Logical Agents

Chapter 6, AIMA2e Chapter 7

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Outline

- ♦ Knowledge-based agents
- ♦ Wumpus world
- \diamondsuit Logic in general—models and entailment
- ♦ Propositional (Boolean) logic
- ♦ Equivalence, validity, satisfiability
- ♦ Inference rules and theorem proving
 - forward chaining
 - backward chaining
 - resolution

Knowledge bases



Knowledge base = set of sentences in a formal language

Declarative approach to building an agent (or other system):

TELL it what it needs to know

Then it can ASK itself what to do—answers should follow from the KB

Agents can be viewed at the knowledge level

i.e., what they know, regardless of how implemented

Or at the implementation level

i.e., data structures in KB and algorithms that manipulate them

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A simple knowledge-based agent

```
function KB-AGENT( percept) returns an action static: KB, a knowledge base t, a counter, initially 0, indicating time Tell(KB, Make-Percept-Sentence(percept, t)) action \leftarrow Ask(KB, Make-Action-Query(t)) Tell(KB, Make-Action-Sentence(action, t)) t \leftarrow t + 1 return action
```

The agent must be able to:

Represent states, actions, etc.

Incorporate new percepts

Update internal representations of the world

Deduce hidden properties of the world

Deduce appropriate actions

Wumpus World PEAS description

Performance measure

gold +1000, death -1000

-1 per step, -10 for using the arrow

Environment

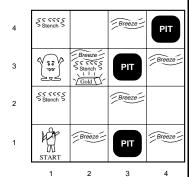
Squares adjacent to wumpus are smelly Squares adjacent to pit are breezy Glitter iff gold is in the same square Shooting kills wumpus if you are facing it Shooting uses up the only arrow Grabbing picks up gold if in same square

Releasing drops the gold in same square

Sensors Breeze, Glitter, Smell

Actuators Left turn, Right turn,

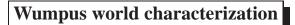
Forward, Grab, Release, Shoot



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Wumpus world characterization

Observable??



Observable?? No—only local perception

<u>Deterministic</u>??

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Wumpus world characterization

Observable?? No—only local perception

<u>Deterministic</u>?? Yes—outcomes exactly specified

Episodic??

Wumpus world characterization

Observable?? No—only local perception

<u>Deterministic??</u> Yes—outcomes exactly specified

Episodic?? No—sequential at the level of actions

Static??

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Wumpus world characterization

Observable?? No—only local perception

<u>Deterministic</u>?? Yes—outcomes exactly specified

Episodic?? No—sequential at the level of actions

Static?? Yes—Wumpus and Pits do not move

Discrete??

Wumpus world characterization

Observable?? No—only local perception

<u>Deterministic</u>?? Yes—outcomes exactly specified

Episodic?? No—sequential at the level of actions

Static?? Yes—Wumpus and Pits do not move

Discrete?? Yes

Single-agent??

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Wumpus world characterization

Observable?? No—only local perception

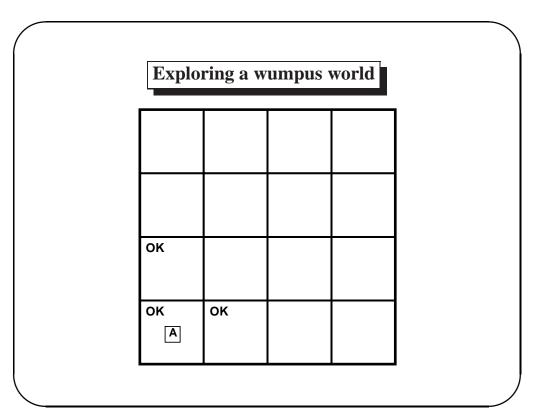
<u>Deterministic</u>?? Yes—outcomes exactly specified

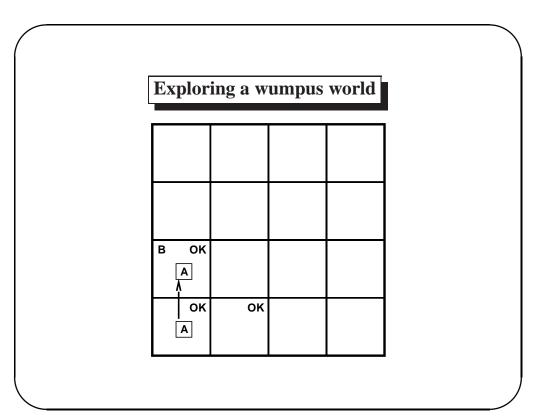
Episodic?? No—sequential at the level of actions

