Time varying image analysis

- Motion detection
- Computing image motion
- Motion estimation
- Egomotion and structure from motion
- Motion classification

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The problems

- Visual surveillance
  - stationary camera watches a workspace – find moving objects and alert an operator
  - moving camera navigates a workspace – find moving objects and alert an operator
- Image coding
  - use image motion to perform more efficient coding of images
- Navigation
  - camera moves through the world - estimate its trajectory
    - » use this to remove unwanted jitter from image sequence - image stabilization and mosaicking
    - » use this to control the movement of a robot through the world

Surveillance example: Adding an object to the scene

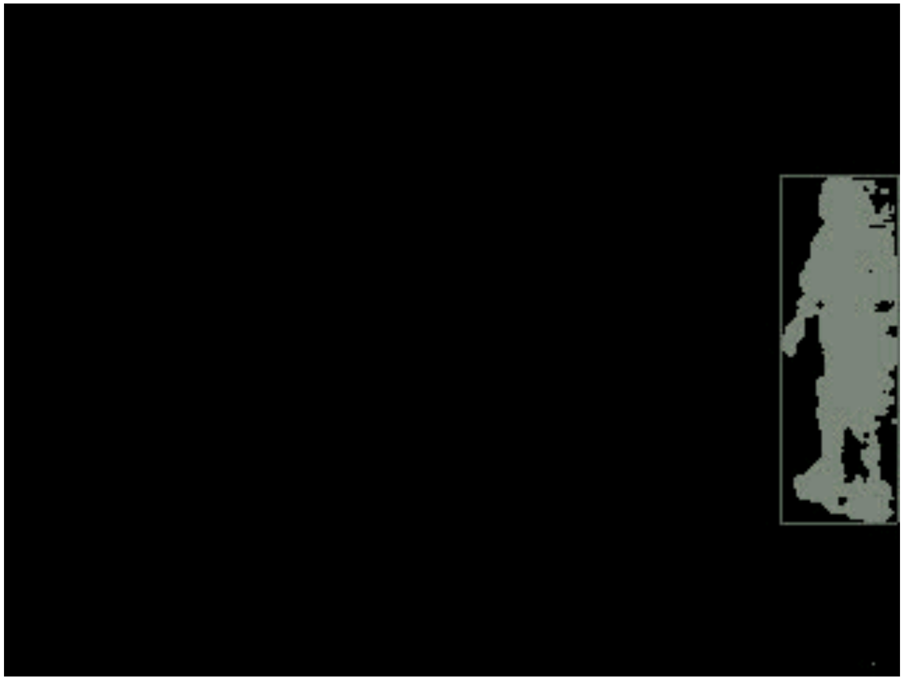

Image Sequence Smoothing

Motion detection

- Frame differencing
  - subtract, on a pixel by pixel basis, consecutive frames in a motion sequence
  - high differences indicate change between the frames due to either motion or changes in illumination
- Problems
  - noise in images can give high differences where there is no motion
    - » compare neighborhoods rather than points
  - as objects move, their homogeneous interiors don't result in changing image intensities over short time periods
    - » motion detected only at boundaries
    - » requires subsequent grouping of moving pixels into objects

Image Differencing

Image Differencing: Results

1 frame difference

5 frame difference

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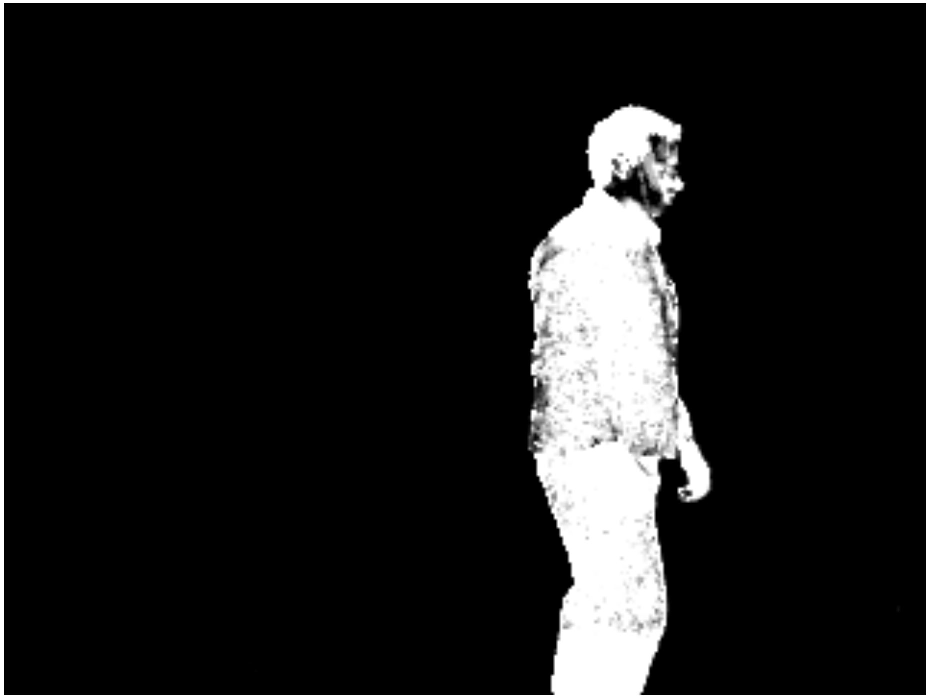
Motion detection

- Background subtraction
  - create an image of the stationary background by averaging a long sequence
    - for any pixel, most measurements will be from the background
    - computing the median measurements, for example, at each pixel, will with high probability assign that pixel the true background intensity - fixed threshold on diffrencing used to find “foreground” pixels
    - can also compute a distribution of background pixels by fitting a mixture of Gaussians to set of intensities and assuming large population is the background - adaptive thresholding to find foreground pixels
  - difference a frame from the known background frame
    - even for interior points of homogeneous objects, likely to detect a difference
    - this will also detect objects that are stationary but different from the background
    - typical algorithm used in surveillance systems
- Motion detection algorithms such as these only work if the camera is stationary and objects are moving against a fixed background

Background Subtraction: Results

Confidence corresponds to gray-level value.
High confidence – bright pixels, low confidence – dark pixels.



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Background modeling: color-based

- At each pixel model colors *(r,g,b)* or gray-level values *g.* The following equations are used to recursively estimate the mean and the variance at each pixel:

$$\mu_{t+1} = \alpha\mu_t + (1-\alpha)z_{t+1}$$

$$\sigma_{t+1}^2 = \alpha(\sigma_t^2 + (\mu_{t+1} - \mu_t)^2) + (1-\alpha)(z_{t+1} - \mu_{t+1})^2$$

where z_{t+1} is the current measurement. The mean μ and the variance σ can both be time varying. The constant α is set empirically to control the rate of adaptation ($0<\alpha<1$).

