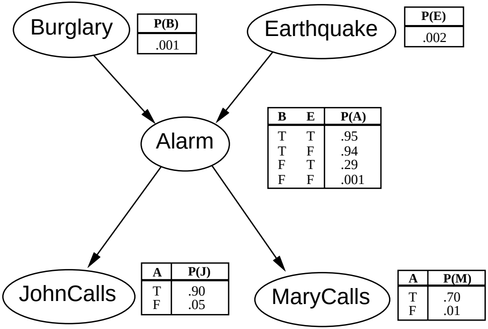
Variable Elimination Example

A typical belief network with conditional probabilities is given in the following figure:



The letters *B,E,A,J,* and *M* stand for *Burglary, Earthquake, Alarm, JohnCalls*, and *MaryCalls*, respectively. All variables (nodes) are Boolean, so the probability of, say, $\neg P(A)$ in any row of its table is $1 - P(A)$. Use *variable elimination* method to compute probability $\mathbf{P}(\mathbf{B}|j, m)$.

a	$P(m\|a) = f_M(a)$
t	0.70
f	0.01

a	$P(j\|a) = f_J(a)$
t	0.90
f	0.05

a	b	e	$P(a\|b,e) = f_A(a,b,e)$
t	t	t	0.95
t	t	f	0.94
t	f	t	0.94
t	f	f	0.94
f	t	t	0.05
f	t	f	0.06
f	f	t	0.71
f	f	f	0.999

b	e	$f_{\bar{A}JM}(b, e)$
t	t	$0.95{*}0.70{*}0.90 + 0.05{*}0.01{*}0.05 = 0.598525$
t	f	$0.94{*}0.70{*}0.90 + 0.06{*}0.01{*}0.05 = 0.59223$
f	t	$0.29{*}0.70{*}0.90 + 0.71{*}0.01{*}0.05 = 0.183055$
f	f	$0.001{*}0.70{*}0.90 + 0.999{*}0.01{*}0.05 = 0.0011295$

e	$P(e) = f_E(e)$
t	0.002
f	0.998

b	$f_{\bar{E}\bar{A}JM}(e)$
t	$0.002{*}0.598525{+}0.998{*}0.59223 = 0.59224259$
f	$0.002{*}0.183055 + 0.998{*}0.0011295 = 0.0014934$

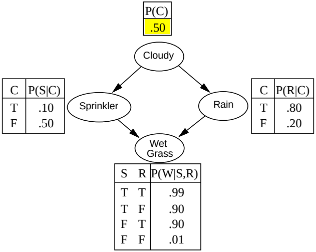
b	$f_B(b)$
t	0.001
f	0.999

b	$f_B(b) \times f_{\bar{E}\bar{A}JM}(e)$
t	$0.001{*}0.59224259 = 0.00059224259$
f	$0.999{*}0.0014934 = 0.0014919066$

$$\mathbf{P}(\mathbf{B}|j, m) = \alpha < 0.00059224259, 0.0014919066 > \approx < 0.2842, 0.7158 >$$

Problem #24

A typical belief network with conditional probabilities is given in the following figure:



The letters *C,R,S* and *W* stand for *Cloudy*, *Rain*, *Sprinkler*, and *Wet Grass*, respectively. All variables (nodes) are Boolean, so the probability of, say, $\neg A$ in any row of its table is $1 - P(A)$.

a) Assuming that the nodes are introduced in the following order *Wet Grass*, *Sprinkler*, *Rain* and *Cloudy* construct a corresponding belief network. Show which probabilities need to be specified.

b) Compute probabilities $\mathbf{P}(W)$ and $\mathbf{P}(S|W)$.

Problem #24: Solution a)

Order: W, S, R, C. Probabilities $\mathbf{P}(\mathbf{W})$, $\mathbf{P}(\mathbf{S}|\mathbf{W})$, $\mathbf{P}(\mathbf{R}|\mathbf{S},\mathbf{W})$, $\mathbf{P}(\mathbf{C}|\mathbf{R},\mathbf{S},\mathbf{W})$

- $P(W)$

Problem #24: Solution a)

Order: W, S, R, C. Probabilities $\mathbf{P}(\mathbf{W})$, $\mathbf{P}(\mathbf{S}|\mathbf{W})$, $\mathbf{P}(\mathbf{R}|\mathbf{S},\mathbf{W})$, $\mathbf{P}(\mathbf{C}|\mathbf{R},\mathbf{S},\mathbf{W})$

- $P(W)$
- $P(S|W) = P(S)$?

Problem #24: Solution a)

Order: W, S, R, C. Probabilities $\mathbf{P}(\mathbf{W})$, $\mathbf{P}(\mathbf{S}|\mathbf{W})$, $\mathbf{P}(\mathbf{R}|\mathbf{S},\mathbf{W})$, $\mathbf{P}(\mathbf{C}|\mathbf{R},\mathbf{S},\mathbf{W})$

- $P(W)$
- $P(S|W) = P(S)$?
- $P(S|W) = P(S)$? No!