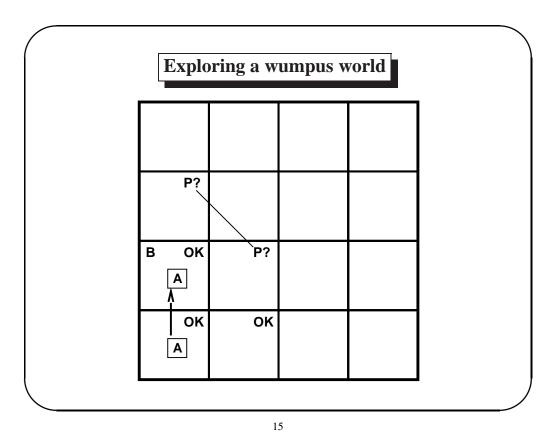
Static?? Yes—Wumpus and Pits do not move

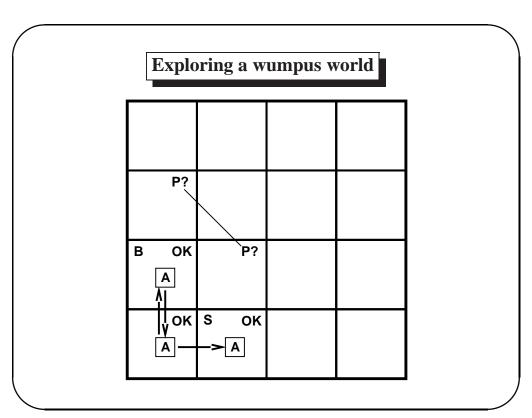
Discrete?? Yes

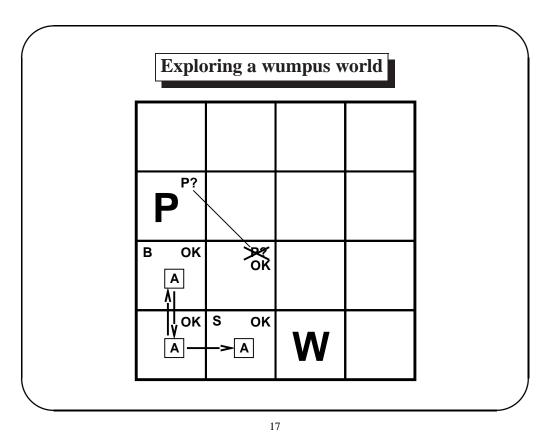
Single-agent?? Yes—Wumpus is essentially a natural feature

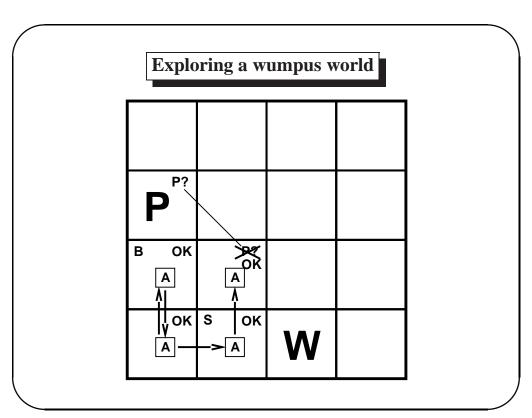


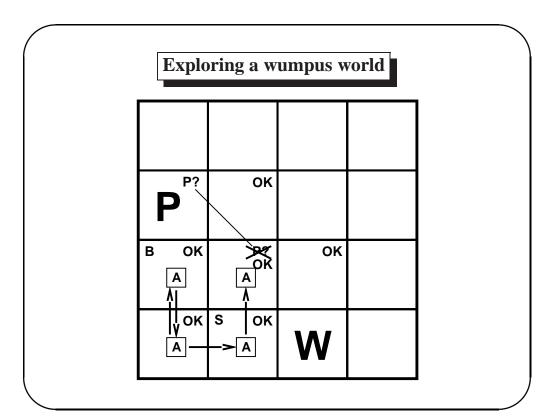


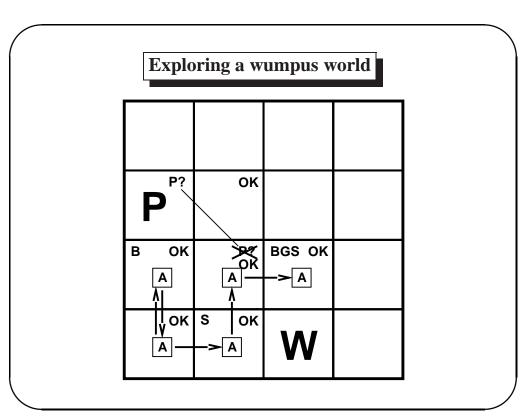




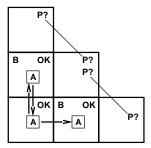






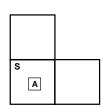


Other tight spots



Breeze in (1,2) and (2,1) \Rightarrow no safe actions

Assuming pits uniformly distributed, (2,2) has pit w/ prob 0.86, vs. 0.31



Smell in (1,1) \Rightarrow cannot move

Can use a strategy of coercion: shoot straight ahead wumpus was there \Rightarrow dead \Rightarrow safe

wumpus wasn't there \Rightarrow safe

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Logic in general

Logics are formal languages for representing information such that conclusions can be drawn

Syntax defines the sentences in the language

Semantics define the "meaning" of sentences; i.e., define truth of a sentence in a world