- A pixel is marked as foreground if given red value *r* (or for any other measurement, say *g* or *b*) we have

$$|r - \mu_t| > 3\max(\sigma_r, \sigma_{rcam})$$

Background model

- σ_{rcam} is the variance of the camera noise, can be estimated from image differences of any two frames.
- If we compute differences for all channels, we can set a pixel as foreground if any of the differences is above the preset threshold.
- Noise can be cleaned using connected component analysis and ignoring small components.
- Similarly we can model the chromaticity values rc, gc and use them for background subtraction:

  $r_c = r/(r+g+b)$, $g_c = g/(r+g+b)$

Background model: edge-based

- Model edges in the image. This can be done two different ways:
  - Compute models for edges in a the average background image
  - Subtract the background (model) image and the new frame; compute edges in the subtraction image; mark all edges that are above a threshold.
    - » The threshold can be learned from examples
    - » The edges can be combined (color edges) or computed separately for all three color channels

Foreground model

- Use either color histograms (4-bit per color), texture features, edge histograms to model the foreground
- Matching the foreground objects between frames: **tracking**
- Can compare foreground regions directly: shift and subtract. SSD or correlation: *M, N* are two foreground regions.

$$SSD = \sum_{i=1}^{n}\sum_{j=1}^{n}[M(i,j) - N(i,j)]^2$$

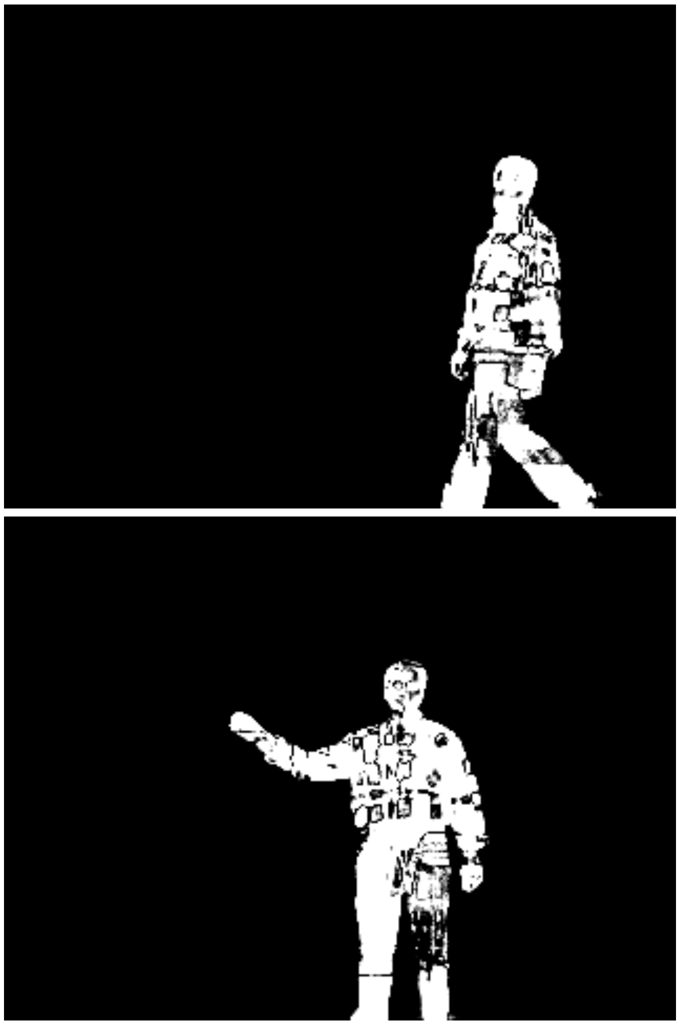
$$C = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} M(i,j)N(i,j)}{[\sum_{i=1}^{n}\sum_{j=1}^{n} M(i,j)^2 \sum_{i=1}^{n}\sum_{j=1}^{n} N(i,j)^2]^{1/2}}$$

