Problem #2

For each of the following pairs of atomic sentences, give the most general unifier, if it exists.

a. *P(A,B,B), P(x,y,z)*

b. *Q(y,G(A,B)), Q(G(x,x),y)*

c. *Older(Father(y),y), Older(Father(x),John)*

d. *Knows(Father(y),y), Knows(x,x)*

e. *Ancestor(x,y), Ancestor(John,Father(John))*

Problem #2

For each of the following pairs of atomic sentences, give the most general unifier, if it exists.

a. *P(A,B,B), P(x,y,z)*

- $\sigma = \{x/A, y/B, z/B\}$

b. *Q(y,G(A,B)), Q(G(x,x),y)*

- No

c. *Older(Father(y),y), Older(Father(x),John)*

- $\sigma = \{y/x, x/John\}$

d. *Knows(Father(y),y), Knows(x,x)*

- No, occurs y in $Father(y)$

e. *Ancestor(x,y), Ancestor(John,Father(John))*

- $\sigma = \{x/John, y/Father(John)\}$

Problem #3

The function *cons(x,y)* denotes list formed by inserting the element x at the head of the list y. We denote the empty list by Nil; the list (2) by $cons(2, Nil)$; the list $(1, 2)$ by $cons(1, cons(2, Nil))$; and so on. The formula $Member(e, l)$ is intended to mean that e is a member of the list l. We have the following axioms:

- $(\forall x, y)[Member(x, cons(x, y))]$
- $(\forall x, y, z)[Member(x, y) \supset Member(x, cons(z, y))]$

Prove $Member(b, cons(a, cons(b, nil)))$ from these axioms by the method of resolution refutation.

Problem #3: Solution

1. $Member(x, cons(x, y))$
2. $\neg Member(x, y) \vee Member(x, cons(z, y))$
3. $\neg Member(b, cons(a, cons(b, nil)))$

Problem #3: Solution

1. $Member(x, cons(x, y))$
2. $\neg Member(x, y) \vee Member(x, cons(z, y))$
3. $\neg Member(b, cons(a, cons(b, nil)))$
4. Resolve 2 and 3: $\sigma = \{x/b, z/a, y/cons(b, nil)\}$
   $\neg Member(b, cons(b, nil))$

Problem #3: Solution

1. $Member(x, cons(x, y))$
2. $\neg Member(x, y) \vee Member(x, cons(z, y))$
3. $\neg Member(b, cons(a, cons(b, nil)))$
4. Resolve 2 and 3: $\sigma = \{x/b, z/a, y/cons(b, nil)\}$
   $\neg Member(b, cons(b, nil))$
5. Resolve 4 and 1: $\sigma = \{x/b, y/nil\}$
   $\{\}$

Problem #4

Dr. Somebody, Dr. Anybody, and Dr. Nobody are computer scientists. We know the following facts about them:

1. Dr. Somebody is an associate professor.
2. Dr. Nobody is an assistant professor and has published papers with Dr. Anybody.
3. Dr. Anybody is either an associate or an assistant professor (but not both) and has published papers with Dr. Somebody.

Use resolution refutation to prove that an assistant professor has published papers with an associate professor; that is, prove $\exists x, y[Assistant(x) \wedge Associate(y) \wedge PPW(x, y)]$.

Problem #4: Solution

Dr. Somebody is an associate professor.

1. $Assoc(S)$

Problem #4: Solution

Dr. Somebody is an associate professor.

1. $Assoc(S)$

Dr. Nobody is an assistant professor and has published papers with Dr. Anybody.

2. $Asst(N)$
3. $PPW(N, A)$

Problem #4: Solution

Dr. Somebody is an associate professor.

1. $Assoc(S)$

Dr. Nobody is an assistant professor and has published papers with Dr. Anybody.

2. $Asst(N)$
3. $PPW(N, A)$

Dr. Anybody is either an associate or an assistant professor (but not both) and has published papers with Dr. Somebody.

4. $Assoc(A) \vee Asst(A)$
5. $\neg Assoc(A) \vee \neg Asst(A)$
6. $PPW(A, S)$

Problem #4: Solution

Dr. Somebody is an associate professor.

1. $Assoc(S)$

Dr. Nobody is an assistant professor and has published papers with Dr. Anybody.

2. $Asst(N)$
3. $PPW(N, A)$

Dr. Anybody is either an associate or an assistant professor (but not both) and has published papers with Dr. Somebody.