E.g., the language of arithmetic

 $x + 2 \ge y$ is a sentence; x2 + y > is not a sentence

 $x+2 \geq y$ is true iff the number x+2 is no less than the number y

 $x+2 \geq y$ is true in a world where $x=7,\;y=1$

 $x + 2 \ge y$ is false in a world where x = 0, y = 6

Entailment

Entailment means that one thing *follows from* another:

$$KB \models \alpha$$

Knowledge base KB entails sentence α

if and only if

 α is true in all worlds where KB is true

E.g., the KB containing "the Giants won" and "the Reds won" entails "Either the Giants won or the Reds won"

E.g., x + y = 4 entails 4 = x + y

Entailment is a relationship between sentences (i.e., *syntax*) that is based on *semantics*

Note: brains process *syntax* (of some sort)

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Models

Logicians typically think in terms of models, which are formally structured worlds with respect to which truth can be evaluated

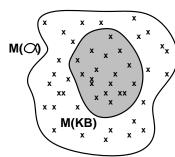
We say m is a model of a sentence α if α is true in m

 $M(\alpha)$ is the set of all models of α

Then $KB \models \alpha$ if and only if $M(KB) \subseteq M(\alpha)$

E.g. KB = Giants won and Reds won

 α = Giants won



Entailment in the wumpus world

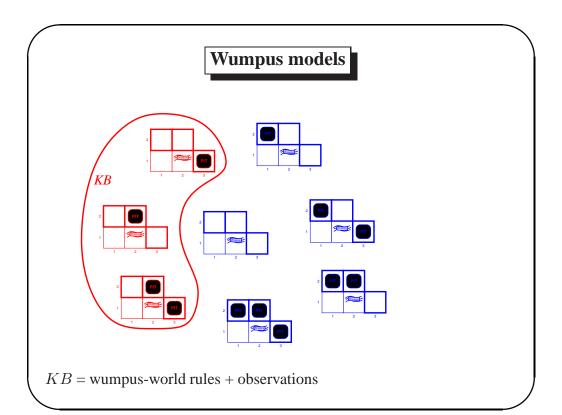
Situation after detecting nothing in [1,1], moving right, breeze in [2,1]

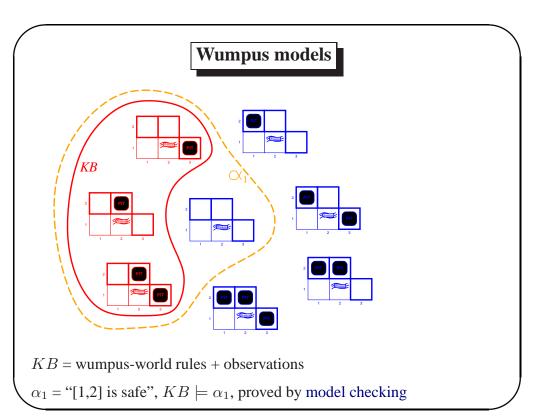
? ? A ?

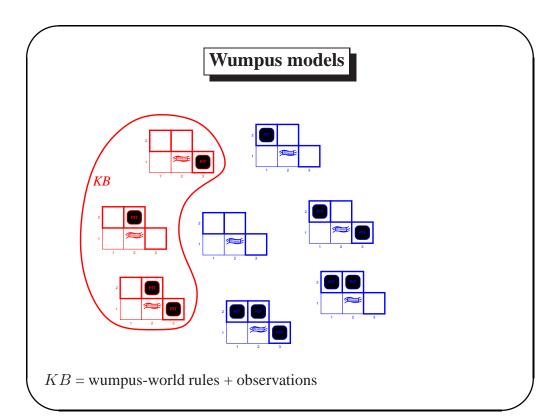
Consider possible models for ?s assuming only pits

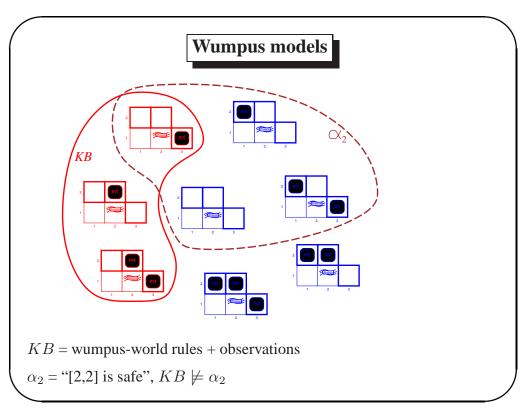
3 Boolean choices \Rightarrow 8 possible models

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Inference

 $KB \vdash_i \alpha = \text{sentence } \alpha \text{ can be derived from } KB \text{ by procedure } i$

Consequences of KB are a haystack; α is a needle.

Entailment = needle in haystack; inference = finding it

Soundness: *i* is sound if

whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$

Completeness: *i* is complete if

whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$

Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.

That is, the procedure will answer any question whose answer follows from what is known by the KB.