A 300-Frame Sequence with a "Busy" Background

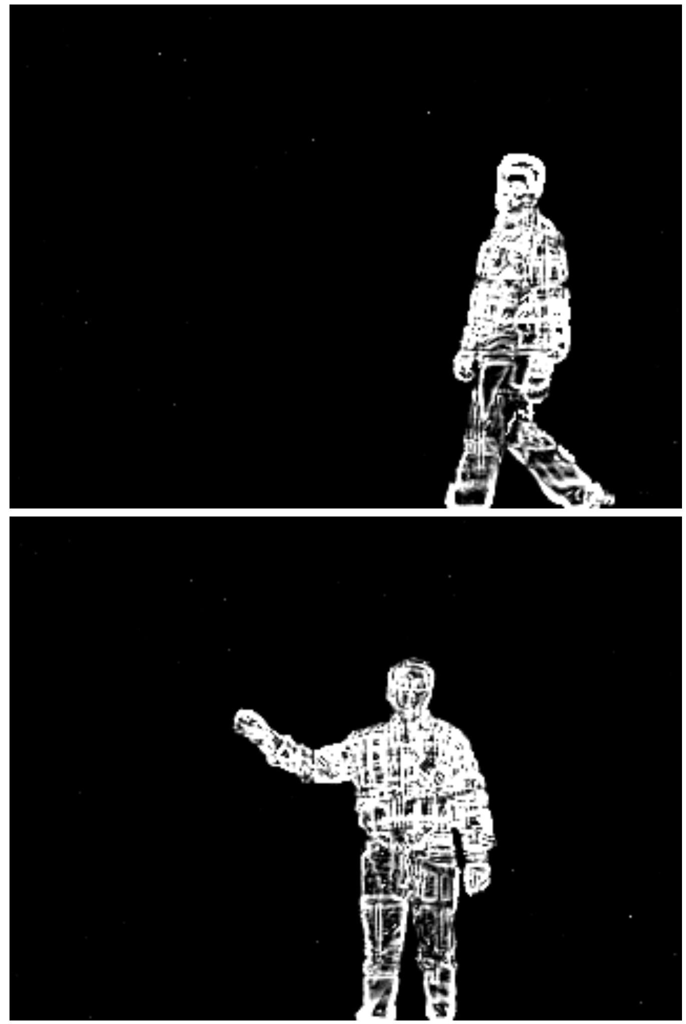
click to start movie

Some Intermediate Maps Used in the Method

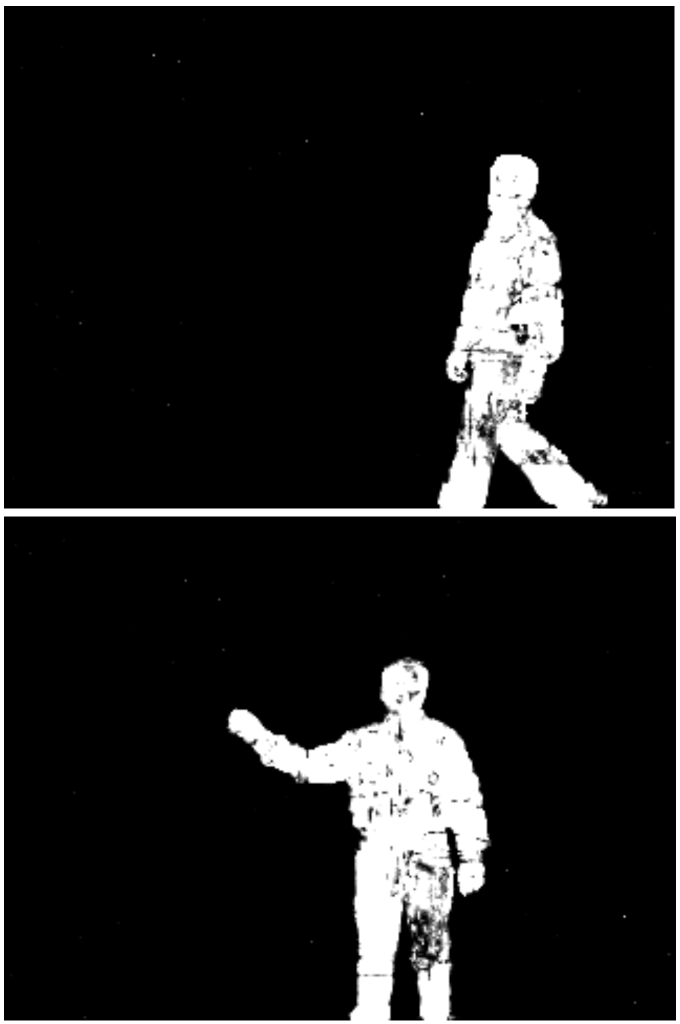
Color-based moving object detection



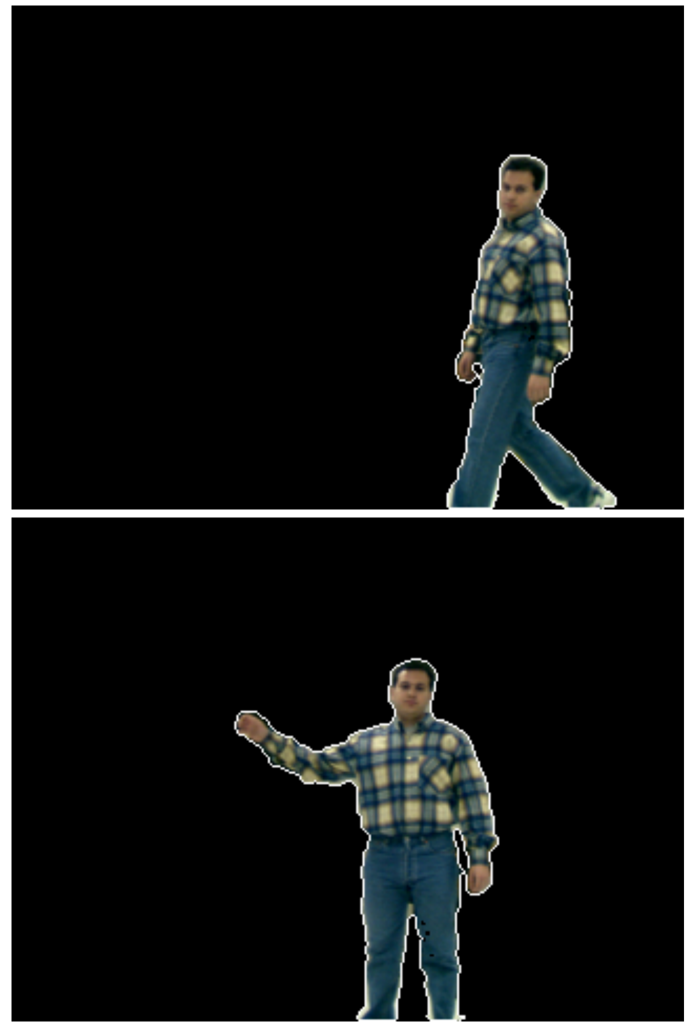
Edge-based moving object detection



Combined color and edge based detection



Detected human



Results for the sequence

click to start movie

Using histograms for background modeling

- Use histograms of small regions to model the background:
  - Color histograms computed for small regions of the “background” image and the current (new) image (reduced color/ 12 bit bit representation)
  - Color edge histograms computed for small regions of the “background” image and the current image (36 bin quantization)

Color Histograms

Reduced color representation =

$C = (R/16) * 256 + (G/16)*16 + (B/16)$

(This results in a 24 -> 12 bit color depth reduction)

This results in a 4096 bin histogram

- lowest 4 bits are less useful
- requires less storage
- faster implementation - easier to compare histograms

Color Edge Histograms

- Use edge detector to compute edges in each color band $(r_x, r_y, g_x, g_y, b_x, b_y)$
- Combine the three color bands into the structure matrix, S, to compute the color edge response
- The edge strength is computed as the larger of the two eigenvalues of S, and the orientation is given by the corresponding eigenvector
- Histogram bin index is determined using edge orientation (36 bins total), and the bin count is incremented using the edge magnitude

Histogram Matching

- Histogram Intersection

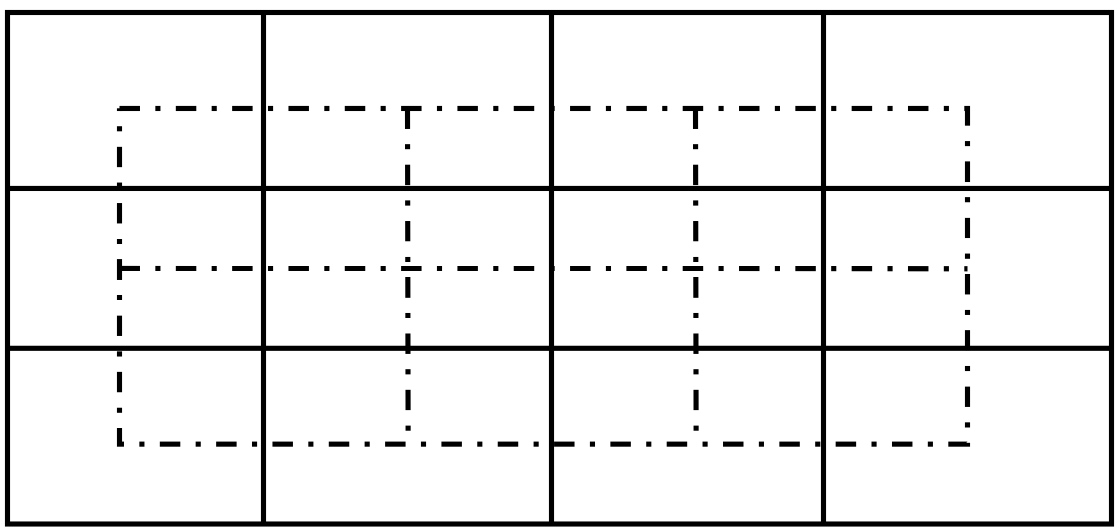
$$I(h_c, h_b) = \frac{\sum_i \min\{h_c(i), h_b(i)\}}{\sum_i \max\{h_c(i), h_b(i)\}}$$