4. $Assoc(A) \vee Asst(A)$
5. $\neg Assoc(A) \vee \neg Asst(A)$
6. $PPW(A, S)$

Prove $\exists x, y[Assistant(x) \wedge Associate(y) \wedge PPW(x, y)]$, negate it:

7. $\neg Asst(x) \vee \neg Assoc(y) \vee \neg PPW(x, y)$

Problem #4: Proof by resolution

1. $Assoc(S)$
2. $Asst(N)$
3. $PPW(N, A)$
4. $Assoc(A) \vee Asst(A)$
5. $\neg Assoc(A) \vee \neg Asst(A)$
6. $PPW(A, S)$
7. $\neg Asst(x) \vee \neg Assoc(y) \vee \neg PPW(x, y)$

Problem #4: Proof by resolution

1. $Assoc(S)$
2. $Asst(N)$
3. $PPW(N, A)$
4. $Assoc(A) \vee Asst(A)$
5. $\neg Assoc(A) \vee \neg Asst(A)$
6. $PPW(A, S)$
7. $\neg Asst(x) \vee \neg Assoc(y) \vee \neg PPW(x, y)$
8. Resolve 1 and 7: $\sigma = \{y/S\}$; $\neg Asst(x) \vee \neg PPW(x, S)$

Problem #4: Proof by resolution

1. $Assoc(S)$
2. $Asst(N)$
3. $PPW(N, A)$
4. $Assoc(A) \vee Asst(A)$
5. $\neg Assoc(A) \vee \neg Asst(A)$
6. $PPW(A, S)$
7. $\neg Asst(x) \vee \neg Assoc(y) \vee \neg PPW(x, y)$
8. Resolve 1 and 7: $\sigma = \{y/S\}$; $\neg Asst(x) \vee \neg PPW(x, S)$
9. Resolve 6 and 8: $\sigma = \{x/A\}$; $\neg Asst(A)$

Problem #4: Proof by resolution

1. $Assoc(S)$
2. $Asst(N)$
3. $PPW(N, A)$
4. $Assoc(A) \vee Asst(A)$
5. $\neg Assoc(A) \vee \neg Asst(A)$
6. $PPW(A, S)$
7. $\neg Asst(x) \vee \neg Assoc(y) \vee \neg PPW(x, y)$
8. Resolve 1 and 7: $\sigma = \{y/S\}$; $\neg Asst(x) \vee \neg PPW(x, S)$
9. Resolve 6 and 8: $\sigma = \{x/A\}$; $\neg Asst(A)$
10. Resolve 4 and 9; $Assoc(A)$

Problem #4: Proof by resolution

1. $Assoc(S)$
2. $Asst(N)$
3. $PPW(N, A)$
4. $Assoc(A) \vee Asst(A)$
5. $\neg Assoc(A) \vee \neg Asst(A)$
6. $PPW(A, S)$
7. $\neg Asst(x) \vee \neg Assoc(y) \vee \neg PPW(x, y)$
8. Resolve 1 and 7: $\sigma = \{y/S\}$; $\neg Asst(x) \vee \neg PPW(x, S)$
9. Resolve 6 and 8: $\sigma = \{x/A\}$; $\neg Asst(A)$
10. Resolve 4 and 9; $Assoc(A)$
11. Resolve 7 and 10; $\sigma = \{y/A\}$ $\neg Asst(x) \vee \neg PPW(x, A)$

Problem #4: Proof by resolution

1. $Assoc(S)$
2. $Asst(N)$
3. $PPW(N, A)$
4. $Assoc(A) \vee Asst(A)$
5. $\neg Assoc(A) \vee \neg Asst(A)$
6. $PPW(A, S)$
7. $\neg Asst(x) \vee \neg Assoc(y) \vee \neg PPW(x, y)$
8. Resolve 1 and 7: $\sigma = \{y/S\}$; $\neg Asst(x) \vee \neg PPW(x, S)$
9. Resolve 6 and 8: $\sigma = \{x/A\}$; $\neg Asst(A)$
10. Resolve 4 and 9; $Assoc(A)$
11. Resolve 7 and 10; $\sigma = \{y/A\}$ $\neg Asst(x) \vee \neg PPW(x, A)$
12. Resolve 2 and 11; $\sigma = \{x/N\}$ $\neg PPW(N, A)$

Problem #4: Proof by resolution

1. $Assoc(S)$
2. $Asst(N)$
3. $PPW(N, A)$
4. $Assoc(A) \vee Asst(A)$
5. $\neg Assoc(A) \vee \neg Asst(A)$
6. $PPW(A, S)$
7. $\neg Asst(x) \vee \neg Assoc(y) \vee \neg PPW(x, y)$
8. Resolve 1 and 7: $\sigma = \{y/S\}$; $\neg Asst(x) \vee \neg PPW(x, S)$
9. Resolve 6 and 8: $\sigma = \{x/A\}$; $\neg Asst(A)$
10. Resolve 4 and 9; $Assoc(A)$
11. Resolve 7 and 10; $\sigma = \{y/A\}$ $\neg Asst(x) \vee \neg PPW(x, A)$
12. Resolve 2 and 11; $\sigma = \{x/N\}$ $\neg PPW(N, A)$
13. resolve 3 and 12; $\{\}$