Problem #24: Solution a)

Order: W, S, R, C. Probabilities $\mathbf{P}(\mathbf{W})$, $\mathbf{P}(\mathbf{S}|\mathbf{W})$, $\mathbf{P}(\mathbf{R}|\mathbf{S},\mathbf{W})$, $\mathbf{P}(\mathbf{C}|\mathbf{R},\mathbf{S},\mathbf{W})$

- $P(W)$
- $P(S|W) = P(S)$?
- $P(S|W) = P(S)$? No!
- $P(R|S,W) = P(R|S)$?

Problem #24: Solution a)

Order: W, S, R, C. Probabilities $\mathbf{P(W)}$, $\mathbf{P(S|W)}$, $\mathbf{P(R|S,W)}$, $\mathbf{P(C|R,S,W)}$

- $P(W)$
- $P(S|W) = P(S)$?
- $P(S|W) = P(S)$? No!
- $P(R|S,W) = P(R|S)$?
- $P(R|S,W) = P(R|S)$? No!

Problem #24: Solution a)

Order: W, S, R, C. Probabilities $\mathbf{P(W)}$, $\mathbf{P(S|W)}$, $\mathbf{P(R|S,W)}$, $\mathbf{P(C|R,S,W)}$

- $P(W)$
- $P(S|W) = P(S)$?
- $P(S|W) = P(S)$? No!
- $P(R|S,W) = P(R|S)$?
- $P(R|S,W) = P(R|S)$? No!
- $P(C|R,S,W) = P(C|R,S)$? $P(C|R,S,W) = P(C|R)$?
- $P(C|R,S,W) = P(C|R,S)$? Yes! $P(C|R,S,W) = P(C|R)$? No!

Problem #24: Solution b)

$$\mathbf{P(W)} = \sum_c \sum_s \sum_r \mathbf{P(c)P(s|c)P(r|c)P(W|s,r)}$$

$$\mathbf{P(W)} = \sum_c \mathbf{P(c)} \sum_s \mathbf{P(s|c)} \sum_r \mathbf{P(r|c)P(W|s,r)}$$

$$\begin{aligned}
P(w) &= P(c)\{P(s|c)[P(r|c)P(w|s,r) + P(\neg r|c)P(w|s\neg r)] + \\
&\quad P(\neg s|c)[P(r|c)P(w|\neg s,r) + P(\neg r|c)P(w|\neg s\neg r)]\} + \\
&\quad P(\neg c)\{P(s|\neg c)[P(r|\neg c)P(w|s,r) + P(\neg r|\neg c)P(w|s\neg r)] + \\
&\quad P(\neg s|\neg c)[P(r|\neg c)P(w|\neg s,r) + P(\neg r|\neg c)P(w|\neg s\neg r)]\} \\
P(w) &= 0.5\{0.1[0.8*0.99 + 0.2*0.9] + 0.9[0.8*0.9 + 0.2*0.01]\} + \\
&\quad 0.5\{0.5[0.2*0.99 + 0.8*0.9] + 0.5[0.2*0.9 + 0.8*0.01]\} \\
P(w) &= 0.5[0.1*1.512 + 0.9*0.722] + 0.5[0.5*0.918 + 0.5*0.188] \\
&\quad 0.5*0.801 + 0.5*0.553 = 0.677 \\
P(\neg w) &= 0.323
\end{aligned}$$

Problem #24: Solution b)

$$\mathbf{P(S|W)} = \mathbf{P(S,W)/P(W)} = \alpha \sum_c \sum_r \mathbf{P(c)P(S|c)P(r|c)P(W|S,r)}$$

$$\mathbf{P(S|W)} \quad = \quad \alpha \sum_c \mathbf{P(c)P(S|c)} \sum_r \mathbf{P(r|c)P(W|S,r)}$$

$$\begin{aligned}
P(s|w) &= \alpha\{P(c)P(s|c)[(P(r|c)P(w|s,r) + P(\neg r|c)P(w|s,\neg r)] + \\
&\quad P(\neg c)P(s|\neg c)[(P(r|\neg c)P(w|s,r) + P(\neg r|\neg c)P(w|s,\neg r)]\} \\
&= \alpha[0.05(0.8 * 0.99 + 0.18) + 0.25(0.2 * 0.99 + 0.72)] = 0.2781 \\
P(\neg s|w) &= \alpha[0.45(0.8 * 0.9 + 0.2 * 0.01) + 0.25(0.2 * 0.9 + 0.8 * 0.01)] \\
P(s|\neg w) &= ? \\
P(\neg s|\neg w) &= ?
\end{aligned}$$

$$P(s|w) + P(\neg s|w) = 1, \quad P(s|\neg w) + P(\neg s|\neg w) = 1,$$