- Chi Squared Formula

$$\chi^2(h_c, h_b) = \sum_i 2 \frac{(h_c(i) - h_b(i))^2}{h_c(i) + h_b(i)}$$

Overall control

- Divide each frame into *40x40* pixel blocks
- To make sure that we do not miss objects on grid block boundaries we tile the frame by overlaying two grids, one of which is shifted by *20* pixels in *x* and *y* directions

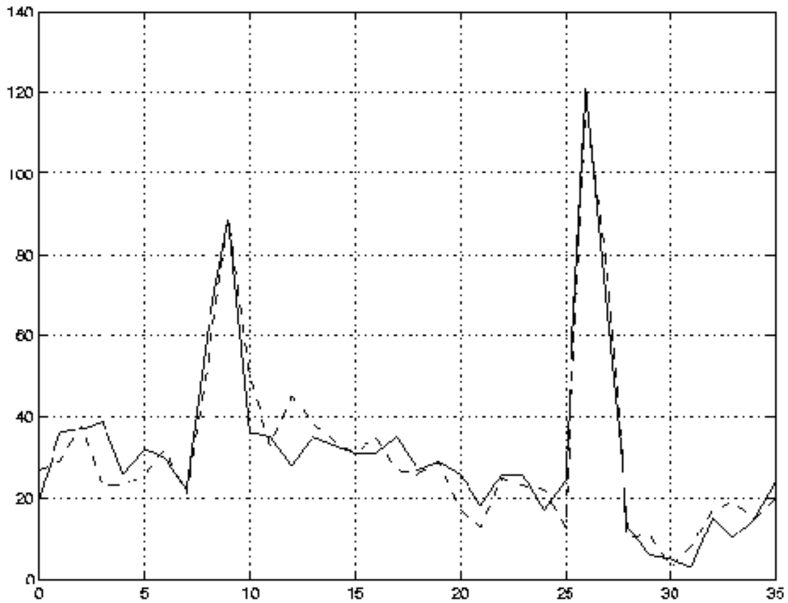


Criteria for block activation

- On a block by block basis, similarity measures between background and foreground histograms are computed
- For histogram intersection: If the similarity is below a threshold, T, then the block contains a foreground object and is activated for display
- For chi squared: If the X^2 measure is greater than a threshold, T, then the block contains a foreground object and is activated for display