Problem #11

Consider the sentence "Heads I win; tail you lose." We can represent this sentence plus associated domain knowledge in the propositional logic using the following proper axioms, where *(H) Heads*, *(T) Tails*, *(W) WinMe*, and *(L) LoseYou* are propositional variables:

$$H \Rightarrow W, \quad T \Rightarrow L, \quad \neg H \Rightarrow T, \quad L \Rightarrow W$$

a) Determine if it is possible to prove W (WinMe) using the rule of inference modus ponens and these four axioms.

b) Convert each of the four axioms to a disjunction of literals.

c) For each of the resulting disjunctions, specify if it is a Horn clause.

d) Determine if it is possible to prove W (WinMe) using the resolution rule of inference and the four axioms written as disjunctions of literals.

Problem #11: Solution

a) Determine if it is possible to prove W (WinMe) using the rule of inference modus ponens and these four axioms.
No. No ground facts!

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a) Determine if it is possible to prove W (WinMe) using the rule of inference modus ponens and these four axioms.
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$$\neg H \vee W, \qquad \neg T \vee L, \qquad H \vee T, \qquad \neg L \vee W$$

Problem #11: Solution

a) Determine if it is possible to prove W (WinMe) using the rule of inference modus ponens and these four axioms.
No. No ground facts!

b) Convert each of the four axioms to a disjunction of literals.

$$\neg H \vee W, \qquad \neg T \vee L, \qquad H \vee T, \qquad \neg L \vee W$$

c) For each of the resulting disjunctions, specify if it is a Horn clause.
Yes, *Yes,* *No,* *Yes*

Problem #11: Solution

a) Determine if it is possible to prove W (WinMe) using the rule of inference modus ponens and these four axioms.
No. No ground facts!

b) Convert each of the four axioms to a disjunction of literals.

$$\neg H \vee W, \qquad \neg T \vee L, \qquad H \vee T, \qquad \neg L \vee W$$

c) For each of the resulting disjunctions, specify if it is a Horn clause.
Yes, *Yes,* *No,* *Yes*

d) Determine if it is possible to prove W (WinMe) using the resolution rule of inference and the four axioms written as disjunctions of literals.

$$1.\ \neg H \vee W, \quad 2.\ \neg T \vee L, \quad 3.\ H \vee T, \quad 4.\ \neg L \vee W \quad 5.\ \neg W$$

Problem #11: Solution

a) Determine if it is possible to prove W (WinMe) using the rule of inference modus ponens and these four axioms.
No. No ground facts!

b) Convert each of the four axioms to a disjunction of literals.

$$\neg H \vee W, \qquad \neg T \vee L, \qquad H \vee T, \qquad \neg L \vee W$$

c) For each of the resulting disjunctions, specify if it is a Horn clause.
Yes, *Yes,* *No,* *Yes*

d) Determine if it is possible to prove W (WinMe) using the resolution rule of inference and the four axioms written as disjunctions of literals.

1. $\neg H \vee W$, 2. $\neg T \vee L$, 3. $H \vee T$, 4. $\neg L \vee W$ 5. $\neg W$

6. 4&5 $\rightarrow \neg L$, 7. 2&6 $\rightarrow \neg T$,
8. 3&7 $\rightarrow H$, 9. 1&8 $\rightarrow W$, 10. 5&9 $\rightarrow \{\}$

Problem #13

Consider the following problem: The rules in the National Zoo forbid visitors to feed animals. A prairie dog, an animal, has some candy, and all of its candy was given to it by BB, who is a Zoo visitor.

a. Represent these facts as first-order logic clauses.
b. Use backward-chaining to prove that BB broke Zoo rules.
c. Use forward-chaining to prove that BB broke Zoo rules.

Problem #13: a

- The rules in the National Zoo forbid visitors to feed animals.
  $Visitor(x) \wedge Animal(y) \wedge Feed(x, y) \Rightarrow BreaksRules(x)$

Problem #13: a

- The rules in the National Zoo forbid visitors to feed animals.

  $Visitor(x) \wedge Animal(y) \wedge Feed(x,y) \Rightarrow BreaksRules(x)$

  $Visitor(x) \wedge Animal(y) \wedge Food(z) \wedge Give(x,y,z) \Rightarrow Feed(x,y)$

- A prairie dog, an animal, has some candy, and all of its candy was given to it by BB, who is a Zoo visitor.