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Propositional logic: Syntax

Propositional logic is the simplest logic—illustrates basic ideas

The proposition symbols P_1 , P_2 etc are sentences

If S is a sentence, $\neg S$ is a sentence (negation)

If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)

If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (disjunction)

If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)

If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

Propositional logic: Semantics

Each model specifies true/false for each proposition symbol

E.g.
$$P_{1,2}$$
 $P_{2,2}$ $P_{3,1}$ $true$ $true$ $false$

(With these symbols, 8 possible models, can be enumerated automatically.)

Rules for evaluating truth with respect to a model m:

$$\neg S$$
 is true iff S is false $S_1 \wedge S_2$ is true iff S_1 is true and S_2 is true $S_1 \vee S_2$ is true iff S_1 is true or S_2 is true $S_1 \Rightarrow S_2$ is true iff S_1 is false or S_2 is true i.e., is false iff S_1 is true and S_2 is false $S_1 \Leftrightarrow S_2$ is true iff $S_1 \Rightarrow S_2$ is true and $S_2 \Rightarrow S_1$ is true

Simple recursive process evaluates an arbitrary sentence, e.g.,

$$\neg P_{1,2} \land (P_{2,2} \lor P_{3,1}) = true \land (false \lor true) = true \land true = true$$

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Truth tables for connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	false	true	false	false	true	true
false	true	true	false	true	true	false
true	false	false	false	true	false	false
true	true	false	true	true	true	true

Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in [i, j]. Let $B_{i,j}$ be true if there is a breeze in [i, j].

$$\neg P_{1,1}$$

$$\neg B_{1,1}$$

$$B_{2,1}$$

"Pits cause breezes in adjacent squares"

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Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in [i, j]. Let $B_{i,j}$ be true if there is a breeze in [i, j].

$$\neg P_{1,1}$$

$$\neg B_{1,1}$$

$$B_{2,1}$$

"Pits cause breezes in adjacent squares"

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

 $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

"A square is breezy if and only if there is an adjacent pit"

Truth tables for inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	KB	α_1
false	true							
false	false	false	false	false	false	true	false	true
:	:	:	:	:	:	:	:	:
false	true	false	false	false	false	false	false	true
false	true	false	false	false	false	true	\underline{true}	\underline{true}
false	true	false	false	false	true	false	\underline{true}	\underline{true}
false	true	false	false	false	true	true	\underline{true}	\underline{true}
false	true	false	false	true	false	false	false	true
:	:	:	:	:	:	:	:	:
true	false	false						

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Inference by enumeration

Depth-first enumeration of all models is sound and complete

```
function TT-ENTAILS?(KB, \alpha) returns true or false

symbols \leftarrow a list of the proposition symbols in KB and \alpha

return TT-CHECK-ALL(KB, \alpha, symbols, [])

function TT-CHECK-ALL(KB, \alpha, symbols, model) returns true or false

if EMPTY?(symbols) then

if PL-TRUE?(KB, model) then return PL-TRUE?(\alpha, model)

else return true

else do

P \leftarrow FIRST(symbols); rest \leftarrow REST(symbols)

return TT-CHECK-ALL(KB, \alpha, rest, EXTEND(P, true, model) and

TT-CHECK-ALL(KB, \alpha, rest, EXTEND(P, false, model)
```

 $O(2^n)$ for n symbols; problem is co-NP-complete

Logical equivalence

Two sentences are logically equivalent iff true in same models:

$$\alpha \equiv \beta$$
 if and only if $\alpha \models \beta$ and $\beta \models \alpha$

$$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha)$$
 commutativity of \wedge

$$(\alpha \vee \beta) \equiv (\beta \vee \alpha)$$
 commutativity of \vee

$$((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma))$$
 associativity of \wedge

$$((\alpha \lor \beta) \lor \gamma) \equiv (\alpha \lor (\beta \lor \gamma))$$
 associativity of \lor

$$\neg(\neg \alpha) \equiv \alpha$$
 double-negation elimination

$$(\alpha \Rightarrow \beta) \equiv (\neg \beta \Rightarrow \neg \alpha)$$
 contraposition

$$(\alpha \Rightarrow \beta) \equiv (\neg \alpha \lor \beta)$$
 implication elimination

$$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha))$$
 biconditional elimination

$$\neg(\alpha \land \beta) \equiv (\neg \alpha \lor \neg \beta)$$
 de Morgan

$$\neg(\alpha \lor \beta) \equiv (\neg \alpha \land \neg \beta)$$
 de Morgan

$$(\alpha \wedge (\beta \vee \gamma)) \quad \equiv \quad ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of} \ \wedge \ \text{over} \ \vee \\$$

$$(\alpha \lor (\beta \land \gamma)) \equiv ((\alpha \lor \beta) \land (\alpha \lor \gamma))$$
 distributivity of \lor over \land

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Validity and satisfiability

A sentence is valid if it is true in *all* models,

e.g.,
$$True$$
, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the Deduction Theorem:

$$KB \models \alpha$$
 if and only if $(KB \Rightarrow \alpha)$ is valid

A sentence is satisfiable if it is true in some model

e.g.,
$$A \vee B$$
, C

A sentence is unsatisfiable if it is true in no models

e.g.,
$$A \wedge \neg A$$

Satisfiability is connected to inference via the following:

$$KB \models \alpha$$
 if and only if $(KB \land \neg \alpha)$ is unsatisfiable

i.e., prove α by *reductio ad absurdum*

Proof methods

Proof methods divide into (roughly) two kinds:

Application of inference rules

- Legitimate (sound) generation of new sentences from old
- Proof = a sequence of inference rule applications

Can use inference rules as operators in a standard search alg.