Examples of edge histograms

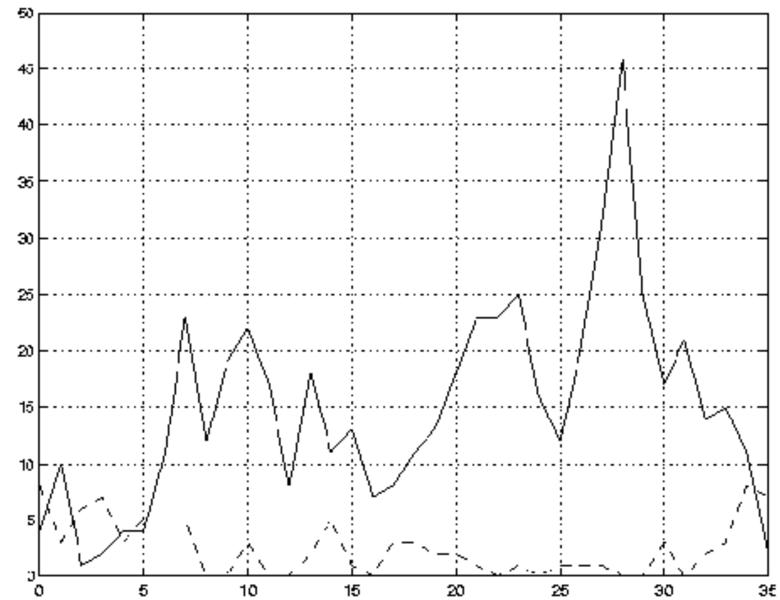
similar histograms



Similarity (inters.) = 92%

$X^2 = 61$

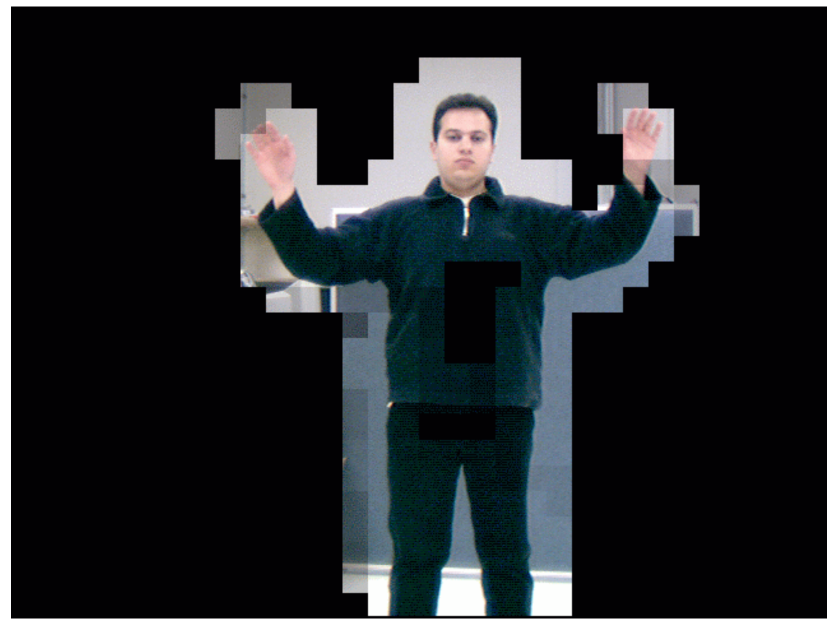
different histograms



Similarity (inters.) = 22%

$X^2 = 828$

Using edge histograms for detection



Moving person in a cluttered scene

Color histogram based detection

Edge histogram-based detection

Surveillance: dropping an object

Surveillance: removing an object

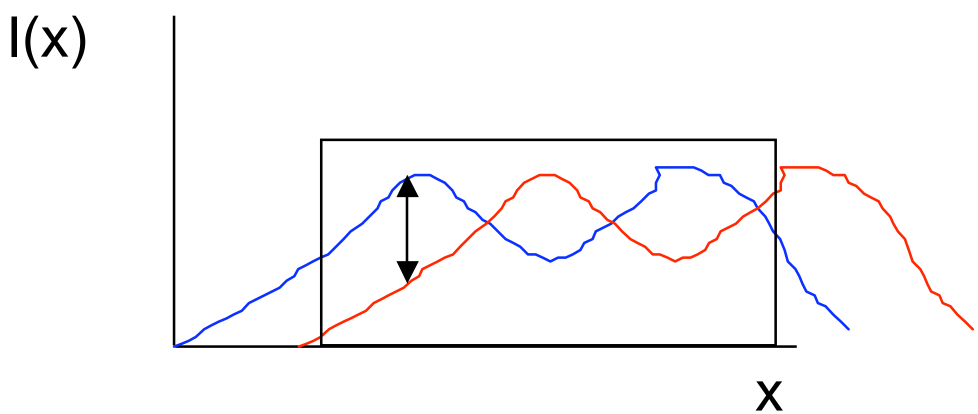
Surveillance: Interacting people

Motion estimation - optic flow

- Optic flow is the 2-D velocity field induced in a dynamic scene due to the projection of moving objects onto the image plane
- Three prevalent approaches to computing optic flow:
  - token matching or correlation
    - » extract features from each frame (grey level windows, edge detection)
    - » match them from frame to frame
  - gradient techniques
    - » relate optic flow to spatial and temporal image derivatives
  - velocity sensitive filters
    - » frequency domain models of motion estimation

A 1-d gradient technique

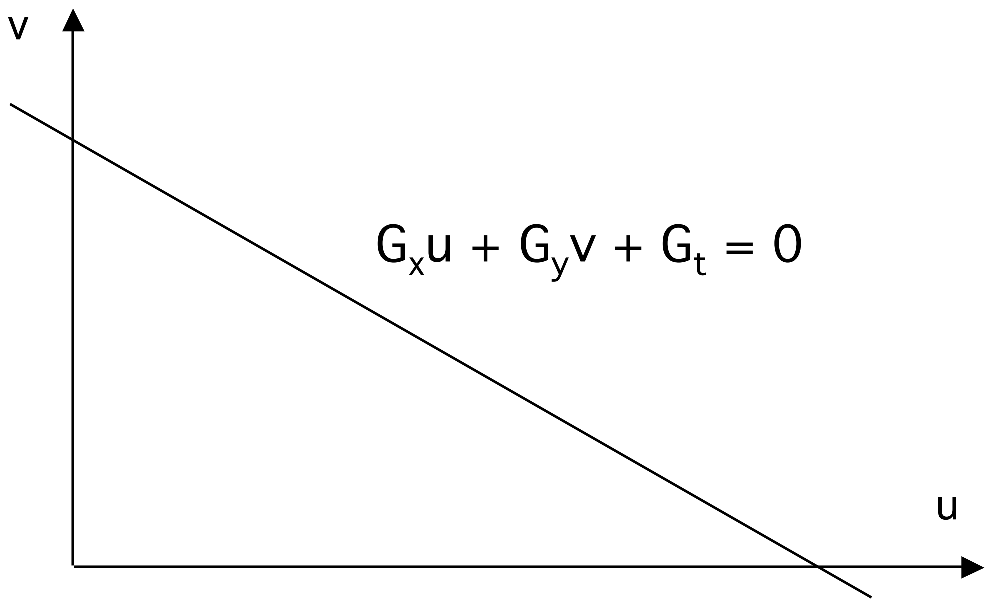
- Suppose we have a 1-D image that changes over time due to a translation of the image
- Suppose we also assume that the image function is, at least over small neighborhoods, well approximated by a linear function.
  - completely characterized by its value and slope
- Can we estimate the motion of the image by comparing its spatial derivative at a point to its temporal derivative?
  - example: spatial derivative is 10 units/pixel and temporal derivative is 20 units/frame
  - then motion is (20 units/frame) / (10 units/pixel) = 2 pixels/frame



Gradient techniques

- Assume $I(x,y,t)$ is a continuous and differentiable function of space and time
- Suppose the brightness pattern is locally displaced by a distance dx, dy over time period dt.
  - this means that as the time varying image evolves, the image brightnesses of points don't change (except for digital sampling effects) as they move in the image
  - $I(x,y,t) = I(x + dx, y + dy, t + dt)$
- We expand I in a Taylor series about (x,y,t) to obtain
  - $I(x + dx, y + dy, t + dt) = I(x,y,t) + dx\, \partial I/\partial x + dy\, \partial I/\partial y + dt\, \partial I/\partial t$ + (higher order terms)
- $dI/dt = [I(x+dx, y+dy, t+dt) - I(x,y,t)]/dt = dx/dt\, \partial I/\partial x + dy/dt\, \partial I/\partial y + \partial I/\partial t = 0$
  - valid only if temporal change is due entirely to motion
- Can rewrite this as $dI/dt = G_x u + G_y v + G_t = 0$. The G's are derivatives measured from the image sequence, and u and v are the unknown optic flow components in the x and y directions, respectively

