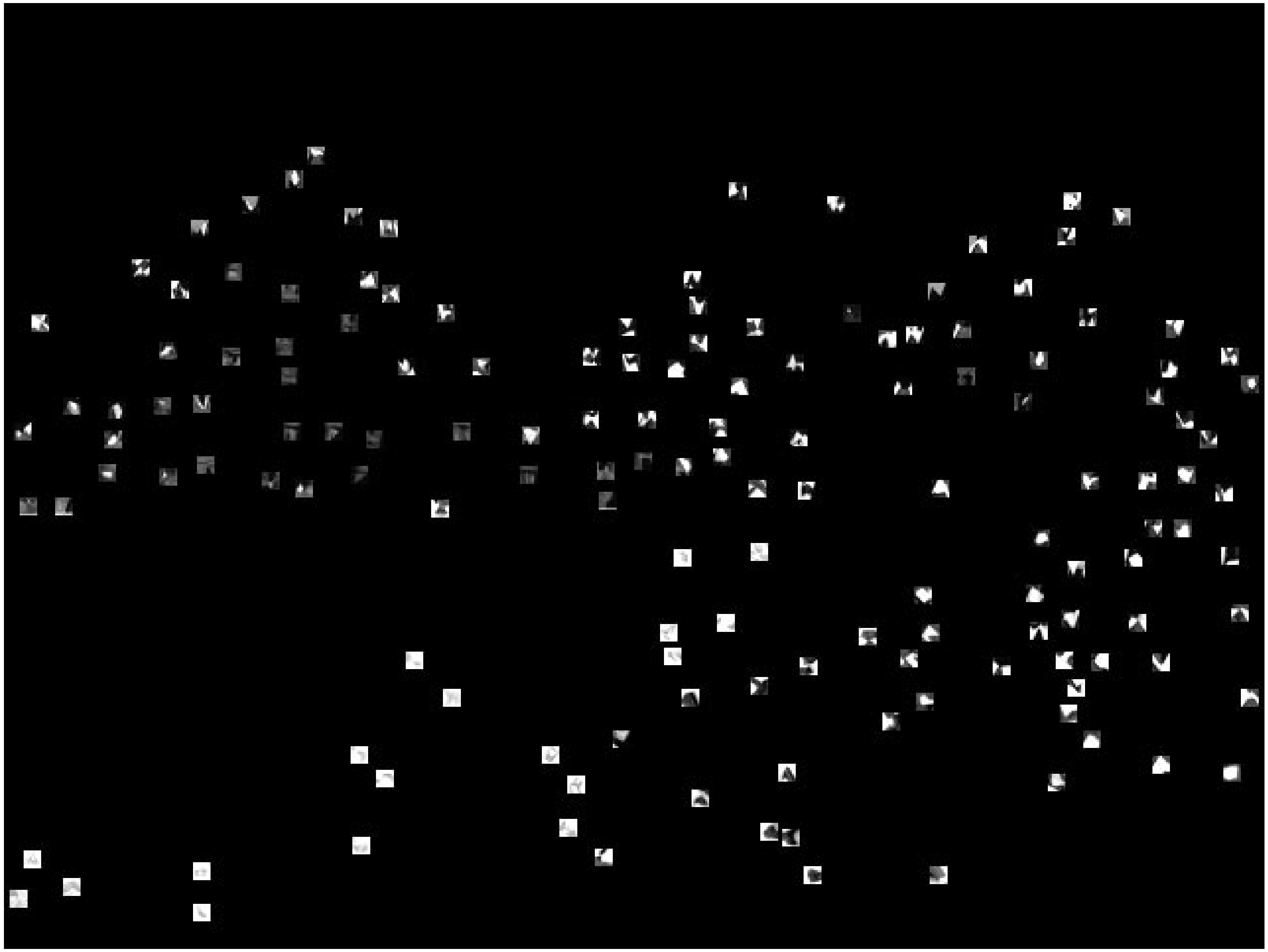
Identify the corner points using the measure described earlier.

Consider all image points for which $\lambda_1 \geq \tau$ and $\lambda_1/\lambda_2 \leq \kappa$. (For example, use τ equal to one-twentieth of the maximum λ_1 in the entire image and we use $\kappa = 2.5$.)