- Typically require translation of sentences into a normal form

Model checking

truth table enumeration (always exponential in *n*) improved backtracking, e.g., Davis–Putnam–Logemann–Loveland heuristic search in model space (sound but incomplete) e.g., min-conflicts-like hill-climbing algorithms

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Forward and backward chaining

Horn Form (restricted)

KB = conjunction of Horn clauses

Horn clause =

♦ proposition symbol; or

 \Diamond (conjunction of symbols) \Rightarrow symbol

E.g., $C \wedge (B \Rightarrow A) \wedge (C \wedge D \Rightarrow B)$

Modus Ponens (for Horn Form): complete for Horn KBs

$$\frac{\alpha_1, \dots, \alpha_n, \qquad \alpha_1 \wedge \dots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

Can be used with forward chaining or backward chaining.

These algorithms are very natural and run in *linear* time

Forward chaining

Idea: fire any rule whose premises are satisfied in the KB, add its conclusion to the KB, until query is found

$$P \Rightarrow Q$$

$$L \wedge M \Rightarrow P$$

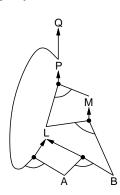
$$B \wedge L \Rightarrow M$$

$$A \wedge P \Rightarrow L$$

$$A \wedge B \Rightarrow L$$

 \boldsymbol{A}

B



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Forward chaining algorithm

```
function PL-FC-ENTAILS?(KB, q) returns true or false
```

local variables: *count*, a table, indexed by clause, initially the number of premises *inferred*, a table, indexed by symbol, each entry initially *false agenda*, a list of symbols, initially the symbols known to be true

while agenda is not empty do

 $p \leftarrow POP(agenda)$

unless inferred[p] do

 $inferred[p] \leftarrow true$

for each Horn clause c in whose premise p appears do

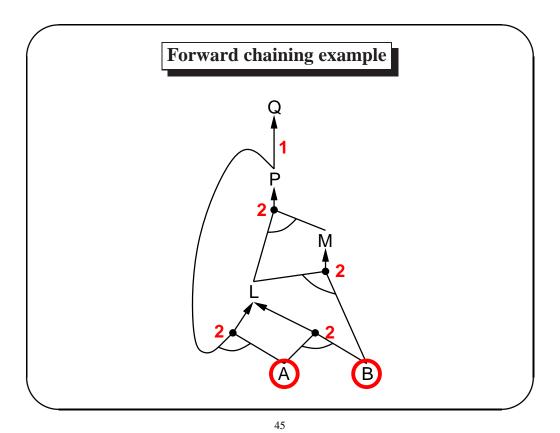
decrement *count*[c]

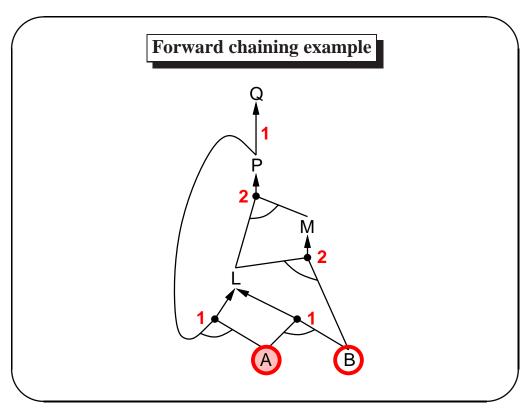
if count[c] = 0 **then do**

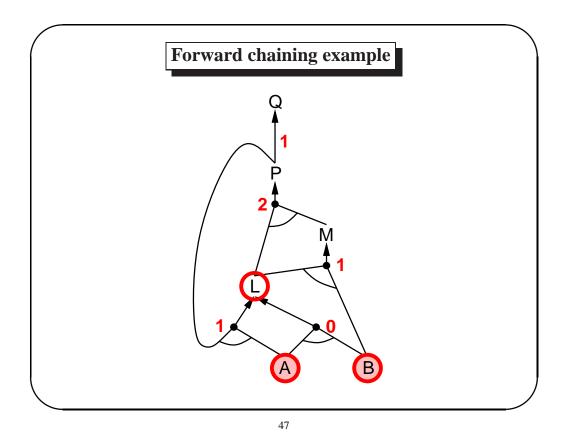
if HEAD[c] = q then return true

PUSH(HEAD[c], agenda)