Motion constraint line

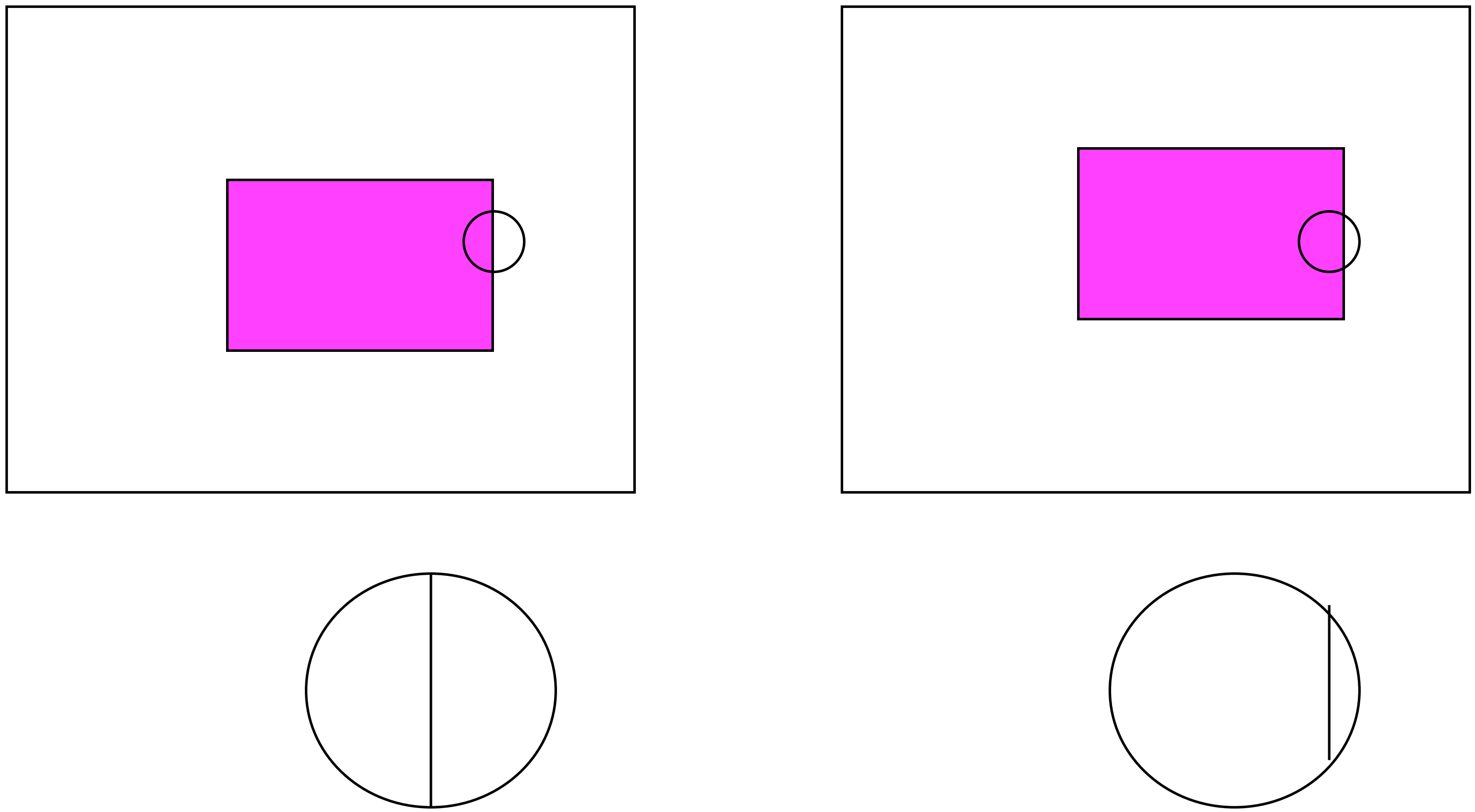


- So, the spatial and temporal derivatives at a point in the image only provide a linear constraint on the optic flow

Motion constraint line

- If G_x and G_y are small, then motion information cannot be accurately determined
- If $G_x = 0$, then $-G_t = G_y v$, so that v is determined, but u is unknown
- If H and L denote the gradient and level directions at a pixel then
  - $G_H = \|\nabla G\|$
  - L is perpendicular to H
  - $G_L = 0$
- Then $G_t = -G_H dh/dt$, where $n_f = dh/dt$ is the displacement in the gradient direction ($\mathbf{h} = \nabla G / \|\nabla G\|$)
  - dh/dt can be recovered by measuring G_t and G_H. It is called **normal flow**
  - but dl/dt cannot be recovered, since $G_L = 0$
  - this is called the aperture problem

Aperture problem



Motion Flow Example: Images

Motion Flow Example: Normal Flow

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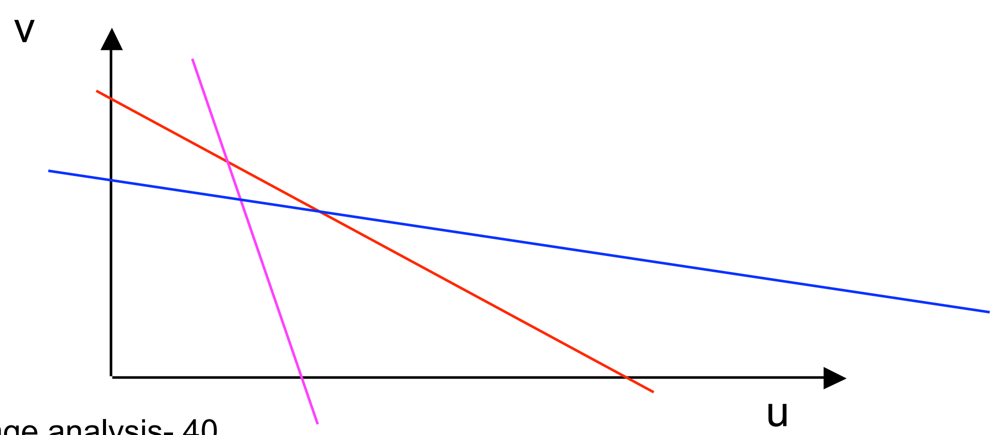
Recovering u and v

- Compute for normal flow in a small image neighborhood
  - $n_f = -G_t/||\nabla G||$
- Solve system of linear equations corresponding to motion constraints in the small neighborhood
  - assume u and v will not vary in that small neighborhood
  - requires that neighborhoods have edges with different orientations, since slope of motion constraint line is determined by image gradient



Recovering u and v

- If the constraint lines in a neighborhood are nearly parallel (i.e., the gradient directions are all similar), then the location of the best fitting (u,v) will be very sensitive to errors in estimating gradient directions.
- More generally, one could fit a parametric form to local neighborhoods of constraint lines, finding parameters that bring constraint lines “nearest” to the estimated motion assigned to each pixel.
  - for example, if we assume that the surface we are viewing in any small image neighborhood is well approximated by a plane, then the optical flow will be a quadratic function of image position in that image neighborhood

A regularization approach

- Many vision problems such as stereo reconstruction of visible surfaces and recovery of optic flow are instances of **ill posed** problems.
- A problem is well posed when its solution:
  - exists
  - is unique, and
  - depends continuously on its initial data
- Any problem that is not well posed is said to be ill posed
- The optic flow problem is to recover both degrees of freedom of motion at each image pixel, given the spatial and temporal derivatives of the image sequence
  - but any solution chosen at each pixel that locally satisfies the motion constraint equation can be used to construct an optic flow field consistent with the derivatives measured
  - therefore, the solution is not unique - how to choose one?