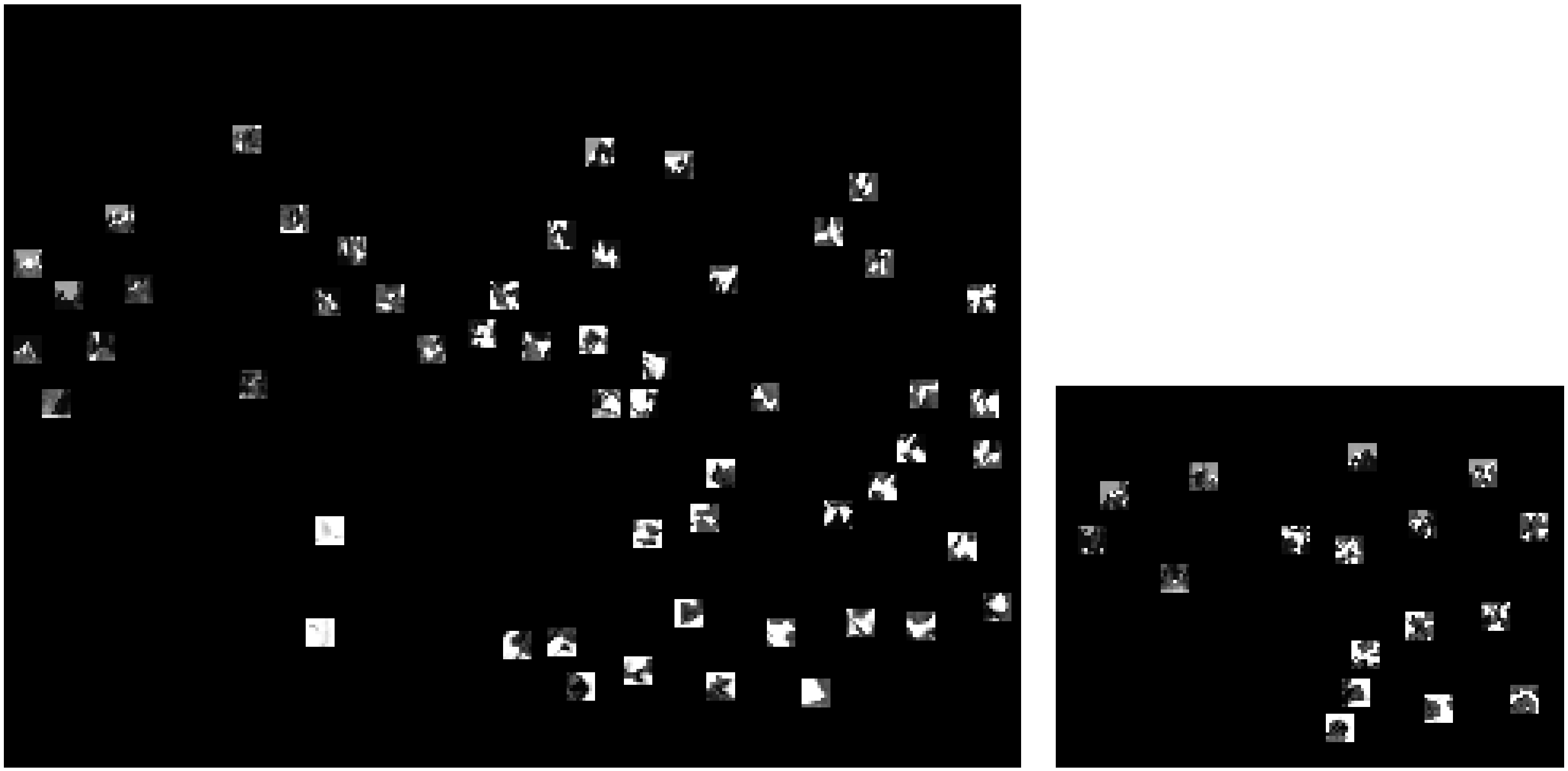
Decreasing τ or increasing κ will typically result in a larger number of candidate points.

Example: selected corner points





Selected corner points for full resolution.



Selected corner points for 1/2 and 1/4 resolutions.