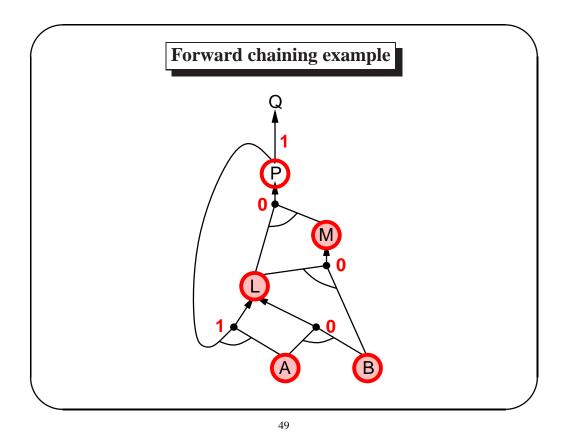
return false

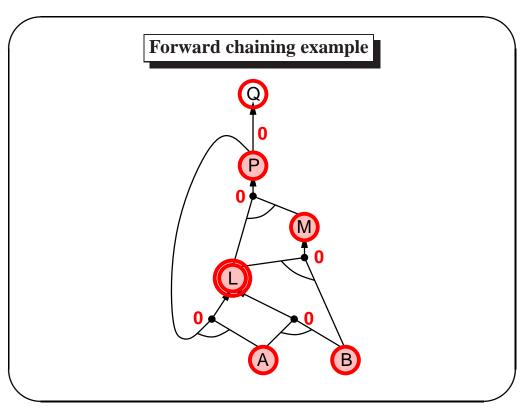


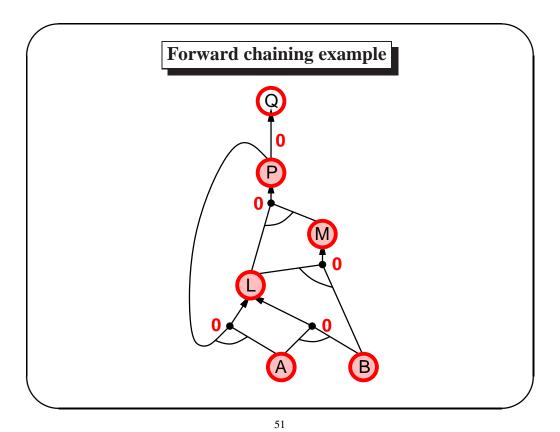


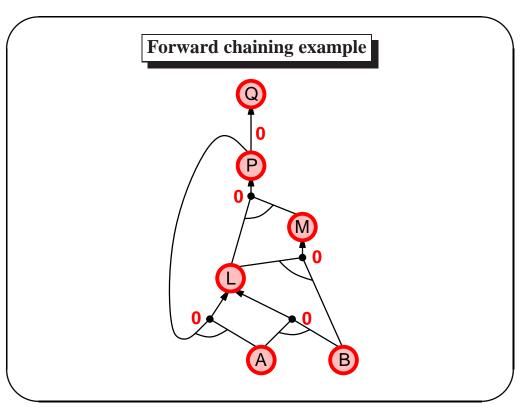


Forward chaining example Q P









Proof of completeness

FC derives every atomic sentence that is entailed by KB

- 1. FC reaches a fixed point where no new atomic sentences are derived
- 2. Consider the final state as a model m, assigning true/false to symbols
- 3. Every clause in the original KB is true in mProof: Suppose a clause $a_1 \wedge \ldots \wedge a_k \Rightarrow b$ is false in mThen $a_1 \wedge \ldots \wedge a_k$ is true in m and b is false in mTherefore the algorithm has not reached a fixed point!
- 4. Hence m is a model of KB
- 5. If $KB \models q, q$ is true in \emph{every} model of KB, including m

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Backward chaining

Idea: work backwards from the query q:

to prove q by BC,

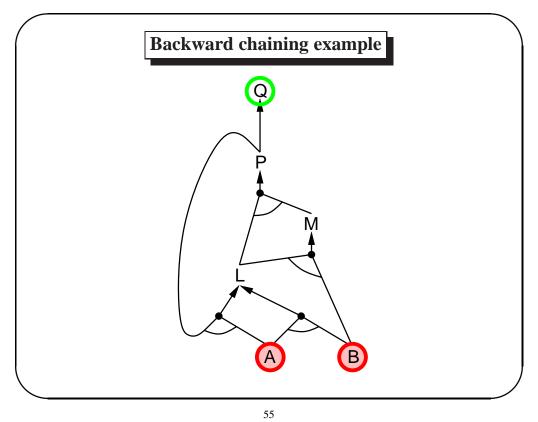
check if q is known already, or

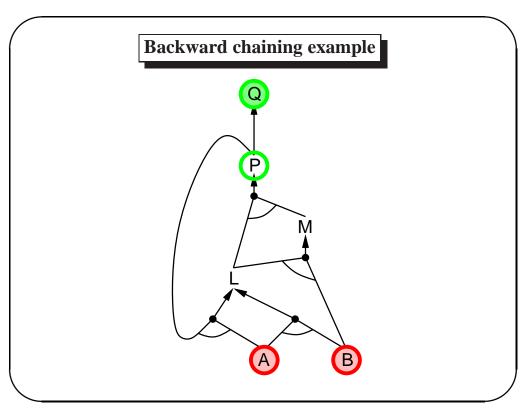
prove by BC all premises of some rule concluding q