Parametric models

$$\begin{bmatrix} \delta x \\ \delta y \end{bmatrix} = \begin{bmatrix} w1 \\ w4 \end{bmatrix} + \begin{bmatrix} w2 & w3 \\ w5 & w6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} x^2 & xy \\ xy & y^2 \end{bmatrix} \begin{bmatrix} w7 \\ w8 \end{bmatrix}$$

- (δx,δy) are flow components – *optical* flow
- This is a quadratic 8 parameter model (moving plane?)
- We can assume simpler models:
  - Constant flow: *w2=w3=w5=w6=w7=w8=0*
  - Rotation, translation, and shear: *w2=w6=w7=s8=0*
  - Divergence, scaling, and translation: *w3=w5=w7=s8=0*
  - Linear affine: *w7=s8=0*

Estimating parametric flow from normal flow

- At each point we measure the normal flow n_f, and the gradient direction (n_x, n_y). We can write for linear affine (6 param.) model:
  - $n_f \approx \delta x n_x + \delta y n_y = w2 n_x x + w3 n_x y + w1 n_x + w5 n_y x + w6 n_x y + w4 n_y = \mathbf{w} \cdot \mathbf{p}$
  - $\mathbf{p} = (n_x \; n_x x \;\; n_x y \; n_y \; n_y x \;\; n_y y)^T$, and
  - $\mathbf{w} = (w1 \; w2 \; w3 \; ww4 \; w5 \; w6)^T$ is the vector of affine parameters
- Computer normal flow at many points and have an overdetermined system of linear equations:
  - P**w** $\approx$ **b**, where P is a matrix whose elements are $\mathbf{p}_i$ computed at points (x_i, y_i).

Estimating parametric flow (cont.)

- **b** is the vector of normal flow values $n_{f,i}$ measured at points (x_i, y_i)

- Solve P**w** $\approx$ **b** using Linear Least Squares method
  - **w**$=(P^TP)^{-1}P^T$**b**
  - in *Matlab* write **w** = P \ **b**;

A regularization approach

- Solution - add a priori knowledge that can choose between the solutions
- Formally, suppose we have an ill posed problem of determining z from data y expressed as
  - $Az = y$, where A is a linear operator (e.g., projection operation in image formation)
- We must choose a quadratic norm $\|\ \|$ and a so-called stabilizing functional $\|Pz\|$ and then find the z that minimizes:
  - $\|Az-y\|^2 + \lambda \|Pz\|^2$
  - λ controls the compromise between the degree of regularization and the closeness of the solution to the input data (the first term).
- T. Poggio, V. Torre and C. Koch, Computational vision and regularization theory, *Nature*, **317**, 1984.

A regularization approach

- For optic flow:
  - the first term is $[dx/dt\ \partial I/\partial x + dy/dt\ \partial I/\partial y + \partial I/\partial t]^2 = [dI/dt]^2$
    - this should, ideally, be zero according to the theory
  - the second term enforces a smoothness constraint on the optic flow field:

    $\varepsilon = (\partial u/\partial x)^2 + (\partial v/\partial x)^2 + (\partial u/\partial y)^2 + (\partial v/\partial y)^2$
  - The regularization problem is then to find a flow field that minimizes

    $[dI/dt]^2 + \lambda\ \varepsilon$
  - This minimization can be done over the entire image using various iterative techniques

Token and correlation methods

- Gradient based methods only work when the motion is “small” so that the derivatives can be reliably computed
  - although for “large” motions, one can employ multiresolution methods
- Tracking algorithms can compute motion when the motion is “large”
  - correlation
  - feature tracking
- Correlation
  - choose a kxk window surrounding a pixel, p, in frame i.
  - compare this window against windows in similar positions in frame i+1
  - The window of best match determines the displacement of p from frame i to frame i+1

Correlation

- Correlation
  - sum of squared gray level differences
  - sum of absolute intensity differences
  - “robust” versions of these sensitive to outliers
- Drawbacks of correlation
  - matching in the presence of rotation is computationally expensive since all orientations of the window must be matched in frame i+1
  - if motion is not constant in the kxk window then the window will be distorted by the motion, so simple correlation methods will fail
    - » this suggests using smaller windows, within which motion will not vary significantly
    - » but smaller windows have less **specificity**, leading to matches more sensitive to noise

Tracking

- Apply a feature detector, such as an edge detector, to each frame of the sequence
  - want features to be distinctive
  - example: patterns of edges or gray levels that are dissimilar to their surrounds (image has a locally small autocorrelation)
- Match these features from frame to frame
  - might assume that nearby features move similarly to help disambiguate matches (but this is not true at motion boundaries)
  - integrate the matching with assumptions about scene structure - e.g., features are all on a plane moving rigidly

Motion estimation – token matching

- Extract features from each frame (grey level windows, edge detection)

$$S = \begin{pmatrix} \Sigma E_x^2 & \Sigma E_x E_y \\ \Sigma E_x E_y & \Sigma E_y^2 \end{pmatrix}$$

E_x and E_y are x and y components of image gradient

  - $\lambda_1 \geq \lambda_2 \geq 0$ are eigenvalues of M
  - If $\lambda_1 = \lambda_2 = 0$, mean squared magnitude of the gradient is 0 (flat, unchanging area in the image)
  - If $\lambda_1 > \lambda_2 = 0$, values do not change in the direction of the corresponding eigenvector (edge)
  - If $\lambda_1 > 0$ and $\lambda_2 > 0$, gray values change in multiple directions (corner)
    - » $\lambda_2 > \tau$, where τ is some threshold

Motion estimation – token matching

- Match them from frame to frame. Detect tokens in the next frame using lower threshold. Why?
  - Minimize SSD (sum of squared differences) over a neighborhood in the new image. M is a small area around the token (5x5,7x7,11x11)

$$SSD = \sum_{i=1}^{n} \sum_{j=1}^{n} [M(i,j) - N(i,j)]^2$$

  - Maximize the correlation over a neighborhood in the new image

$$C = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} M(i,j) N(i,j)}{[\sum_{i=1}^{n} \sum_{j=1}^{n} M(i,j)^2 \sum_{i=1}^{n} \sum_{j=1}^{n} N(i,j)^2]^{1/2}}$$

Multiresolution methods

- Consider using edges as features for a tracking algorithm for motion estimation. What should the scale of the edge detector be?
  - small scale
    - » many edges are detected
    - » easily confused with one another
    - » computationally costly matching problem
  - coarse scale
    - » relatively few edges identified
    - » localized only poorly, so motion estimates have high errors
    - » simple matching problem

Multiresolution methods

- Multiresolution - process the image over a range of scales, using the results at coarser scales to guide the analysis at finer scales
  - detect edges at a coarse scale
  - estimate motion by tracking
  - use these estimates as initial conditions for matching edges at next finest scale
- These are also called **focusing** methods or **scale space** methods
  - can also apply to gradient based motion estimators

3-D motion and optical flow

- Assume a camera moving in a static environment
- A rigid body motion of the camera can be expressed as a translation and a rotation about an axis through the origin.
- Let
  - **t** be the translational component of the camera motion
  - ω be the angular velocity
  - **r** be the column vector $[X\ Y\ Z]^T$
- Then the velocity of r with respect to the XYZ coordinate system is

  $$\mathbf{V} = -\mathbf{t} + \omega \times \mathbf{r}$$
- Let the components of
  - **t** $= [U\ V\ W]^T$
  - **w** $= [A\ B\ C]^T$