  $Animal(PrairieDog)$
  $Food(Candy)$
  $Visitor(BB)$
  $Give(BB, PrairieDog, Candy)$

Problem #13: b

$BreaksRules(x)$

$Feed(x, y)$

$Visitor(x)$ $Animal(y)$ $Food(z)$ $Give(x, y, z)$

Problem #13: b

$$BreaksRules(x)$$

$$\{x/BB, y/PrairieDog, z/Candy\}$$

$$Feed(x, y)$$

$Visitor(x)$ $Animal(y)$ $Food(z)$ $Give(x, y, z)$

$Visitor(BB)$ $Animal(PrairieDog)$ $Food(Candy)$

$Give(BB, PrairieDog, Candy)$

Problem #13: b

$$BreaksRules(x)$$

$$\{x/BB, y/PrairieDog, z/Candy\}$$

$$Feed(x, y)$$
$$Feed(BB, PrairieDog)$$

$Visitor(x)$	$Animal(y)$	$Food(z)$	$Give(x, y, z)$
$Visitor(BB)$	$Animal(PrairieDog)$	$Food(Candy)$	$Give(BB, PrairieDog, Candy)$

Problem #13: b

$$BreaksRules(x)$$
$$BreaksRules(BB)$$
$$\{x/BB, y/PrairieDog, z/Candy\}$$

$$Feed(x, y)$$
$$Feed(BB, PrairieDog)$$

$$Visitor(x) \quad Animal(y) \quad Food(z) \quad Give(x, y, z)$$
$$Visitor(BB) \quad Animal(PrairieDog) \quad Food(Candy)$$
$$Give(BB, PrairieDog, Candy)$$

Problem #5

We are given the following paragraph:

> Tony, Mike, and John belong to the Alpine Club. Every member of the Alpine Club is either a skier or a mountain climber or both. No mountain climber likes rain, and all skiers like snow. Mike dislikes whatever Tony likes and likes whatever Tony dislikes. Tony dislikes rain and snow.

Represent this information by predicate-calculus sentences in such a way that you can represent the question "Is there a member of the alpine club who is a skier but not a mountain climber?" as a predicate-calculus expression. Use resolution refutation to answer it.

Problem #7

Consider the following statements:

(1) Whoever can read is literate ($\forall x[R(x) \Rightarrow L(x)]$).

(2) Horses are not literate ($\forall x[H(x) \Rightarrow \neg L(x)]$).

(3) Some horses are intelligent ($\exists x[H(x) \wedge I(x)]$).

Use these statements to prove the following statement by resolution:

(4) Some who are intelligent cannot read ($\exists x[I(x) \wedge \neg R(x)]$).

Problem #9

We represent the statement that everything is representable in the predicate calculus as $\forall x, \mathit{represents}(pc, x)$. General Problem Solver (GPS) is a system for automated problem solving. We represent the statement that all problems representable in predicate calculus are solvable using GPS as $\forall x, (\mathit{problem}(x) \land \mathit{represents}(pc, x)) \Rightarrow \mathit{solves}(gps, x)$. Now using these two statements and the fact that the Traveling Salesperson Problem (TSP) is a problem ($\mathit{problem}(tsp)$), prove that GPS solves it.

Problem #10

Attempt to unify the following pairs of expressions. Either show their most general unifiers or explain why they will not unify.

(a) p(X,Y) and p(a,Z)

(b) p(X,X) and p(a,b)

(c) ancestor(X,Y) and ancestor(bill,father(bill))

(d) ancestor(X,father(X)) and ancestor(david,george)

Problem #12

We have defined four different binary logical connectives ($\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$).

a. Are there any others that might be useful?
b. How many binary connnectives can there be?
c. Why are some of them not very useful?

Problem #15

Sam, Clyde, and Oscar are rabbits. We know the following facts about them:

1. Sam is pink.
2. Clyde is gray and likes Oscar.
3. Oscar is either pink or gray (but not both) and likes Sam.

Use resolution refutation to prove that a gray rabbit likes a pink rabbit; that is, prove $(\exists x, y)[Gray(x) \wedge Pink(y) \wedge Likes(x, y)]$.

Problem #16

The function *cons(x,y)* denotes list formed by inserting the element x at the head of the list y. We denote the empty list by *nil*; the list (2) by $cons(2, nil)$; the list $(1, 2)$ by $cons(1, cons(2, nil))$; and so on. The formula $Last(l, e)$ is intended to mean that e is the last element of the list l. We have the following axioms:

- $(\forall u)[Last(cons(u, nil), u)]$
- $(\forall x, y, z)[Last(y, z) \supset Last(cons(x, y), z)]$

1. Prove the following theorem from these axioms by the method of resolution refutation:

$$(\exists v)[Last(cons(2, cons(1, nil)), v)]$$

2. Use answer extraction to find v, the last element of the list $(2, 1)$.

Problem #19

Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following sentences?

a. $(A \wedge B) \vee (B \wedge C)$

b. $A \vee B$

c. $A \Leftrightarrow B \Leftrightarrow C$

Hint: In each sentence all four propositions have to be considered.