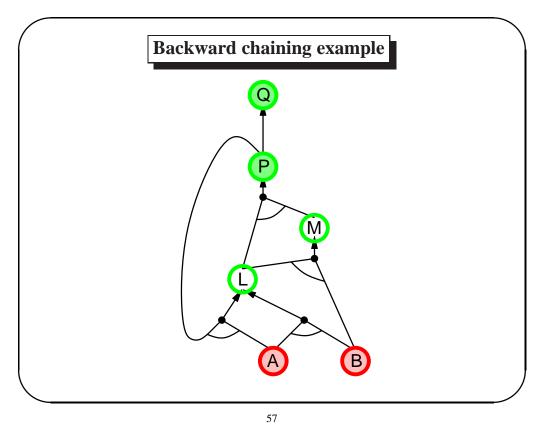
Avoid loops: check if new subgoal is already on the goal stack

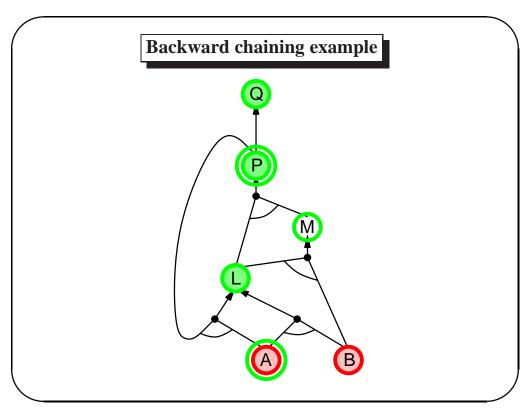
Avoid repeated work: check if new subgoal

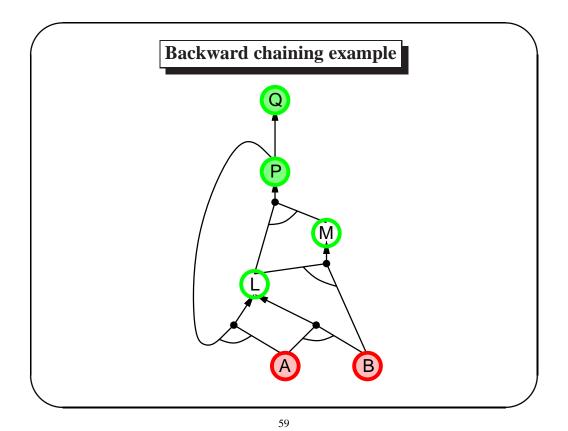
- 1) has already been proved true, or
- 2) has already failed