3-D Motion and Optic Flow

- Rewrite in component form:

  $X' = -U - BZ + CY$

  $Y' = -V - CX + AZ$

  $Z' = -W - AY + BX$

  where the differentiation is with respect to time
- The optic flow at a point (x,y) is (u,v) where

  $u = x', x = fX/Z$

  $v = y', y = fY/Z$
- Differentiating x and y with respect to time, we obtain

  $u = X'/Z - XZ'/Z^2 = (-U/Z - B + Cy) - x(-W/Z - Ay + Bx)$

  $v = Y'/Z - YZ'/Z^2 = (-V/Z - Cx + A) - y(-W/Z - Ay + Bx)$

3-D Motion and Optic Flow

- These can be written in the form

  $u = u_t + u_r$

  $v = v_t + v_r$

- (u_t, v_t) denotes the translational component of the optic flow
- (u_r, v_r) denotes the rotational component of the optic flow

  $u_t = [-U + xW]/Z$

  $v_t = [-V + yW]/Z$

  $u_r = Axy - B(x^2 + 1) + Cy$

  $v_r = A(y^2 + 1) - Bxy - Cx$

- Notice that the rotational part is independent of Z - it just depends on the image location of a point
- So, all information about the structure of the scene is revealed through the translational component

Special case of a plane in motion

- Suppose we are looking at a plane while the camera moves
  - $Z = Z_0 + pX + qY$
- Then for any point on this plane
  - $Z - pX - qY = Z_0$
  - $1 - p(X/Z) - p(Y/Z) = Z_0/Z$
  - $1/Z = [1 - pX/Z - qY/Z]/Z_0 = [1 - px - qy]/Z_0$
- So, we can rewrite the translational components of motion for a plane as:

  $u_t = [-U + xW][1 - px - qy]/Z_0 = [-U/Z_0 + xW/Z_0]\ [1 - px - qy]$

  $v_t = [-V + yW][1 - px - qy]/Z_0 = [-V/Z_0 + xW/Z_0]\ [1 - px - qy]$
- These are quadratic equations in x and y
- So, if we can compute the translational component of the optic flow at “enough” points from a planar surface, then we can recover the translational motion (with unknown scaling) and the orientation of the plane being viewed.

Pure translation

- When camera motion is only translation, then we have

  $u_t = [-U + xW]/Z$

  $v_t = [-V + yW]/Z$
- Consider the special point $(u,v) = (U/W, V/W)$.
  - This is the “image” of the velocity vector onto the image plane
  - The motion at this point must be 0 since the surface point along this ray stays on the ray as the camera moves (also our equations evaluate to 0 at $(U/W, V/W)$)
- Consider the line connecting any other (x,y) to $(x + u_t, y + v_t)$
  - The slope of this line is $v_t/u_t = [x-u]/[y-v]$
  - So, the line must pass through (u, v)
- All of the optic flow vectors are concurrent, and pass through the special point (u,v) which is called the **focus of expansion (contraction)**

Pure translation

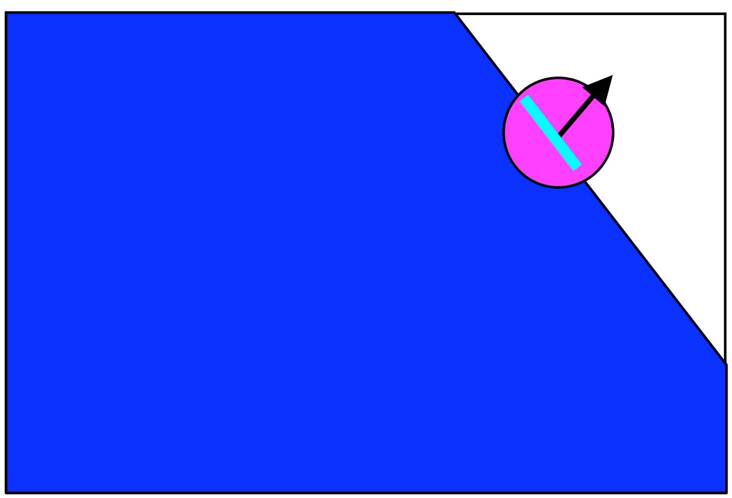
- Another way to look at it
  - Let $\Delta t = 1$, so that the image center at time t moves from (0,0,0) to (U,V,W) at time t+1
  - Think of the two images as a stereo pair
  - The location of the projection of (U,V,W), the lens center at time t+1 (the "right" image), in the image at time t (the left image) is at location (U/W, V/W) = (u,v)
  - All conjugate lines at time t must pass through this point
  - So, given a point (x,y) at time t, the location of its corresponding point at time t+1 in the **original** coordinate system must line on the line connecting (x,y) to (u,v)
- So, if we know the optic flow at two points in the case of pure translation, we can find the focus of expansion
  - in practice want more than two points

Pure translation

- Can we recover the third component of motion, W?
- No, because the same optic flow field can be generated by two similar surfaces undergoing similar motions (U,V and W always occur in ratio with Z).

Normal flows and camera motion estimation

- If we can compute optic flow at a point, then the foe is constrained to lie on the extension of the optic flow vector
- But the aperture problem makes it difficult to compute optic flow without making assumptions of smoothness or surface order
- Normal flow (the component of flow in the gradient direction) can be locally computed at a pixel without such assumptions
- Can we recover camera motion from normal flow?



Identifying the FOE from normal flow

- Assume that the foe is within the field of view of the camera
- For each point, p, in the image

  For each normal flow vector, **n,**

    If p lies in the "correct" halfplane of **n,** then score a vote for p

  The FOE is the centroid of the connected component of highest scoring points (might be a single pixel, but ordinarily will not be).
- Alternative code - maintain an array of counters in register with the image

  For each normal flow vector,**n,**

    Increment the counters corresponding to all pixels in the "correct" halfplane of **n**

  Search the array of counters for the connected component of highest vote count
- For an image containing N normal flow vectors and mxm pixels, both algorithms are (m^2N), but (2) is more efficient

Identifying the FOE from normal flow

- What if the FOE is outside the field of view of the camera?
- The image plane is a bad place to represent the FOE to begin with
  - FOE indicates the direction of translational motion
  - Pixels in a perspective projection image do not correspond to equal angular samples of directions
    - » in the periphery, a pixel corresponds to a wide range of directions
  - Solution - represent the array of accumulators as a sphere, with an equiangular sampling of the surface of the sphere
    - » Each normal vector will then cast votes for all samples in a hemisphere
    - » Simple mathematical relationship between the spherical coordinate system of the array of counters, and the image coordinate system