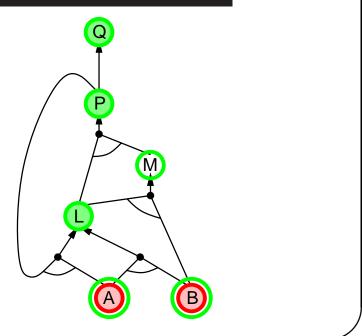


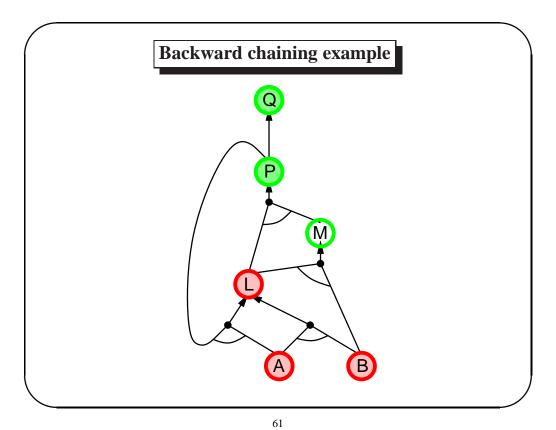




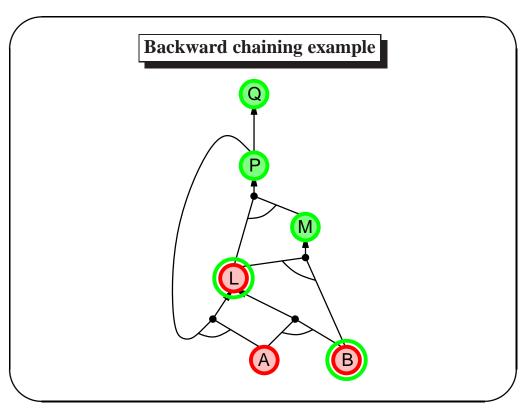


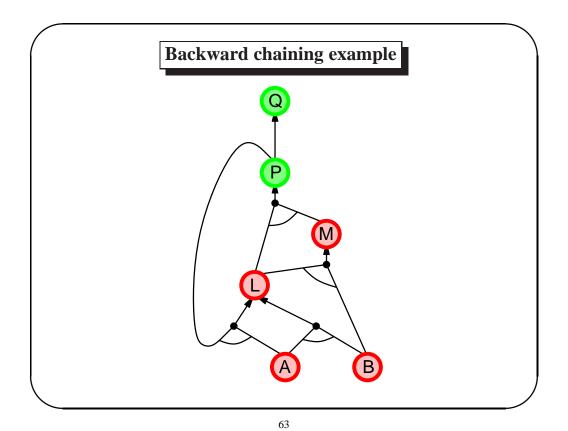
Backward chaining example



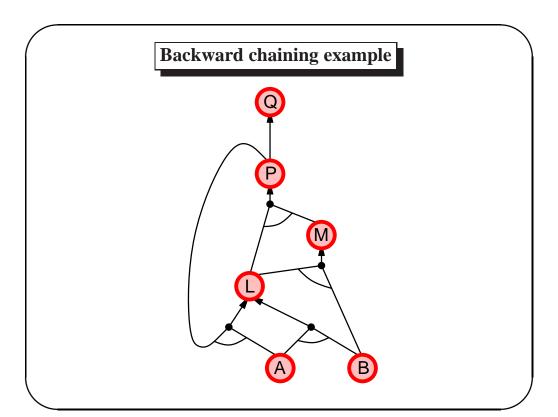


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Backward chaining example A B B B B B B B



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Forward vs. backward chaining

FC is data-driven, cf. automatic, unconscious processing, e.g., object recognition, routine decisions

May do lots of work that is irrelevant to the goal

BC is goal-driven, appropriate for problem-solving,

e.g., Where are my keys? How do I get into a PhD program?

Complexity of BC can be *much less* than linear in size of KB

Resolution

Conjunctive Normal Form (CNF—universal)

conjunction of disjunctions of literals

clauses

E.g.,
$$(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$$

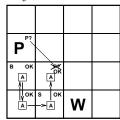
Resolution inference rule (for CNF): complete for propositional logic

$$\frac{\ell_1 \vee \dots \vee \ell_k, \quad m_1 \vee \dots \vee m_n}{\ell_1 \vee \dots \vee \ell_{i-1} \vee \ell_{i+1} \vee \dots \vee \ell_k \vee m_1 \vee \dots \vee m_{j-1} \vee m_{j+1} \vee \dots \vee m_n}$$

where ℓ_i and m_j are complementary literals. E.g.,

$$\frac{P_{1,3} \vee P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$$

Resolution is sound and complete for propositional logic



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Conversion to CNF

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

1. Eliminate \Leftrightarrow , replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)$.

$$(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$$

2. Eliminate \Rightarrow , replacing $\alpha \Rightarrow \beta$ with $\neg \alpha \lor \beta$.

$$(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg (P_{1,2} \lor P_{2,1}) \lor B_{1,1})$$

3. Move ¬ inwards using de Morgan's rules and double-negation:

$$(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land ((\neg P_{1,2} \land \neg P_{2,1}) \lor B_{1,1})$$

4. Apply distributivity law (\lor over \land) and flatten:

$$(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg P_{1,2} \lor B_{1,1}) \land (\neg P_{2,1} \lor B_{1,1})$$

Resolution algorithm

Proof by contradiction, i.e., show $KB \wedge \neg \alpha$ unsatisfiable

function PL-RESOLUTION(KB, α) **returns** true or false

clauses ← the set of clauses in the CNF representation of $KB \land \neg \alpha$ new ← $\{\ \}$

loop do

for each C_i , C_j in clauses do

 $resolvents \leftarrow PL-RESOLVE(C_i, C_j)$

if resolvents contains the empty clause then return true

 $new \leftarrow new \cup resolvents$

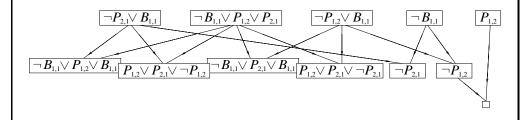
if $new \subseteq clauses$ **then return** false

 $\mathit{clauses} \leftarrow \mathit{clauses} \cup \mathit{new}$

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Resolution example

$$KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1} \alpha = \neg P_{1,2}$$



Summary

Logical agents apply inference to a knowledge base to derive new information and make decisions

Basic concepts of logic:

- syntax: formal structure of sentences
- semantics: truth of sentences wrt models
- entailment: necessary truth of one sentence given another
- inference: deriving sentences from other sentences
- soundess: derivations produce only entailed sentences
- completeness: derivations can produce all entailed sentences

Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.

Forward, backward chaining are linear-time, complete for Horn clauses Resolution is complete for propositional logic

Propositional logic lacks expressive power